Journal of Astronomical History and Heritage 5(1):35-40 2002

The contribution of José Luis Sérsic to celestial mechanics

Adrián Brunini and Octavio Ismael Miloni

Facultad de Ciencias Astronómicas y Geofísicas,
Universidad Nacional de La Plata & IALP-CONICET
Paseo del Bosque, La Plata (1900), Argentina
E-mail: abrunini@fcaglp.unlp.edu.ar

Abstract:
In this paper, we shall review some aspects of José Luis Sersic's Ph.D. thesis: "Application of a certain type of canonical transformations to Celestial Mechanics", presented in 1956 to obtain the degree of Doctor in Philosophy in Astronomy of the National University of La Plata (La Plata, Argentina). We have found that Sérsic's work shares deep similarity to the method now known as Hori's Method although this was published ten years later. We shall discuss possible connections between both works, and the circumstances whereby Sérsic's work remained hidden to the international astronomical community.

Keywords: *celestial mechanics, J L Sérsic, Hori's Method, La Plata National University*

1 INTRODUCTION

José Luis Sérsic (Figure 1) was one of Argentina's most distinguished astronomers. Sérsic (1933-1993) received his Ph.D. in Astronomy from the Escuela Superior de Ciencias Astronómicas (High School of Astronomy) at the La Plata National University (Argentina), now the Facultad de Ciencias Astronómicas y Geofísicas of this University. His thesis advisor was Reynaldo Cesco, who at that time was Professor of Celestial Mechanics at this institution.

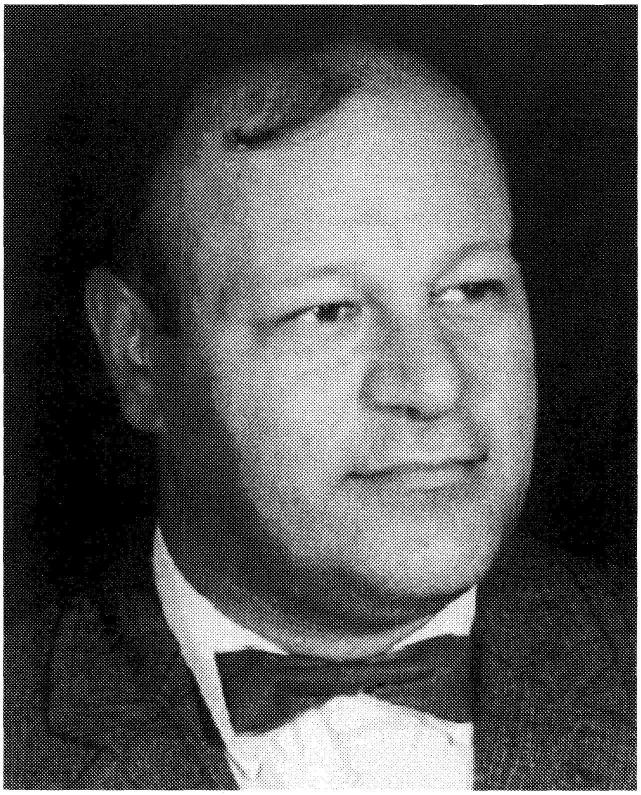

Figure 1. José Luis Sérsic (1933-1993).

Sérsic's Ph.D. thesis, titled "Application of a certain type of canonical transformations to Celestial Mechanics" written in Spanish (Sérsic, 1969), was presented to a thesis tribunal comprising Professors Reynaldo Cesco, Livio Gratton, and Pedro Zadunaisky on 1956 August 7.

Soon after graduating, Sérsic decided to focus his research efforts on problems associated with extragalactic astronomy. One year later he moved to the National Observatory of Cordoba (Argentina), dedicating the rest of his professional life to this field of astronomy, in which he received international recognition.

In our opinion, this change of research interest was one of the reasons (but perhaps not the most important one) why Sérsic's Ph.D. thesis remained almost unknown to the international community of celestial mechanicians. Although it was reproduced in Spanish, in the *Contributions of the La Plata Astronomical Observatory*, a publication with a wide international circulation, unfortunately this only occurred in 1969, thirteen years after the thesis had been approved (Sérsic, 1969). This delay was probably due to problems with the University publisher. Following, in Table 1, is a translated table of the contents of Sérsic's thesis, which may be found in the Library of the Facultad de Ciencias Astronómicas y Geofísicas of La Plata University (and we are happy to furnish copies of it upon request).

Table 1: A table of contents of Sérsic's Ph.D. thesis (translated from the Spanish original).

CONTENTS

1. Introduction.
2. General considerations.
3. Canonical transformations.
4. Canonical operators.
5. Extension of the operation.
6. Nature of the transformations.
7. Properties of the transformations.
8. Extension to more than two variables.
9. Transformation with a functional parameter.
10. Associated system of differential equations.
11. Analyticity of the transformation groups.
12. Convergence of the transformation groups.
13. Convergence of special groups.
14. Direct integration of the canonical equations.
15. Dynamical systems depending on one parameter.
16. Formal integration.
17. Intermediary orbit.
18. Determination of the characteristic function.
19. Convergence of the expansions of V and K.
20. Theorem of Poincaré.
21. Existence of periodic solutions.
22. Computation of periodic solutions.
23. General expressions for the solutions.

We are convinced that Sérsic's thesis was of a great originality at the time of its presentation, and that even now most of its contents will be of interest to celestial mechanicians. Its impact on perturbation theories could have been very significant. But this is not the only reason for presenting this review on Sérsic's work: in 1966, G Hori published a celebrated paper on a perturbative method known today as Hori's Method, and we believe that this is very similar to what Sérsic wrote in his Ph.D. thesis one decade earlier.

In this paper, we shall present the most important aspects of Sérsic's thesis, emphasizing those points that are similar to Hori's Method (although we shall not reproduce Hori's well-known results, which may be found in his paper). Our aim is to make colleagues more aware of J L Sérsic's contribution and also of the importance of celestial mechanics at La Plata during the mid-twentieth century.

2 THE SÉRSIC DOCTORAL THESIS

In this section we shall describe the main contents of Sérsic's thesis in the context of Hori's work.

The first two Sections of Sérsic's thesis are devoted to general considerations, presenting the basic aspects of Hamiltonian dynamics. Questions such as the definition of canonical transformations and the invariance of the Poisson brackets under these kinds of transformations are the main points reviewed there.

After this, in the third Section (Canonical Operators) Sérsic presents the basic ideas upon which his method of canonical transformations is based. There, at variance with the method proposed by Poincaré (1892) – a set of canonical transformations in implicit form – Sérsic presents a canonical transformation from the couple of conjugate variables (q,p) to a new one (q',p') by means of an *explicit transformation*:

$$q' = S(q)$$
$$p' = T(p)$$

where S and T are arbitrary functions that can be expanded in powers of a certain parameter α (not yet specified), in the form

$$S = 1 + \alpha\,\sigma_1 + \alpha^2\,\sigma_2 + \alpha^3\,\sigma_3 + ...$$
$$T = 1 + \alpha\,\tau_1 + \alpha^2\,\tau_2 + \alpha^3\,\tau_3 + ...$$

where the σ_j and τ_j are operators over p and q. Thus, formally, the transformations will be

$$q' = q + \alpha\,\sigma_1\,q + \alpha^2\,\sigma_2\,q + \alpha^3\,\sigma_3\,q + ...$$
$$p' = p + \alpha\,\tau_1\,p + \alpha^2\,\tau_2\,p + \alpha^3\,\tau_3\,p + ...$$

By application of properties of the Poisson brackets, such as its invariance under canonical transformations, Sérsic arrives to a particular form for the operators S and T:

$$S \equiv T.$$

He also found recurrent relations for the determination of the operators σ_j and τ_j $(\sigma_j \equiv \tau_j)$:

$$\sigma_1 = V(q\,,p)$$

$$\sigma_{i+1} = \frac{1}{(i+1)!}\,[V(q,p), \sigma_i\,],$$

where V is some defined function in phase space. The brackets denote the well-known Poisson brackets.

As it can be appreciated, in this Section, the author presented a kind of canonical transformation never applied before to problems of celestial mechanics or even classical dynamics. Nevertheless, at the time this methodology was widely used in quantum mechanics, as Sérsic himself points out in the introduction to his thesis, where he states: "The possibility to express the previously mentioned transformations in the form of operators may be found in the chapter V of the Principles of Quantum Mechanics of P A M Dirac, where an intensive use of the Poisson brackets is found, and operators of the form $e^{iS/h}$ are also used ...".

Having defined the operators S and T, Sérsic writes them in an exponential form (as in Dirac's book):

$$S \equiv T \equiv E^{\alpha V} \equiv 1 + \alpha\,[V,\,] + \frac{\alpha^2}{2!}[V,[V,]] + \frac{\alpha^3}{3!}\,V,[V,[V,]] + ...$$

where E is the basis of the natural logarithms.

With this form for the operators, the transformations in the variables q and p are written in a much more compact form:

$$q' = E^{\alpha V}\,q$$
$$p' = E^{\alpha V}\,p$$

In Section 4 the operator of the transformation is applied to arbitrary functions H of (q,p). In the following Sections 5 to 12 Sérsic analyses general properties of the operators, such as composition of operators, analytic domains, convergence regions, etc..

Section 13, Direct integration of Dynamical Systems, is completely devoted to presenting the formal integration of systems of canonical equations

$$\frac{dp}{dt} = [H,q]$$

$$\frac{dq}{dt} = [H,p]$$

in the same notation of operators.

Finally, in Section 14, Dynamical systems depending on one parameter, the perturbation theory takes its form. In this section, Sérsic applies the preceding results to a Hamiltonian of the form

$$H = H_0 + \alpha H_1 + \alpha^2 H_2 + \ldots$$

where α is a small parameter, so H_0 is the principal part Hamiltonian, being the rest a small perturbation. Usually when $\alpha = 0$ the solution of the problem is known (as in the case of the three-body problem).

Sérsic points out that the central problem is to search for the function V, in such a way that the system can be integrated. This is the scope of Section 17 (Determination of the characteristic function). Writing the new Hamiltonian as

$$K = K_0 + \alpha K_1 + \alpha^2 K_2 + \ldots$$

Sérsic arrives to the set of equations

$$K_0 = H_0$$
$$K_1 = H_1 + [H_0,V_1]$$
$$K_2 = H_2 + [H_0, V_2] + 0.5\,[H_1 + K_1, V_1]$$
$$\cdots = \cdots$$

which are completely identical to the equations 16'-20' in Hori's paper. The characteristic function is equivalent to the so-called *determining function, S,* in Hori's theory.

Through the operator, Sérsic also defines an *intermediary orbit* that is a concept like (although not totally equal to) the one defined as 'fictitious time' in Hori's equations 23 and 24.

The following chapters are devoted to using the method developed in the thesis to analyse the existence of periodic orbits in generic cases, and in the last chapter, Sérsic applies his results to a specific Hamiltonian.

3 DISCUSSION AND CONCLUDING REMARKS

To what extent are the Ph.D. thesis of Sérsic and the method of Hori the same? Hori (1966) presented his method to obtain canonical transformations and, for the integration of the specific problem of the motion of an artificial satellite in the field of an oblate planet, he determined the appropriate generating function through the 'averaging' principle proposed by Poincaré (1892) and used by Von Zeipel (Danby 1992). All the applications of Hori's Method where made in connection with this technique of averaging.

Therefore, to start our discussion we formulate the following question: Is the search for these kinds of canonical transformations with Lie generators the method referred to as Hori's Method in the literature or is it the search for the appropriate generating functions to particular problems of celestial mechanics? The answer to this question can be searched for within the title of Hori's paper itself, (General perturbations with unspecified canonical variables) as well as within the abstract of this paper ("A theorem by Lie in canonical transformations is applied to the theory of general perturbations ..."). The abstract indicates that Hori's paper is centred on the presentation of the canonical transformation with Lie generator and not on the search for the generating function. We believe that this fact is explicitly stated by Deprit (1969), Campbell and Jefferys (1970), and Henrard and Roels (1973), to cite just three papers that appeared soon after the publication of Hori's paper.

The work of José Luis Sérsic remained hidden to the eyes of the rest of the scientific world for almost half of a century. The particular circumstances of this unfortunate fact are not completely clear. However, some comments may help to shed light on this situation. Publication of research carried out at the Observatorio Astronómico de La Plata in international journals only become a common practice in the 1960's. It is worth mentioning that Sérsic's thesis advisor, Reynaldo Cesco, almost never published his results in journals with an international circulation, and this circumstance may have conspired against the publication of his disciple's doctoral thesis. There is also the already-mentioned fact that soon after the presentation of the doctoral thesis Sérsic transferred his research allegiance from celestial mechanics to astrophysics.

We have found few references to Sérsic's Ph.D. thesis in the literature, and they all appear in papers written by Professor Sylvio Ferraz-Mello following a visit to La Plata in 1987. We know that Ferraz-Mello obtained a reprint of Sérsic's thesis at this time, and it is interesting that in all of his references to Sérsic's work Ferraz-Mello presents it as an antecedent of the application of Lie series to celestial mechanics rather than as a precursor of Hori's Method (e.g. see Ferraz-Mello, 1997).

The last and perhaps most difficult question to answer is this: To what extent were the developments that Hori presented in his celebrated paper of 1966 completely independent of the results contained in Sérsic's Ph.D. thesis? To our knowledge, there is no direct evidence that Hori knew of Sérsic's work, but in Sérsic's 'Introduction' there is a sentence that may possibly suggest a connection between the two works: "It is, however, in the method of Peano-Baker and in its applications to Celestial Mechanics made by Yusuke Hagihara where the logic antecedents of this work must be searched for ... ". It is likely that G Hori attended courses in advanced celestial mechanics offered by Hagihara (Kozai, 1998), and he perhaps is the source of inspiration that links both works.

We wish to end this paper by proposing that in future *Hori's Method* should be known as the *Lie-Sérsic-Hori Method* (or simply the *L-S-H Method*) as a part tribute to the beautiful piece of work that was independently developed by José Luis Sérsic in 1956.

4 ACKNOWLEDGEMENTS

AB wishes to thank S Ferraz-Mello for suggesting this investigation to him many years ago.

5 REFERENCES

Campbell, J.A. and Jefferys, W.H., 1970. Equivalence of the perturbation theories of Hori and Deprit. *Celestial Mechanics*, **2**:467-473.

Danby, J.M.A., 1992. *Fundamentals of Celestial Mechanics*. Willmann-Bell, Richmond.

Deprit, A., 1969. Canonical transformation depending on a small parameter. *Celestial Mechanics*, **1**:12-30.

Ferraz-Mello, S., 1997. On Hamiltonian averaging theories and resonance. *Celestial Mechanics*, **66**:39-50.

Henrard, J. and Roels, J., 1973. Equivalence for Lie transformations. *Celestial Mechanics*, **10**:497-512.

Hori, G., 1966. General perturbations with unspecified canonical variables. *Publications of the Astronomical Society of Japan*, **18**:287-296.

Kozai, Y., 1998. The development of celestial mechanics in Japan. *Planetary and Space Science*, **46**:1031-1036.

Poincaré, H., 1892. *Les Méthodes Nouvelles de la Mécanique Céleste*. I Gautier- Villars et Fils, Paris.

Sérsic, J.L., 1969. Aplicación de un cierto tipo de transformaciones canónicas a la Mecánica Celeste. *Serie Astronómica. Observatorio Astronómico de La Plata*, **XXXV**:5-30.

Adrián Brunini is a Professor in the Facultad de Ciencias Astronómicas y Geofísicas of La Plata (Argentina), where he conducts research on planetary sciences as an independent researcher of the National Research Council (CONICET). He is Head of the Department of Celestial Mechanics and a researcher of the La Plata Astrophysical Institute, and has more than 50 publications.

Octavio Miloni is a graduate student in the Facultad de Ciencias Astronómicas y Geofísicas of La Plata (Argentina). At present he is at Sâo Paulo (Brazil), carrying out doctoral studies on resonances and averaging of Hamiltonian systems under the supervision of Professor Sylvio Ferraz-Mello.