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MAGNITUDE SYSTEMS IN OLD STAR CATALOGUES

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Abstract: The current system of stellar magnitudes originally introduced by Hipparchus was strictly defined by Norman Pogson in 1856. He based his system on Ptolemy's star catalogue, the *Almagest*, recorded in about AD137, and defined the magnitude-intensity relationship on a logarithmic scale.

Stellar magnitudes observed with the naked eye recorded in seven old star catalogues were analyzed in order to examine the visual magnitude systems. Although psychophysicists have proposed that human visual sensitivity follows a power-law scale, it is shown here that the degree of agreement is far better for a logarithmic scale than for a power-law scale. It is also found that light ratios in each star catalogue are nearly equal to 2.512, if the brightest (1st magnitude) and the faintest (6th magnitude and dimmer) stars are excluded from the study. This means that the visual magnitudes in the old star catalogues agree fully with Pogson's logarithmic scale.

Keywords: stars, historical star catalogues, stellar magnitude system, visual magnitude estimates, astronomical photometry

1 INTRODUCTION

The concept of magnitudes was introduced by Hipparchus in the second century BC (Werner and Schmeidler, 1986; Zissell, 1998). Hipparchus compiled his catalogue of 850 stars with ecliptic coordinates and visual magnitudes. This work was triggered by the discovery and the observation of a nova (not yet explained) in the constellation Scorpius in 134BC. He started to record the coordinates and magnitudes of fixed stars in order to aid discoveries of such objects, and to record the brightness of each star. He defined the brightest twenty stars as 1st magnitude, Polaris and stars of the Great Dipper in Ursa Major as 2nd magnitude, and stars at the observable limit of the naked eye as 6th magnitude. The work of Hipparchus was lost over the years, but Hipparchus' magnitude system came down through subsequent star catalogues such as the *Almagest*. The observational data in the *Almagest* are found to be partly dependent on those of Hipparchus (Duke, 2002; cf. Evans, 1987; Rawlins, 1982).

In the nineteenth century, astronomers tried to define the magnitude system more precisely and quantitatively, based on simple arbitrary visual estimates. Because of the advent of visual photometry, some refinements have been necessary to extend the magnitude scale to fainter objects (see Peirce, 1878). Sir William and Sir John Herschel tried to deduce the form of the magnitude-intensity relationship, and Sir John concluded that a logarithmic law was preferable to a power law and that the light ratio, R, corresponded to 2.551. However, they did not have access to the intensive photometric measurements required in order to relate the visual magnitude scale to stellar intensities. Stenheil made photometric observations in 1836, deduced a logarithmic form for the magnitude-intensity relation, and calculated a value of $R = 2.831$. In the 1850s, Fechner (1860) and Weber explained the logarithmic law on the basis of physiological principles. Applying the laws of the visual senses to stellar magnitudes, Fechner derived a logarithmic relation. This finding was supported by other researchers who investigated the magnitude-intensity relationship (e.g. see Young, 1990).

In 1856, Pogson used the data in Ptolemy's star catalogue, the *Almagest*, to propose adopting a light ratio $R = 2.512$ for two stars that differ in brightness by one magnitude, and he defined the magnitude as

$$m = -(1/\log R)\log I \qquad (1)$$

In the case of $R = 2.512$, this formula could be transformed into

$$m = -2.5\log I \qquad (2)$$

This definition is well-known as the Pogson scale, and it is still used in stellar photometry.

In the 1960s, psychophysicists demonstrated that the human eye's response to light follows a power law (e.g. Stevens, 1961; 1975), and on this basis visual magnitude estimates should also follow a power law (Schulman & Cox, 1997). On the other hand, astrophysicist, Hearnshaw (1996; 1999), examined the *Almagest*, and he showed that the magnitudes listed there agreed with a logarithmic scale, and that the light ratio (R) was 3.42, far larger than the value derived by Pogson.

In order to determine whether visual magnitude estimates fit a logarithmic or a power law, we intend to investigate the magnitude systems in a number of old star catalogues, using data in *Sky Catalogue 2000.0* (Hirshfeld et al., 1991) for 'modern' stellar magnitudes (henceforth referred to as V magnitudes). The following old star catalogues contain stellar magnitudes estimated by eye and graded from 1 to 6 on the basis of the Hipparchus system:

1. The *Almagest* (Ptolemy, AD127–141)
2. *Kitāb Ṣuwar al-Kawākib* (al-Ṣūfī, 986)
3. *Ulugh Beg's Catalogue of Stars* (1437)
4. *Astronomiae Instauratae Progymnasmata* (Brahe, 1602)
5. *Uranometria* (Bayer, 1603)
6. *Historia Coelestis Britannica* (Flamsteed, 1725)
7. *Uranometria Nova* (Argelander, 1843)

A detailed discussion of these seven catalogues is included in Fujiwara et al. (2004).

In this paper, we present the results of our study of magnitude systems in these old star catalogues. The characteristics of each of the catalogues are discussed in Section 2, below, and the magnitude data and associated analyses are found in Section 3. We graphically present and compare the historical magnitude data with logarithmic and power-law scales in Section 4.1, and light ratios (*R*) are described in Section 4.2. Conclusions are presented in Section 5.

2 CHARACTERISTICS OF THE OLD STAR CATALOGUES

The *Almagest* provides one of the earliest quantitative studies on the brightness of the stars, and was written by Ptolemy (or Claudius Ptolemaeus) in the second century AD (Schmidt, 1994; Zinner, 1926). It comprises thirteen books, and books VII and VIII discuss the fixed stars. It was necessary to establish the coordinates of the stars near the ecliptic in order to observe the positions of the planets. These two books contain a catalogue of 1,022 stars, complete with magnitudes and ecliptic latitudes and longitudes, and the stars are arranged according the classical forty-eight constellations of antiquity. In our study, we only investigate the stars listed in books VII and VIII, and although Ptolemy's original catalogue has not survived, there are numerous manuscript copies dating from the ninth to sixteenth centuries AD. Extensive philological studies of the *Almagest* were conducted by Kunitzsh (1986) and Toomer (1998), and we used the star catalogues in these two works and extracted visual magnitudes for 1,022 stars. Ptolemy's own recorded observations expended from AD127 to 141, and the mean epoch of his catalogue is about AD137.

Kitāb Ṣuwar al-Kawākib (henceforth '*Ṣuwar al-Kawākib*'), which means book on the constellations of the fixed stars, was written in Arabic in the tenth century AD by Abu'l-Ḥusayn 'Abd al-Raḥmān ibn 'Umar al-Ṣūfī (903–986). Al-Ṣūfī is best known for his observations and descriptions of the fixed stars. In this work, he presents the results of his own observations, noting where they differ from or add to data in Ptolemy's star catalogue. In al-Ṣūfī's book—as in the *Almagest*—the forty-eight Ptolemaic constellations are presented in direction order: first the boreal constellations, then the twelve constellations on the ecliptic, and finally the austral constellations. In the table of stars, he lists the magnitude and celestial latitude and longitude of each star. The epoch of this star catalogue is AD964. For each star, he adopts a precession correction of 1° in 66 years, and simply adds a constant of 12° 42′ to Ptolemy's longitudes. However, it is important to stress that the magnitudes represent the results of his own observations. This catalogue represents the only significant independent work on stellar magnitudes between classical times and the Middle Ages; most other medieval astronomers merely reproduced data from Ptolemy's star catalogue. Since the art of printing had yet to be developed, our most serious concern was that scribal errors may have crept into the various manuscripts. Consequently, we examined various manuscripts and other literature relevant to al-Ṣūfī work (e.g. al-Bīrūnī, 1030; Schjellerup, 1874), but we ended up relying mainly upon *Ṣuwar al-Kawākib* (al-Ṣūfī, 986), which was published in Hyderabad in 1954.

At Samarkand, in AD1420, Ulugh Beg (1394–1449) founded a *madrasa*, or institution of higher learning, in which astronomy was the most important subject. A major outcome of the scientific work of Ulugh Beg and his school was the mathematical tables called the *Zīj* of Ulugh Beg or the *Zīj-i Gurgāni* ('Guragon'the title of Genghis Khan's son-in-law, was sometimes also used by Ulugh Beg). This work was originally written in the Tadzhil language, and includes calendrical calculations, planetary tables and a star catalogue that is based on astronomical observations made at Samarkand in about AD1437. Knobel (1917) accessed all contemporary Persian manuscripts available in Great Britain, and published the *Zīj-i Gurgāni* as "Ulugh Beg's Catalogue of Stars". For the purposes of this study, we used the French edition (which also contains some English passages).

Tycho Brahe (1546–1601), the well-known Danish astronomer, observed a supernova in Cassiopeia (the so-called 'Tycho's Nova') from 11 November 1572 until March 1574 (see Stephenson and Green, 2002: 91-95), and recorded his observations in two books (Brahe, 1573; 1602). The latter work, *Astronomiae Instauratae Progymnasmata,* was published posthumously, and in it Brahe included solar and lunar theories and a catalogue with the positions and magnitudes of 777 fixed stars. The data in this catalogue are based upon Tycho's own observations (Dreyer, 1890), and are of high precision. For example, stellar positions were determined to within one minute of arc. In our study, we used a reprinted edition of the *Astronomiae Instauratae Progymnasmata.*

The German astronomer, Johann Bayer (1572–1625), introduced a new system of naming fixed stars in his *Uranometria*, which was published in 1603. In Ptolemy's much earlier *Almagest* there are forty-eight constellations, and stars are usually identified by number and elaborate descriptions. For example, α UMi was described as "The star at the end of the tail of the Little Bear." This was a cumbersome method, and did not always direct each observer to the same star! Bayer decided to reform the system by unambiguously and succinctly identifying every star visible to the naked eye. In each constellation, he assigned Greek letters to the naked eye stars, in approximate order of magnitude, and continued on using the Latin alphabet when necessary (see Figure 1). Bayer's nomenclature is still widely used today. In the *Uranometria*, Bayer adds twelve southern constellations to Ptolemy's original forty-eight, and he depicts the positions and magnitudes of about 1,200 individual stars (with magnitudes for the southern stars drawn from data provided by Dutch explorers). For our study, we used Bayer's reprinted edition, which includes the twelve southern constellations.

John Flamsteed (1646–1719) was the first Astronomer Royal at the Royal Observatory, Greenwich, where he carried out 20,000 observations of almost 3,000 stars. His accumulating observational data were presented in the *Historia Coelestis*, which was published in 1712, and in the *Historia Coelestis Britannica*, which appeared posthumously in 1725. This latter work comprises three volumes, and the third and final volume contains his star catalogue (the earlier volumes 1 and 2 contain planetary data). The catalogue contains magnitude estimates, plus equatorial and ecliptic positions; the mean epoch is AD1689. We used the original copy of the *Historia Coelestis Britannica* held in the Paris Observatory for our study.

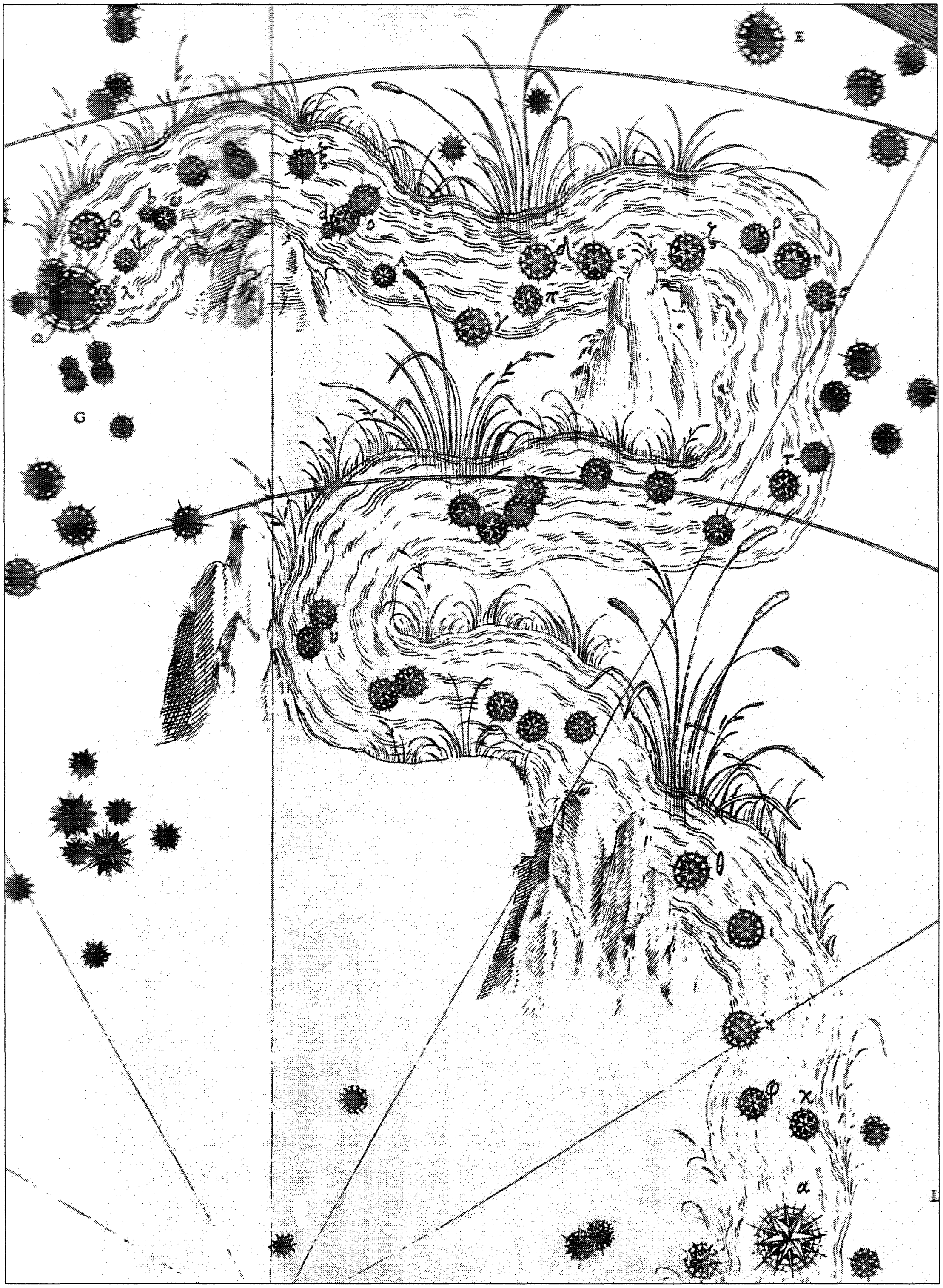

Figure 1: Map of the constellation of Eridanus in *Uranometria* (Bayer, 1603), showing stars labeled with all twenty-four Greek letters, plus four letters drawn from the Latin alphabet.

Finally, in 1843 the Prussian astronomer Friedrich Wilhelm August Argelander (1799–1875) published his *Uranometria Nova*, which contained the equatorial coordinates and magnitudes of 3,256 stars that he observed with the naked eye from Bonn. Argelander was not so concerned about stellar positions; the main feature of his work was that he recorded *all* stars visible to the naked eye, settled on a nomenclature that has survived through to the present day, and clearly delineated the boundaries of the various constellations. We accessed one of Argelander's original 1843 published catalogues for this study.

3 DATA AND ANALYSIS

Before we could use the data contained in the old catalogues we had to examine the characteristics of each, and apply corrections to the published magnitudes. This was especially relevant in the case of Ulugh Beg's catalogue, which some scholars believe was copied from al-Ṣūfī's earlier work (e.g. see Knobel, 1917). To the contrary, our investigations revealed that although he was possibly influenced by al-Ṣūfī's efforts, Ulugh Beg did not directly duplicate his catalogue. Consequently, we were able to demonstrate the independence of all seven early catalogues, and the consistency of their magnitude data (see Fujiwara et al., 2004).

In the old star catalogues, magnitude classes were recorded by numbers (1–6) and plus or minus signs which indicated 'a little brighter than' or 'a little dimmer than', respectively. To quantify these magnitude descriptions, we subtracted or added 0.33 according to the plus or minus sign respectively. For example, we assigned 2.67 for 3+, and 3.33 for 3–.

We also had to omit unsuitable stars. Firstly, there were bright stars in *Sky Catalogue 2000.0* with $V < 1$, such as α CMa (Sirius) or α Boo (Arcturus), because when the various old catalogues were recorded there was no concept of a zero or a minus magnitude. Secondly, we eliminated stars that we could not readily identify. For example, Bayer recorded the six stars, π^1, π^2, π^3, π^4, π^5 and π^6 Ori as just the one star, π Ori, so we were unable to assign individual magnitudes to each of the components. As for the constellation 'Argo', it was divided by Lacaille into the constellations of Puppis, Vela, and Carina during the eighteenth century, so we simply omitted them from our analysis. We also had to leave out double and multiple stars with separation distances >1′ (the limit of the resolving power of the naked eye) that were recorded as single objects. For example, the angular separation of α^1 Cap and α^2 Cap is 7′ so if a catalogue recorded it as a single α Cap it was rejected. For close double stars (with separations of <1′) we accepted the combined single magnitudes listed in the old catalogues, and compared these estimates with the combined *V* magnitudes. Finally, we omitted known variable stars where $\Delta m_V > 0.5$, namely, ε Aur, δ Cep, μ Cep, o Cet (Mira), χ Cyg, β Lyr, β Per (Algol), ζ Phe and λ Tau. For further details see Fujiwara et al., 2004.

After this culling process, we ended up with a sample of 2,124 naked-eye stars (see Table 1). In this table, the catalogues are listed in column 1, with the numbers referring to those given here in Section 1; the observational epoch (rather than the publication date) of each catalogue is shown in column 2; and the total number of stars (n) in each is listed in column 3.

Table 1: Catalogue, observational epoch and number of sampled stars.

Catalogue	Epoch (AD)	n
1	137	910
2	964	911
3	1437	889
4	1572	658
5	1603	949
6	1689	1003
7	1843	1946

Atmospheric extinction is an important subject, and needs to be taken into consideration. We computed the mean differences, $m - V$, and standard deviation, σ, for each star catalogue, binned by declination. The mean difference, $m - V$, corresponds to the possible extinction, and since it was always smaller than σ we decided not to make any corrections to the magnitude data in the seven catalogues.

We then proceeded to compare the stellar magnitudes in the seven old star catalogues with the Pogson magnitude system. First we investigated the magnitude data in the old catalogues on the basis of a logarithmic magnitude scale (Figure 2). In this figure, the magnitudes of the stars as recorded in each star catalogue are shown as dotted lines that are equivalent to the Pogson scale, whereas the solid lines indicate linear regressions. Then we plotted the same magnitude data on a power-law scale (Figure 3). In these plots, a 'pure' power-law function, i.e.

$$\log m = aV + b \iff$$

$$m = 10^{aV+b} = 10^b(10^V)^a = 10^b(I/I_0)^{-2.5a} \qquad (3)$$

is drawn as a straight line, whereas a 'shifted' power-law formula, such as the function suggested by Schulman and Cox (1997),

$$m = 5.5556(2.512^{(-0.5)(6-V)}) + 0.4444, \qquad (4)$$

is indicated by a curved line. Because a shifted power-law function proved inadequate, we chose to use a pure power-law function for our study. Details of these two power-law functions are given in Appendix 1.

4 RESULTS AND DISCUSSION

4.1 Logarithmic versus Power Law Scales

In this study, we compared the magnitude systems based on a logarithmic scale with that of a power-law scale. As shown in Figure 2, on a logarithmic scale the magnitude data recorded in old catalogues correspond exactly with the Pogson logarithmic scale (the dotted line). Our regression fits (the solid lines) for each catalogue are almost identical to the dotted lines.

On the other hand, Figure 3 indicates that the function suggested by Schulman and Cox (the dotted lines) in no way fits the magnitude data in the old star catalogues. Relative to the power-law regressions shown by the solid lines, at dimmer magnitudes (i.e. m = 3–6), the regressed functions do fit the magnitude data, but at brighter magnitudes (m = 1–3) they deviate notably. This indicates that the power-law regressions do not fit the data.

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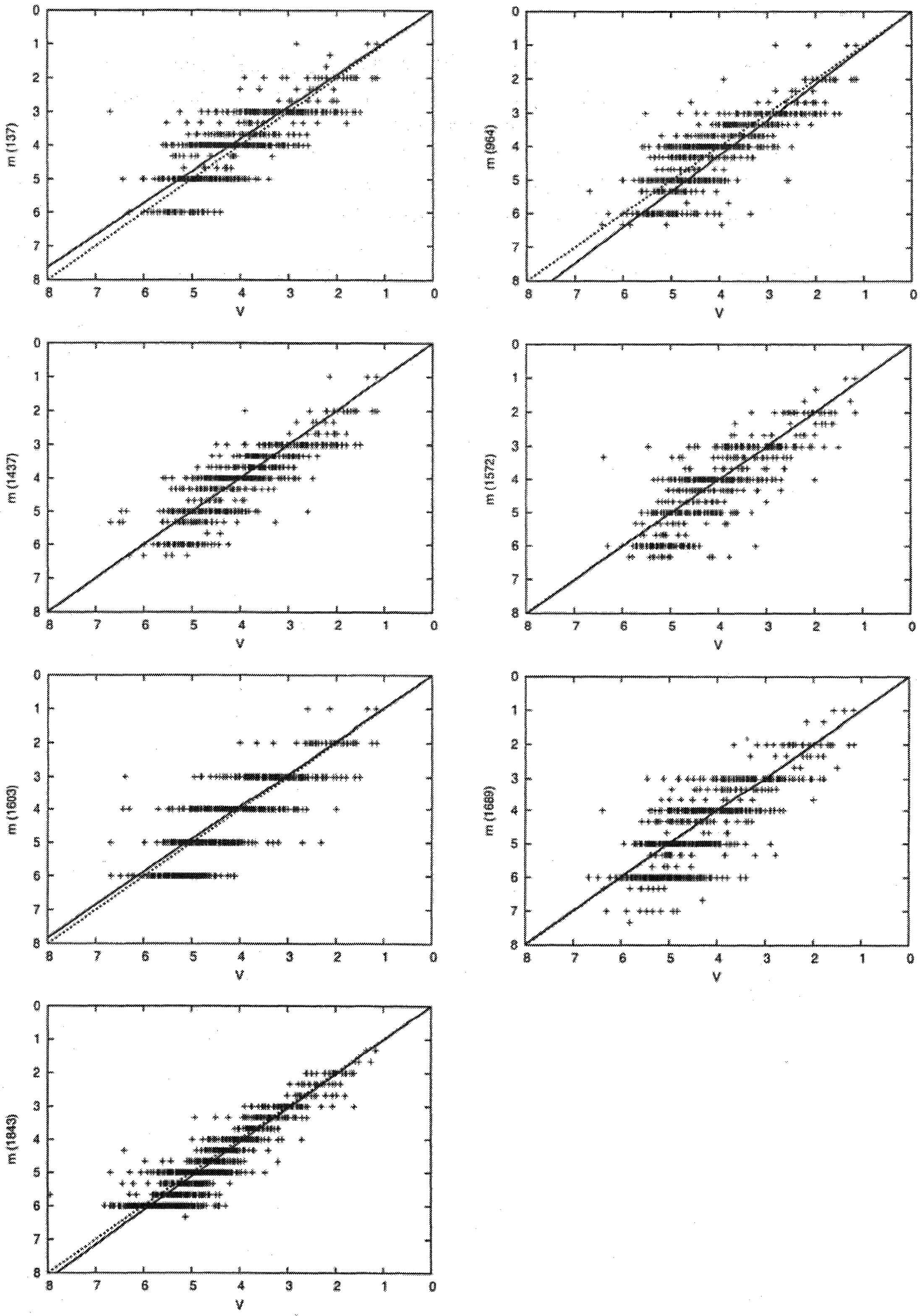

Figure 2: Magnitude systems on the logarithmic scale, where m(137) refers to the *Almagest* magnitudes, m(1437) to Ulugh Beg's catalogue, and so on, while V magnitudes are those listed in the *Star Catalogue 2000.0*. The dotted lines indicate Pogson's scale and the solid lines are the linear regressions.

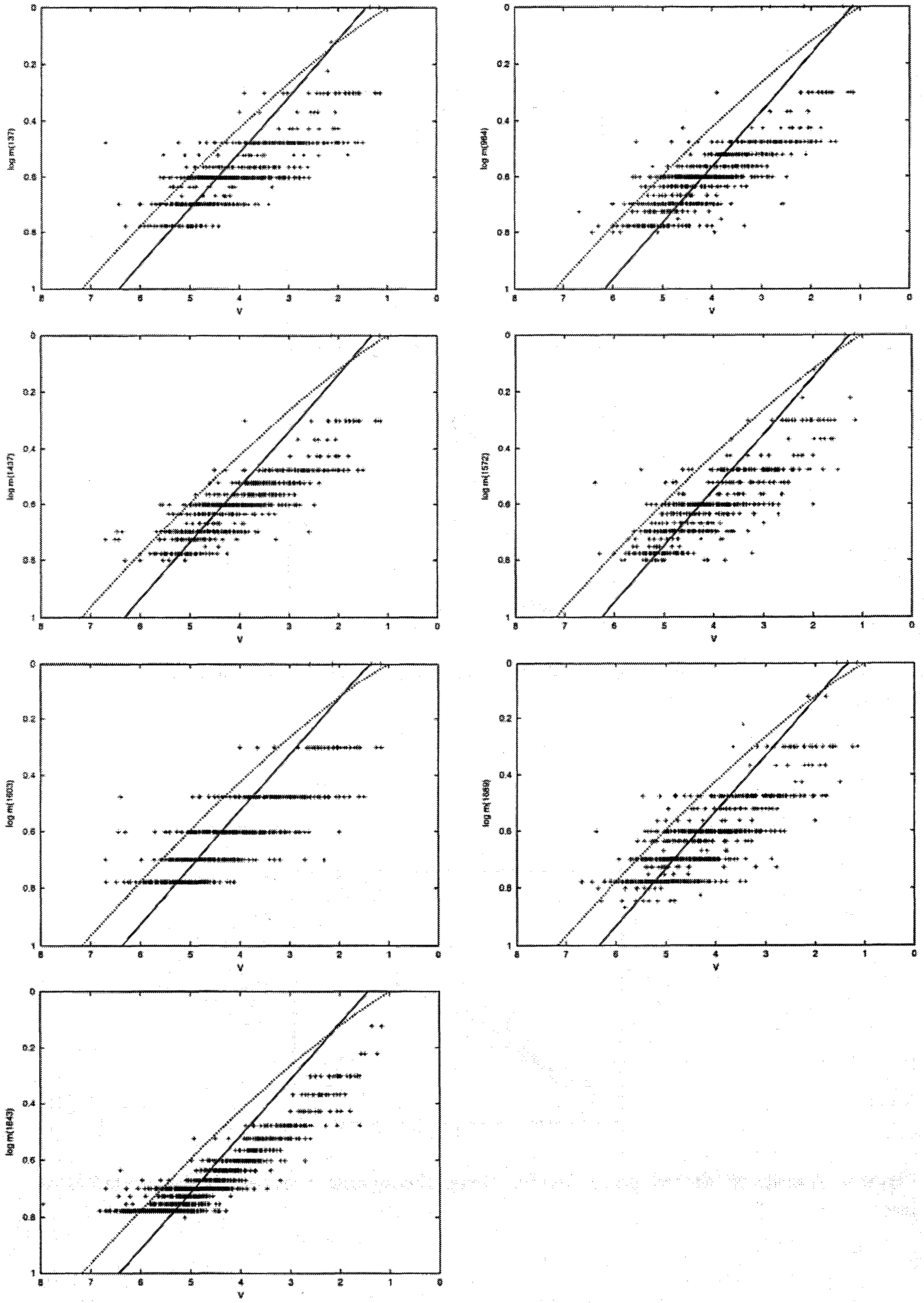

Figure 3: Magnitude systems on the power law scale. The dotted lines indicate the Schulman-Cox function and the solid lines are the power-law regressions.

In order to investigate which scale the magnitude data in the old star catalogues best fit, we carried out chi-squared (χ^2) tests. Table 2 contains the reduced chi-squares; column 1 lists the catalogues, while the reduced logarithmic and power-law chi-squares are shown in column 2 and 3 respectively. If a regressed function fits the data, the chi-squared value should be small. As shown in this Table, all of the logarithmic chi-squared values are small, indicating that the linear regressions are correct. In comparison, the power-law chi-squares values are large, which indicates that the power-law regressions do not fit the data. This indicates that the magnitudes in all of the old star catalogues do not follow a power-law but they do conform to a logarithmic scale.

Table 2: Reduced chi-square in old star catalogues on logarithmic and power-law scale.

Catalogue	χ^2 (logarithmic)	χ^2 (power-law)
1	0.3627	1.1396
2	0.2720	0.4851
3	0.2853	1.0845
4	0.3740	1.0179
5	0.4296	1.2828
6	0.3885	1.0976
7	0.1419	0.9286

4.2 Examination of the Light Ratio (*R*)

In Section 4.1, the magnitude systems were found to fit a logarithmic scale. Subsequently, we examined light ratios (*R*) of magnitude systems in the old star catalogues. In Table 3, we calculate *R* for each star catalogue using the linear regressions shown in Figure 2. Each *R* in the old star catalogues approximates Pogson's $R = 2.512$.

In calculating the value of *R* some data points had to be excluded, as discussed in Section 3. Firstly, values of *V* for stars recorded as 1st magnitude in the old star catalogues were reduced to fainter magnitudes, given that stars with $V < 1$ were omitted from our study. In addition, the average *V* of stars recorded as magnitude 6 was increased toward the brighter magnitude. Considering the present ranges of each magnitude, 3rd magnitude includes stars of 2.5-3.4 and 6th magnitude stars of 5.5-6.4. However, ancient observers defined the limit of naked eye visibility as magnitude 6, so their records rarely contain stars of $V > 6.0$. In the old star catalogues, only those stars with values of V = 5.5-6.0 magnitude stars were estimated as magnitude 6; observers recorded only the brightest stars in the range of the 6th magnitude. In the *Historia Coelestis Britannica*, Flamsteed (1689) recorded stars of the 7th magnitude, which he did not define at the time. The magnitudes of these stars were therefore considered to be imprecise, and they were not included in our study.

Table 3: The light ratio (*R*) in old star catalogues.

Catalogue	*R*
1	2.615
2	2.360
3	2.505
4	2.495
5	2.554
6	2.509
7	2.451

In Figure 4, we compare distributions of dispersion ($m_{137} - V$) in the *Almagest* for stars of 3rd and 6th magnitude. The distribution of 3rd magnitude stars is symmetry around $V = 3.0$, while the 6th magnitude curve is asymmetry and appears to be truncated at the fainter end of the distribution. Near the observable limit ($V = 6.0$), recorded and non-recorded stars should be represented on a 50:50 basis (the so-called 'Malmquist bias'). Consequently, there should be more stars of magnitude 6, and the peak in the distribution of 6th magnitude stars should be at 6.0. As shown in Figure 4, distributions of 6th and 3rd magnitude stars have a common shape for the brighter part ($\Delta m > 0$) of the histograms. On the other hand, for $\Delta m \leq 0$ the distribution of 6th magnitude stars is almost suppressed, and the peak is moved to the brighter part. Meanwhile, Table 4 indicates that the standard deviation (σ) of the 6th magnitude stars in the *Almagest* is much smaller than other values listed, which supports the view that some of the 6th magnitude stars are missing. This is probably due to the observable limit of the naked eye, and the 6th magnitude in the old star catalogues should correspond to the same class in Pogson's system. As well as 6th magnitude stars, the distribution of the 5th magnitude stars should also be truncated, but most of this distribution is within the range $V < 6.0$, and does not effect the results (for detailed values of σ see Table 3 in Fujiwara *et. al.*, 2004).

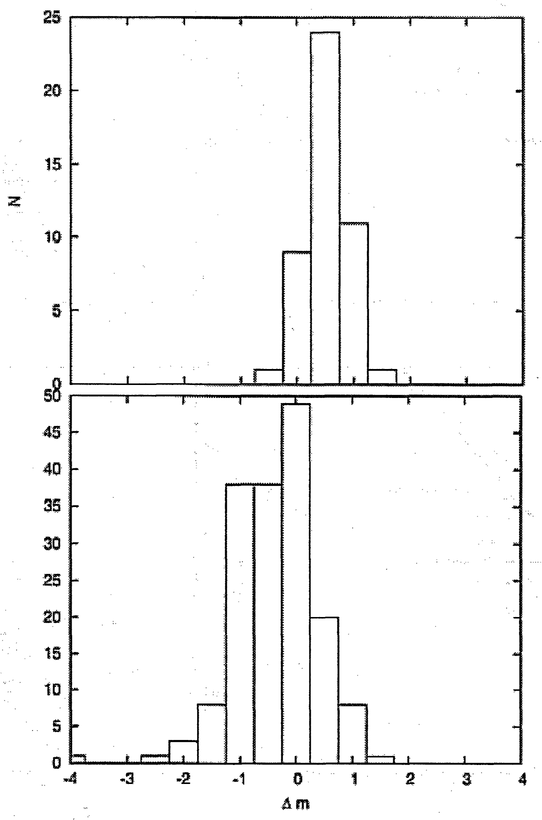

Figure 4: Differences in the distributions of dispersions between 6th magnitude stars (upper) and 3rd magnitude stars (lower).

Table 4: Standard deviation (σ) for each recorded magnitude in the *Almagest*.

Magnitude	σ
1st	0.91
2nd	0.66
3rd	0.73
4th	0.56
5th	0.51
6th	0.39

Relative to the magnitude system in the *Almagest*, Hearnshaw (1996) gave a logarithmic light ratio of R = 3.26, and later he revised this to R = 3.42 (Hearnshaw, 1999). Both values deviate from the Pogson system. Using all magnitudes recorded in the *Almagest*, we confirmed that R corresponds with Hearnshaw's findings. However, we suggest that the difference between Pogson's and Hearnshaw's values should be ascribed to marginal magnitudes, namely the brightest (1st magnitude) stars and the dimmest (6th and 7th magnitude) stars. In order to determine the real system of magnitude, we omitted the inaccurate 1st and 6th magnitude stars when investigating the linear regressions. As shown in Figure 2 and in Table 3, the magnitude systems in the old star catalogues, including the *Almagest*, do fit Pogson's scale of R = 2.512. We also note that the linear regressions in Figure 2 and values of R in Table 3 differ for stars in the al-Ṣūfī and Ulugh Beg catalogues, supporting the view that they represent from two quite independent observational datasets.

5 CONCLUSIONS

We show that there is agreement between the magnitude scales in the seven old star catalogues and the current system suggested by Pogson, and that the magnitudes in the old star catalogues fit a logarithmic scale. This is in spite of the suggestion by psychophysicists—based on visual sensitivity—that it should reflect a power law. Possibly our finding relates to the definition of the magnitudes in the old catalogues. As noted in Section 1, 1st, 2nd and 6th magnitudes were defined by Hipparchus. If the brightness of the stars of 1st magnitude in ~2.5 times greater than that of 2nd magnitude stars, and 100 times greater than stars of 6th magnitude, then the logarithmic function is a natural consequence. It is no surprise, then, that stars in the old catalogues do not follow a power law when their magnitudes are plotted against V magnitudes in the *Sky Catalogue 2000.0* (as in Figure 3).

It was also found that 6th magnitude stars in the old star catalogues exhibited a Malmquist bias, suggesting that stars in this magnitude range should not be used in determining the current magnitude system. Likewise, 1st magnitude stars also introduce a bias. When these two magnitude classes were omitted, the linear regressions presented here show that magnitudes of stars in the seven old catalogues are consistent with the light ratio of R = 2.512 suggested originally by Pogson.

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APPENDIX 1: A 'PURE' AND 'SHIFTED' POWER-LAW FUNCTION

A power-law response for visual sensitivity can be formulated in two ways. Firstly, if the zero point of the visual magnitude, m, is corresponding to the unit of the intensity, I, the relationship should be written as

$$m = BI^A \quad \Leftrightarrow \quad \log m = aV + b, \qquad (5)$$

where a is positive and A is negative. We call this a 'pure' power-law function, and it has two free parameters.

On the other hand, the zero point of m can be chosen arbitrarily. Thus,

$$m = BI^A + C \quad \Leftrightarrow \quad \log\,(m - c) = aV + b, \qquad (6)$$

also represents a power-law response, and we call this a 'shifted' power law. It has three free parameters, but if $m = 1$ at $V = 1$ and $m = 6$ at $V = 6$ are required, the freedom is reduced to one (i.e., the function is determined by a given a). The function of Schulman and Cox (SC) can be obtained from Equation (6) by setting a at 0.2.

The discussion in Schulman and Cox (1997) indicates that a should be positive, and they set the value at 0.2. As can be seen in Figure 3, however, the data can only be represented by a concave line in this plot, while the function of SC is a convex line. The boundary between a concave and a convex line is $a = 0.1556 = (\log 6)/5$, where c becomes zero (the pure power law, which in this Figure is represented by a straight line).

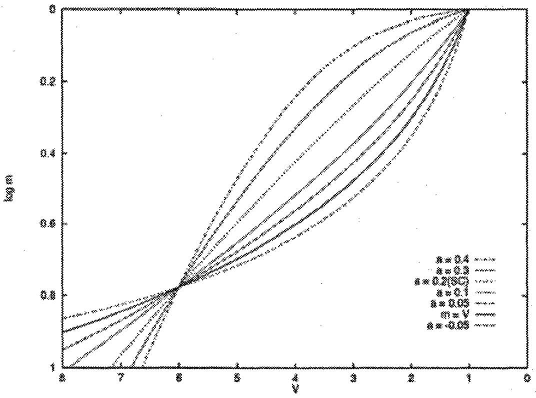

Figure 5: A series of 'shifted' power-law functions. The logarithmmic function (solid line) is also plotted.

Figure 5 shows a series of 'shifted' power-law functions with some values of a. When $a \to +0$ or $a \to -0$, the logarithmic function ($m = V$) is an asymptote. Both $a \to +0$ and $a \to -0$ in Equation 5, however, lead to a singular function (there is no solution for $a = 0$), and the best fit is not far from the logarithmic function (cf. Figure 2). Thus the 'shifted' power-law formula is inadequate as a fitting function. This is the primary reason we opted for a 'pure' power law in our study.

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