Journal of Astronomical History and Heritage 5(2):115-121 2002

Analysis of dates and lunar phase records in *Wucheng*

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Abstract
There is a series of dates with month, sexagenary day and lunar phase in the ancient archive *Wucheng*, which are important clues to date King Wu's conquest. This paper mathematically analyzes the relationship between the explanation of the lunar phase terms and the date of the historic event. Combining with other related astronomical records, we offered some possible suggestions as to the date of King Wu's conquest.

Keywords: *chronology, ancient astronomical records, history of astronomy, Chinese history*

1 INTRODUCTION

In the eleventh century BC, King Wu conquered King Zhou and set up the West Zhou Dynasty. This was an important event in Chinese history and many records relating to it have survived. However, the date of King Wu's conquest is still a puzzle, even though many historians have studied it over the last two thousand years. According to a recent review (Beijing Normal University, 1997), forty-four different conclusions, dating between 1127-1018 BC, have been published.

Many ancient records mention King Wu's conquest, and one of the key ones is *Wucheng*. It is a section of the famous classic *Shujing*, which collected the government statements in early times. A more complete fragment of *Wucheng* has survived in *Hanshu-Lulizhi*, and is a diary-style story on this war:

> The first month, day Renchen (29), it was Pang-Sipo. The next day was Guisi (30); the morning, King Wu departed from Zhou [state] to crusade against King Zhou [of Shang State]. Counting from the day Ji-Sipo of the second month, the fifth day was Jiazi (1) when King Zhou was killed. Counting from the day Jipang-Shengpo of the fourth month, the sixth day was Gengxu (47) when King Wu prayed at the grand temple of Zhou (state). The next day, Xinhai (48), the King prayed to the sky. The fifth day Yimao (52) sacrifice prisoners of the war at the temple.

Here the dates were counted with a sexagenary cycle, and we have placed the order numbers in parentheses. Usually the first month is the one that contains the winter solstice. A lunar phase system was used to determine the lunar day in West Zhou, and this is seen in bronze inscriptions and in historical records. Lunar phase terms used are Chu-ji, Ji-sheng-ba, Ji-wang and Ji-sipo, but the exact meaning of these is difficult to determine and many different hypotheses have been offered (e.g. see Zheng, 1981). From the text we see that each term must be a certain lunar day, on which a particular lunar phase could be seen. Other dates in the month were then counted from these days.

From *Wucheng* we can deduce three dates with sexagenary days and lunar phases. They are:

A: day Renchen (29) of 1st month Pang-sipo
B: day Gengshen (57) of 2nd month Ji-sipo
C: day Yisi (42) of 4th month Ji-pang-shengpo.

In the sexagenary series, Ji-sipo of the 2nd month is 57, so Ji-sipo of the 1st month would be 57 – 29.5 = 27.5 (a month is 29.5 days). Pang-sipo (29) was 1.5 day latter than Ji-sipo in the 1st month. Ji-sipo of the 4th month would be 57 + (2 × 29.5) – 60 = 56. Ji-pang-shengpo (42) was 14 days before Ji-sipo. Here we have found that Ji-sipo and Ji-shengpo were the opposite lunar phase (14 or 15 days different), and that Pang was 1 or 2 day latter than Ji. In Chinese

language, *sheng* (live) and *si* (death) are opposites; *Ji* means 'already'; *Pang* means 'beside' or 'next'. Here we show that the linguistic and mathematical meanings are in accord.

If we can determine the year of *Wucheng*, we can establish the meaning of these lunar phase terms. Normally an historian would select a particular year for King Wu's conquest and then prove that this date adequately explains the lunar terms, but in this investigation we shall discuss all dates that fit certain hypotheses relating to these lunar terms.

2 METHODOLOGY

Lunar calendar day (lcd) is a convenient way to express lunar phases, in which the New Moon day (when the Sun and the Moon are in conjunction) is number 1. There are three different types of hypotheses relating the lunar phase terms:

Hypothesis 1. Jisipo is the New Moon day (lcd 1) and Jishengpo is the Full Moon day (lcd 16).

Hypothesis 2. Jishengpo is the First Quarter day (lcd 8) and Jisipo is the Last Quarter day (lcd 23).

Hypothesis 3. Jishengpo is the day when the New Moon is first seen (lcd.3) and Jisipo is the first day when the loss of the Full Moon begins to be noticed (lcd 18).

The following method has been used to analyze the lunar data in *Wucheng*:

1) The sexagenary number of the 1st day of the 1st month is derived from day Renchen (29) Pang Sipo according to an hypothesis of lunar terms. The sexagenary number of the 1st day of the 2nd month is derived by adding 29 or 30 days, then the lunar calendar day number (lcd) of day Gengshen (57) of the 2nd month. The sexagenary number of the 1st day of the 4th month is derived by adding 59 days, then the lcd of day Yisi (42). These are then listed in Tables 1a, 2a, and 3a (for the three different hypotheses). We then check to see whether the three days fit the lunar hypothesis. Finally we derive the sexagenary number of the 1st day of the 1st month.
2) With modern astronomical knowledge we can compute a New Moon list (e.g. see Zhang, 1990), which includes every New Moon day, its Julian date (year, month and day) and the sexagenary number. Winter solstices are also listed. We can now find in which years the 1st day of the 1st month has the given sexagenary number, and then the New Moon days of the 2nd, 3rd, and 4th months, their Julian dates and their sexagenary numbers. These are listed in Tables 1b, 2b, and 3b.
3) We then insert the three dates in *Wucheng* in Tables 1b, 2b, and 3b, and see if they fit the lunar hypothesis, and if they have 'Jupiter in the constellation Chunhuo' and 'The Sun in the constellation Ximu'.

Our analysis assumes that the date of King Wu's conquest was 1085-1020 BC, and that the 1st day of the 1st month was between November 1 to January 31 (based on the knowledge that the winter solstice (December 30) was in the first month at that time). *Guoyu* records that "Jupiter was in constellation Chunhuo; the Sun was in constellation Ximu." and we note them in passing in Tables 1b, 2b, and 3b under sub-columns J (Jupiter) and S (Sun) – but for details see Liu and Zhou (2001).

3 THE DIFFERENT HYPOTHESES

3.1 Hypothesis 1

In this hypothesis, Jisipo is the New Moon day (lcd 1), Jishengpo is the Full Moon day (lcd 16), and Pang-sipo therefore is lcd 2 or 3. We list lcd 0, 1, 2, 3, 4, and 5 in the second line of Table 1a for a wider discussion. If Renchen (29) Pangsipo is lcd 2, the sexagenary number of its previous day must be (28). In the same way, we get the numbers in the first line. One month (29.5 days) latter we get the third line; then another month latter we get the fifth line, the numbers of the 1st days of next two months. For the same example they are 28 + 29.5 = 57.5 and (57.5 + 29.5) – 60 = 27. If the 1st day of the 2nd month is (58) and Gengshen is (57), then Ji-sipo must be lcd 0. In the same way, we get the lcd numbers in the 4th line. If the 1st day of the 3rd month is (27) and Yisi is (42), then Jipang-shengpo must be lcd 16. In the same way, we get the lcd numbers in the 6th line.

We have noted that there is no Yisi (42) day in the 4th month, so we have to suppose that '4th month' is a mistake and that the original record should have been '3rd month'. This is an important weakness of Hypothesis 1.

Table 1a. The relationship between dates in *Wucheng*, Hypothesis1

1st day of 1st month: 60 No.	(30)	(29)	(28)	(27)	(26)	(25)
Renchen (29) Pangsipo: lcd	0	1	2	3	4	5
1st day of 2nd month: 60 No.	(60)	(59)	(58)	(57)	(56)	(55)
Gengshen (57) Jisipo: lcd	-2	-1	0	1	2	3
1st day of 3rd month: 60 No.	(29)	(28)	(27)	(26)	(25)	(24)
Yisi (42) Jipangshengpo: lcd	14	15	16	17	18	19

From Table 1a we found that if Renshen is (29), then Pangsipo is one of lcd 0-5, Jisipo will be in lcd -1-3, and Jipangshengpo in lcd 14-19. They are all consistent with Hypothesis 1. On the other hand, the sexagenary number of the 1st day of the 1st month must be in (30-25), so we looked for such dates in a New Moon list and inserted them and the first days of the 2nd and 3rd months in Table 1b. Accordingly, the lcd numbers of the three dates are also entered in columns A (Renchen Pangsipo), B (Gengshen Jisipo) and C (Yisi Jipangshengpo). In Table 1b, if Jisipo is lcd 2, 1, or 0 we consider it is good and insert ● in the sub-column L (lunar phase). If Jisipo is 3 or −1, it is acceptable and we insert ○. In the same way, Jishengpo lcd 15, 16, or 17 is good (●) while lcd 14 or 18 are acceptable (○). In passing, the Sun and Jupiter rankings are also listed under S and J.

Table 1b. The dates fitting *Wucheng*, Hypothesis 1

No.	1st day of 1st month	A	1st day o 2nd month	B	1st day o 3rd month	C	L	S	J
1	1081.12.15 BC (27)	3	1.14(57)	1	2.12(26)	17	●	○	○
2	1076.11.20 BC (28)	2	12.20(58)	0	1.18(27)	16	●	●	
3	1075.1.18 BC (27)	3	2.17(57)	1	3.18(26)	17	●		
4	1071.12.25 BC (29)	1	1.23(58)	0	2.22(28)	15	●		●
5	1066.11.30 BC (30)	0	12.30(60)	-2	1.28(29)	14		●	
6	1065.1.28 BC (29)	1	2.27(59)	-1	3.27(28)	15	○		
7	1060.12.23 BC (25)	5	1.21(54)	4	2.20(24)	19			○
8	1055.11.28 BC (26)	4	12.28(56)	2	1.27(26)	17	●	●	
9	1054.1.27 BC (26)	4	2.25(55)	3	3.27(25)	18	○		
10	1050.11.4 BC (28)	2	12.3(57)	1	1.2(27)	16	●	○	
11	1049.1.2 BC (27)	3	1.31(56)	2	3.1(25)	18	○		
12	1045.12.7 BC (28)	2	1.6(58)	0	2.4(27)	16	●	●	
13	1040.11.13 BC (30)	0	12.12(59)	-1	1.11(29)	14	○	●	
14	1039.1.11 BC (29)	1	2.9(58)	0	3.10(27)	16	●		
15	1035.12.16 BC (29)	1	1.15(59)	-1	2.14(29)	14	○	○	●
16	1029.11.11 BC (26)	4	12.10(55)	3	1.9(25)	18	○	●	
17	1028.1.9 BC (25)	5	2.8(55)	3	3.9(24)	19			
18	1024.12.15 BC (26)	4	1.13(55)	3	2.12(25)	18	○	○	

Under Hypothesis 1, no one group of dates supports all three conditions. Groups 1, 4, and 15 are better than others. Group 4 seems to be the best one. We pay more attention to Jupiter than to the Sun (see Liu and Zhou, 2001).

3.2 Hypothesis 2

In this hypothesis, Jishengpo is the First Quarter day (lcd 8), Jisipo is the Last Quarter day (lcd 23), and Pangsipo must be in lcd 24. Also, we discuss a wider range, lcd 22-27. In the same way as Hypothesis 1, we derive Table 2a. On this occasion there is a Yisi (42) day in the 4th month. However, according to the text, the day Yimao (52) was counted from Yisi (42). If Yisi was during the First Quarter (lcd 7 or 8) and Yimao is lcd 17 or 18, why did not the ancient people count Yimao from Jiwang since Jiwang is a lunar term with a well-known meaning (i.e. Full Moon)? This is a weakness of Hypothesis 2. Employing the same method as previously, we prepared Table 2b, using ● for 'good' if Jisipo is in lcd 22, 23, or 24 and Jishengpo is in lcd 7, 8, or 9, and ○ for 'acceptable' if Jisipo is in lcd 21 or 25 and Jishengpo is in lcd 6 or 10.

Under Hypothesis 2, no one group of dates supports all three conditions. Groups 4, 12 and 16 are better than others, and Group 4 seems to be the best one.

Table 2a. The relationship between dates in *Wucheng*, Hypothesis 2

1st day of 1st month: 60 No.	(8)	(7)	(6)	(5)	(4)	(3)
Renchen (29) Pangsipo: lcd	22	23	24	25	26	27
1st day of 2nd month: 60 No.	(38)	(37)	(36)	(35)	(34)	(33)
Gengshen (57) Jisipo: lcd	20	21	22	23	24	25
1st day of 4th month: 60 No.	(37)	(36)	(35)	(34)	(33)	(32)
Yisi (42) Jipangshengpo: lcd	6	7	8	9	10	11

Table 2b. The dates fitting *Wucheng*, Hypothesis 2

No.	1st day of 1st month	A	1st day o 2nd month	B	1st day o 4th month	C	L	S	J
1	1082.11.27 BC (3)	27	12.27(33)	25	2.24(32)	11		●	●
2	1081.1.26 BC (3)	27	2.24(32)	26	4.23(31)	12			●
3	1077.12.30 BC (3)	27	1.29(33)	25	3.29(32)	11			
4	1072.12.6 BC (5)	25	1.5(35))	23	3.5(34)	9	●	●	○
5	1067.11.11 BC (6)	24	12.11(36)	22	2.8(35)	8	●	●	
6	1066.1.9 BC (5)	25	2.8(35)	23	4.8(34)	9	●		
7	1062.12.15 BC (6)	24	1.14(36)	22	3.14(36)	7	●	○	
8	1057.11.20 BC (8)	22	12.20(38)	20	2.17(37)	6		●	
9	1056.1.18 BC (7)	23	2.17(37)	21	4.16(35)	8	○		
10	1052.12.24 BC (8)	22	1.23(38)	20	3.23(37)	6			
11	1051.12.14 BC (3)	27	1.12(32)	26	3.12(31)	12		○	
12	1046.11.19 BC (4)	26	12.19(34)	24	2.16(33)	10	○	●	
13	1045.1.18 BC (4)	26	2.16(33)	25	4.15(32)	11			
14	1041.12.23 BC (5)	25	1.21(34)	24	3.21(33)	10	○		
15	1036.11.28 BC (6)	24	12.27(35)	23	2.25(35)	8	●	●	
16	1035.1.26 BC (5)	25	2.25(35)	23	4.25(34)	9	●		○
17	1031.11.4 BC (8)	22	12.3(37)	21	1.31(36)	7	○	○	
18	1030.1.2 BC (7)	23	1.31(26)	22	3.31(35)	8	●		
19	1026.12.7 BC (7)	23	1.6(37)	21	3.5(36)	7	○	●	
20	1020.11.1 BC (3)	27	12.1(33)	25	1.30(33)	10	○	○	
21	1020.12.31 BC (3)	27	1.30(33)	25	3.29(31)	12			

3.3 Hypothesis 3

In this hypothesis, Jishengpo is the day when the New Moon begins to be seen (lcd 3), Jisipo is the first day when the loss of the Full Moon begins to be noted (lcd 18), and Pangsipo must be in lcd 19. Also, we discuss a wider range, lcd 17-22. In the same way as for Hypothesis 1 we derive Table 3a. Unlike Hypotheses 1 and 2, this hypothesis has no obvious weaknesses. Using the same methodology, we have prepared Table 3b, and inserted ● for 'good' if Jisipo is in lcd 17, 18, or 19 and Jipangshengpo is in lcd 3, 4, or 5, and ○ for 'acceptable' if Jisipo is in lcd 16 or 20 and Jishengpo is in lcd 2 or 6. Under Hypothesis 3, group 10 supports all three conditions, while groups 1 and 11 are also acceptable. In this context it is important to point out that Hypothesis 3 is supported by independent investigations using data drawn from astronomy (Jing, 1999), palaeography (Wu, 2000), and bronze inscriptions (Chen, 2000).

4 THE RELATIONSHIP BETWEEN LUNAR TERMS AND THE YEAR OF KING WU'S CONQUEST

We have shown above that if we have an explanation for the lunar terms, we can derive one or several groups of dates for King Wu's conquest. On the other hand, if we suggest a specific year for King Wu's conquest, this would provide an explanation of the lunar terms. Since Jishengpo and Jisipo are opposite lunar phases, we only need to discuss one of them, and we choose Jisipo because it is more popular in bronze inscriptions. For example, our Hypotheses 1, 2 and 3 can all be indicated by Jisipo 1, 23, 18 (lcd).

Table 3a. The relationship between dates in *Wucheng*, Hypothesis 3

1st day of 1st month: 60 No.	(13)	(12)	(11)	(10)	(9)	(8)
Renchen (29) Pangsipo: lcd	17	18	19	20	21	22
1st day of 2nd month: 60 No.	(42)	(41)	(40)	(39)	(38)	
Gengshen (57) Jisipo: lcd	16	17	18	19	20	
1st day of 3rd month: 60 No.	(41)	(40)	(39)	(38)	(37)	
Yisi (42) Jipangshengpo: lcd	2	3	4	5	6	

Table 3b. The dates fitting *Wucheng*, Hypothesis 3

No.	1st day of the 1s month	A	1st day o 2nd month	B	1st day o 4th month	C	L	S	J
1	1083.12. 8 BC (9)	21	1.7(39)	19	3.6(37)	6	○	●	●
2	1078.11.13 BC (10)	20	12.12(39)	19	2.9(38)	5	●	●	
3	1077. 1.11 BC (9)	21	2.9(38)	20	4.9(38)	5	○		
4	1073.12.17 BC (11)	19	1.15(40)	18	3.15(39)	4	●	○	
5	1068.11.22 BC (12)	18	12.21(41)	17	2.18(40)	3	●	●	
6	1067. 1.20 BC (11)	19	2.18(40)	18	4.18(39)	4	●		
7	1063.12.26 BC (12)	18	1.25(42)	16	3.25(41)	2	○		
8	1057. 1.30 BC (13)	17	2.28(42)	16	4.27(41)	2	○	○	
9	1052.12.24 BC (8)	22	1.23(38)	20	3.23(37)	6	○		
10	1047.11.30 BC (10)	20	12.30(40)	18	2.27(39)	4	●	●	●
11	1046. 1.28 BC (9)	21	2.27(39)	19	4.26(37)	6	○		●
12	1042.11. 5 BC (11)	19	12.4(40)	18	2.1(39)	4	●	○	
13	1041. 1. 3 BC (10)	20	2.1(39)	19	4.1(39)	4	●		
14	1037.12. 9 BC (12)	18	1.7(41)	17	3.8(41)	2	○	●	
15	1032.11.14 BC (13)	17	12.14(43)	15	2.10(41)	2	○	●	
16	1031. 1.12 BC (12)	18	2.10(41)	17	4.10(40)	3	●	○	
17	1027.12.18 BC (13)	17	1.17(43)	15	3.17(42)	1		○	
18	1021.11.12 BC (9)	21	12.12(39)	19	2.9(38)	5	●	●	
19	1020. 1.11 BC (9)	21	2.9(38)	20	4.9(37)	6	○		

Now our discussion is confined to one date: the day Gengshen (57) in the 2nd month Jisipo. Conventionally, the 1st month contains the winter solstice. We do not know exactly which New Moon belongs to the 2nd month, therefore we searched for Gengshen (57) in the winter solstice month and the following three months and found its lcd numbers (obviously Gengshen appears twice in this range). Inserting them in Table 4, we get different lunar phases for Jisipo in different years. In Table 4, 'a' stands for the month that contains winter solstice, 'b' is the next month, and so on.

It is easy to find the corresponding years for different lunar term definitions. For example, for Hypothesis 1, Jisipo lcd 0, 1 and 2, we can easily find 1085, 1080, 1075, 1070, 1054, 1049, 1044, and 1039 BC. They are exactly the same as we get from Table 1b. On the other hand, if we determine that 1027 BC was the year of King Wu's conquest, we find that Jisipo was lcd 9. That means that Jisipo was the First Quarter day and Jishengpo the Last Quarter day. Of course, the lunar term hypotheses also have to fit other conditions implied in *Wucheng*, and we have showed the weaknesses of Hypotheses 1 and 2 in Section 3.

Table 4 lists only four months. Had we decided to accept a wider suggestion about the definition of the 1st month, then the table would have been enlarged to the right until all twelve months were listed and it linked to the following year. We have found a cycle of approximately five years, because two months (59 days) differ from the 60-day cycle by just one day, and after five years there is one month difference.

5 CONCLUDING REMARKS

In this paper, we have mathematically analyzed the relationship between lunar calendar days and the sexagenary days of the dates in *Wucheng*. Using three popular hypotheses for different lunar phase terms, plus ancient records relating to the Sun and Jupiter, we obtained three series

Table 4. The lcd numbers corresponding to day Gengshen in the 2nd month Jisipo

BC	a	b	C	d	BC	a	b	c	d	BC	a	b	c	d	BC	a	b	c	d
1087	21		21		1070		0		0	1053	8		8		1036		17		17
1086		27		28	1069		6		6	1052	14		14		1035		23		23
1085	2		3		1068		11		12	1051		20		21	1034	29		29	
1084		9		10	1067	17		18		1050		26		27	1033		5		5
1083		14		15	1066	22		23		1049	1		2		1032	10		11	
1082		19		21	1065		29		30	1048		7		9	1031	15		17	
1081	25		26		1064	4		5		1047		13		14	1030	21		22	
1080		1		2	1063	9		10		1046	18		19		1029		27		29
1079		7		8	1062		16		17	1045	24		25		1028	3		3	
1078	13		14		1061		22		22	1044		0		1	1027		9		10
1077	19		20		1060		28		28	1043	6		7		1026		15		16
1076		25		26	1059		4		5	1042	12		13		1025		21		22
1075	0		1		1058		9		10	1041	18		19		1024	27		28	
1074	5		7		1057	15		16		1040		24		25	1023		3		4
1073		12		13	1056	20		21		1039		29	0		1022		8		9
1072		18		19	1055	26		27		1038	4		5		1021	13		15	
1071		23		24	1054	2		3		1037		11		12	1020	19		20	
BC	a	b	c	d	BC	a	b	c	d	BC	a	b	c	d	BC	a	b	c	d

of results for the date of King Wu's conquest. Our work was only a small part of a much larger project, and the Xia-Shang-Zhou Chronology Project recently accepted 1046 January 20 BC as the date of King Wu's conquest on the basis of our research and other astronomical, historical and archaeological information (The Expert Group ..., 2000). Although this is a pleasing solution to what has been a complex long-term problem, further works remains to be done.

6 ACKNOWLEDGEMENTS

I wish to thank Dr Wayne Orchiston (Anglo-Australian Observatory) for his kind help in refining the English in this paper.

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