# VARIATIONS IN THE EARTH'S CLOCK ERROR ΔT BETWEEN AD 300 AND 800 AS DEDUCED FROM OBSERVATIONS OF SOLAR AND LUNAR ECLIPSES

# F. Richard Stephenson

Department of East Asian Studies, Durham University, Elvet Hill, Durham DH1 3TH, England; and Centre for Astronomy, James Cook University, Townsville, Queensland 4811, Australia.

E-mail: f.r.stephenson@durham.ac.uk

Abstract: Historical observations of solar and lunar eclipses provide the most effective method of tracing fluctuations in the Earth's rate of rotation in the pre-telescopic period. However, the temporal distribution of these data—some of which extend back to 700 BC—is far from uniform. Between AD 300 and 800, no more than about fifty usable observations are preserved. Analysis of these data enables variations in the Earth's clock error, ΔT, to be enumerated with tolerable precision during this interval. It is shown that departures from a parabolic fit are fairly small.

Keywords: China, ΔT, Earth rotation, eclipses, Europe

# 1 INTRODUCTION

As has been appreciated since the eighteenth century, the length of the day (LOD) is by no means constant. Variations in the LOD are produced by lunar and solar tides, as well as by several non-tidal mechanisms. These variations are relatively small, with amplitudes at the millisecond level. However, over many centuries their cumulative effect—the Earth's rotational clock error, usually known as  $\Delta T$ —may amount to several hours.  $\Delta T$  is defined as the difference between Terrestrial Time (TT: based on the motion of the Moon and planets) and Universal Time (UT: as measured by the Earth's rotation).

For most of the period since the advent of telescopic astronomy, timings of occultations of stars by the Moon are the most effective way of determining  $\Delta T$ . These observations enable the variations in this parameter to be charted in detail since around AD 1800 and with fair precision over the previous two centuries (Stephenson and Morrison, 1984). However, in the pre-telescopic period observations of eclipses have proved to be much more useful than occultations. The determination of  $\Delta T$  over the past few millennia is not only of considerable geophysical significance. Accurate knowledge of the Earth's clock error also enables improved computation of the circumstances of ancient eclipses and other astronomical phenomena, which can be of help in investigating the reliability and accuracy of early observations and also dating historical events.

Numerous observations of solar and lunar eclipses are recorded in ancient and medieval history: notably from Babylon, China, Europe and the Arab lands. Several hundred of these observations are sufficiently precise to enable variations in the Earth's rate of rotation to be investigated in detail. (For a comprehensive discussion see the author's monograph: Stephenson, 1997). Timings of solar and lunar eclipses yield individual results for  $\Delta T$ , while untimed observations of total or near-total solar eclipses enable limits to be assigned to the value of  $\Delta T$ . However, untimed reports of lunar eclipse are of little utility since the degree of obscuration of the Moon is independent of  $\Delta T$ .

Analysis of the various eclipse observations indicates that throughout the interval from 700 BC to AD

1600 the parameter  $\Delta T$  approximated to the following parabola:

$$\Delta T = 32t^2 \sec \tag{1}$$

Here t is measured in centuries from the standard reference epoch AD 1820. By fitting spline functions to the data, long-term variations about the mean parabola have been detected (e.g. Morrison and Stephenson, 2001; Stephenson and Morrison, 1995). However, the results of these investigations reveal that the amplitude of the variations did not exceed about 500 sec at any time during the period covered by the observations (see, also, Morrison and Stephenson, 2005; Stephenson, 2006).

Most of the extant eclipse observations originate from two distinct periods: from 700 BC to 50 BC and again from AD 800 to 1600. As a result, the form of the  $\Delta T$  variations during both of these periods is fairly well-defined. However, between about 50 BC and AD 800, there is a significant lacuna in the available data. This deficiency is particularly serious between 50 BC and AD 300—during which period scarcely any eclipse observations of real value in the study of Earth's past rotation are preserved. The subsequent interval of 500 years is more productive, but even over this period no more than about 50 useful records are extant. Hence in order to study the behaviour of  $\Delta T$  between AD 300 and 800 it is necessary to investigate each available observation with special care. These data are the subject of the present paper.

In our detailed studies of variations in the Earth's rate of rotation since 700 BC, Leslie Morrison and I included nearly forty eclipse observations made between AD 300 and 800 (e.g. Morrison and Stephenson, 2001; Stephenson and Morrison, 1995). The various data from this period which we investigated originated mainly from China, but a few observations from Europe were also included. In the current paper, I have reanalysed each of the individual observations which we used in our previous researches, reinterpreting some of the records. I have also included some hitherto unused data—also from China. During the selected period, there are no useful observations from Korea, while only a single (brief) report of a total solar eclipse from Japan (in AD 628) merits consideration.

For all eclipse computations, I have adopted a value for the lunar acceleration of -26.0 arcsec/cy<sup>2</sup>; this is very close to the results derived from lunar laser ranging (Chapront, Chapront-Touze and Francou, 2002; Williams and Dickey, 2003).

#### **2 EUROPEAN RECORDS**

Between AD 300 and 800, it would appear that usable observations of only two eclipses—both solar—were recorded in European history. In each case the place of observation is well-established and the date is secure.

#### 2.1 AD 364 June 16

This eclipse was observed by the astronomer Theon of Alexandria, who measured the times of the principal phases. Theon's observations are recorded in Book 6 of his Commentary on Ptolemy's Almagest, which was written around AD 370. In reporting the eclipse, Theon closely followed the style of the eclipse records cited by Ptolemy himself. Thus Theon quoted the year as the 1,112th year from the era of Nabonassar, which corresponds to AD 364; Theon also cited the date within that year as the 22nd day of the ancient Egyptian month Payni. The equivalent date on the Julian Calendar may thus be reduced to AD 364 June 16. Modern astronomical computations confirm that this eclipse date is exactly correct. The place of observation is certainly Alexandria (latitude = 31.22° N, longitude = 29.92° E), where Theon lived. The degree of obscuration of the Sun would be guite small at Alexandria; for example, using equation (1) the computed magnitude there would be only about 0.38. However, Theon does not give an estimate of the eclipse magnitude, concentrating instead on the measured times.

Fotheringham (1920) translated the relevant part of Theon's report as follows:

And moreover, we observed with the greatest certainty the time of beginning of contact, reckoned by civil and apparent time, as  $2\frac{5}{6}$  equinoctial hours after midday, and the time of the middle of the eclipse as  $3\frac{4}{5}$  hours, and the time of complete restoration as  $4\frac{1}{2}$  hours approximately after the said midday on the 22nd of Payni.

In his translation, based on the Basel edition of Theon —which was published in AD 1538—Fotheringham noted that the time of first contact was measured in equinoctial (= equal) hours. However, Rome (1950) remarks that the Basel edition is rather unreliable. He asserts that in the best manuscript of Theon's Commentary the term rendered 'equinoctial' (ismerinai) is absent. Rome further points out that Theon, in comparing his measurements of the times of the various phases with the calculated figures based on the tables in Ptolemy's Almagest, specifically used seasonal (emkairikai) hours. There are thus sound reasons for assuming that all three times measured by Theon were quoted in seasonal (i.e. unequal) hours—each equal to one-twelfth of the interval from sunrise to sunset. In my previous publications—both alone and jointly with Leslie Morrison—I was unaware of the Rome (1950) reference, and thus incorrectly assumed that Theon had used equal hours.

The interval from sunrise to sunset at Alexandria on the day of the eclipse may be calculated as 14.20 h; hence the length of each (daylight) seasonal hour would be 1.18 h. The local apparent times of the three phases at Alexandria may thus be deduced from Theon's measurements as 15.35 h, 16.50 h and 17.32 h respectively. Comparing the measured times with their computed equivalents, the three  $\Delta T$  results derived from Theon's individual measurements are respect-tively: 6,800 sec, 6,000 sec and 6,100 sec.

Theon also observed the total lunar eclipse of AD 364 November 26, but unfortunately he does not record any measurements in his *Commentary*. Instead he merely asserts that the observed times agree with calculation based on Ptolemy's tables. No other eclipse observations (solar or lunar) are recorded by Theon.

#### 2.2 AD 484 January 14

This eclipse, which was total on the Earth's surface, was observed to be extremely large at Athens. The event was recorded by Marinus Neapolitanus in his *Life of Proclus*. The eminent philosopher Proclus was Head of the Platonic Academy at Athens. On his death, which occurred on AD 485 April 17, his pupil Marinus succeeded him in this position. In describing the eclipse, Marinus—who may well have been an eyewitness—wrote as follows:

A year before his death there were various omens. There was an eclipse of the Sun which was so pronounced as to turn day into night and the darkness was deep enough for the stars to be visible; it occurred in the eastern horn of the sign of Capricorn. (Trans. Rosan, 1949: 34).

The only large eclipse visible at Athens—the sole feasible place of observation—for several years around this time took place on AD 484 January 14, and thus not much more than a year before Proclus died; at the time, the Sun (longitude = 295°) was indeed in the sign Capricorn (longitude range 270–300°). Hence there can be no doubt about the identity of the eclipse. Preliminary computation using equation (1) indicates that maximum phase occurred close to sunrise. At Athens in January the Sun remains hidden for more than half an hour after sunrise owing to the proximity of Mt Hymettus, so that only the later stages of a sunrise eclipse would be directly visible. This would account for the lack of reference to the degree of obscuration of the Sun at maximal phase.

It would appear from the description—"day into night" and "darkness deep enough for the stars (plural: *asteres*) to be visible"—that the eclipse was either total or on the verge of totality at Athens. This is confirmed by consideration of the visibility of planets and stars. Computation reveals that no planet or star brighter than magnitude 0 would be above the visible horizon around the time of greatest phase. Mercury (magnitude = +0.3, located 25° to the west of the Sun) would be well placed for observation. However, Venus—some 10° to the east of the Sun—would be hidden behind Mt Hymettus.

If the eclipse were fully total at Athens (latitude =  $37.98^{\circ}$  N, longitude =  $23.73^{\circ}$  E), a value of  $\Delta T$  between 4,500 sec and 5,450 sec would be required. For  $\Delta T$  within this range, the solar altitude at maximal phase would have been low: between  $0.3^{\circ}$  and  $3.6^{\circ}$ . If an eclipse of magnitude at least 0.99 is assumed, the derived  $\Delta T$  limits should be slightly widened to between 4,150 and 5,800 sec. It seems unlikely that a significantly smaller phase could have produced the observed effects.

# 3 CHINESE RECORDS OF SOLAR AND LUNAR ECLIPSES

At this period, the principal sources of Chinese eclipse records are the official dynastic histories. In these works, observations of eclipses—of both Sun and Moon—are mainly quoted in treatises devoted to a specific subject: usually monographs on astronomy/astrology (tianwen zhi), the calendar (luli zhi) or the 'five elements' (wuxing zhi). (N.B. the five elements were water, metal, fire, wood and earth). The eclipse reports in these treatises may be assumed to be based on the records of the court astronomers who made their observations from the imperial observatory at the capital. The records often give careful descriptions of eclipses, sometimes quoting technical information such as the R.A. of the Sun and an estimate of the magnitude and time of occurrence. Reports of solar eclipses are also regularly cited in the imperial annals (benji). However, these entries are often devoid of technical details: see Section 4 below.

#### 3.1 Chinese Timings of Solar Eclipses

The earliest record of an eclipse time in Chinese history dates from 134 BC. This was of a solar eclipse and the time of end was estimated to the nearest shi (double hour). Each of the twelve double hours was of equal length. The first shi, named zi, was centred on midnight; it covered the interval from 11 p.m. to 1 a.m. For a full list of the double hours see Table 1. In the centuries following 134 BC, occasional solar eclipse times were quoted to the nearest shi, but prior to AD 585 there are only two instances in which higher precision is used: in AD 193 and 493. The eclipse of AD 493 was said to begin at the start of the double hour wei (13-15 h). However, in AD 193-and on several dates from AD 585 onwards—solar eclipse times are expressed in both double hours and ke (marks). There were 100 marks in a full day and night; the ke was thus equal to 0.24 h. For a detailed discussion, see Stephenson (1997: 274ff) and Steele (2000: 178ff).

Between AD 300 and 800, four eclipses of the Sun were timed to the nearest mark, as cited in the official histories of the period. The Julian dates of these events are as follows: AD 585 July 31, 586 December 16, 594 July 23 and 761 August 5. The first three eclipses occurred during the Sui Dynasty and are reported in the calendar treatise (Chapter 17) of the Suishu. At this period, the Chinese capital was the then recentlyconstructed city of Daxing (now Xi'an: latitude =  $34.27^{\circ}$  N, longitude =  $108.90^{\circ}$  E). The eclipse of AD 761, which was total, took place during the subsequent Tang Dynasty. Careful timings of the various stages of this event are reported in the astronomical treatise (Chapter 36) of the Jiu Tangshu. At this date the capital was Chang'an, built on the site of the Sui capital of Daxing. Only four years prior to the eclipse (AD 757), this city had been restored as the seat of government after its capture by rebels in AD 755.

Previously I had rejected most of the above solar timings since I was under the impression that their interpretation was ambiguous. However, I have since benefitted considerably from discussions with Donald Starr of the Department of East Asian Studies, Durham University. The following revised translations owe much to his help:

- (i) AD 585 July 31: Kaihuang reign period, 5th year, 6th month, 30th day ... The Sun began to be eclipsed after 6 marks in the hour of *wu*; the loss began from the NW edge; it was ½ (eclipsed). Then after 1 mark in the hour of *wei* it began to reappear. At 5 marks (in the hour of *wei*) it was restored to fullness. (*Suishu*: 17).
- (ii) AD 586 December 16: Kaihuang reign period, 6th year,  $10^{th}$  month, 30th day, dingchou ... It was seen during the observations that when the Sun rose 1 zhang (roughly  $10^{\circ}$ ?) above the mountains, at 2 marks in the hour of chen, the eclipse began. The loss began from due west; it was  $\frac{2}{3}$  eclipsed. After 2 marks in the hour of chen it began to reappear. At the entrance to 3 marks in the hour of si it was restored to fullness. (Suishu: 17).
- (iii) AD 594 July 23: Kaihuang reign period, 14th year, 7th month, the first day of the month ... After the 3rd mark in the hour of *wei* the Sun began to be eclipsed. It began from the NW. The Sun was about half eclipsed; then it entered the clouds and was not seen. The eclipse was briefly seen again but it still had not reached fullness. Then it was immediately obscured by clouds. (*Suishu*: 17).
- (iv) AD 761 August 5: Shangyuan reign period, 2nd year, 7th month, day *guiwei*, the first day of the month. The Sun was eclipsed; the large stars were all seen. The Astronomer Royal, Chu Dan, reported: "On day *guiwei* the Sun diminished. Precisely (*zheng*) after 6 marks in the hour of *chen*, the loss began. Precisely after 1 mark in the hour of *si* it was total. At 1 mark before the hour of *wu* it was restored to fullness." (*Jiu Tangshu*: 36).

Table 1: The twelve double hours.

Double Hour	Local Time (h)
zi	23 - 01
chou	01 - 03
yin	03 - 05
mao	05 - 07
chen	07 - 09
si	09 - 11
wu	11 - 13
wei	13 - 15
shen	15 - 17
yu	17 - 19
xu	19 - 21
hai	21 - 23

For a fifth solar eclipse, observed in AD 768, only an *interval* of time is accurately recorded; the actual local time is only approximately cited, rendering the observation of no real value in the determination of  $\Delta T$ . Although I shall not discuss this eclipse further, I give below a translation of the text:

AD 768 March 23. Dali reign period, 3rd year, 3rd month, day *yisi*, the first day of the month. From the hour of *wu* it was deficient. After a further 1 mark (*zhi hou yi ke*) it was eclipsed to the extent of 6½ tenths. (*Jiu Tangshu*: 36).

As Needham *et al.* (1986, 199ff.) have emphasized, *ke* were time-intervals, rather than specific moments. Hence an observation said to be made 'at' a particular mark could have been made at any time between the beginning and end of that mark. Under these circumstances, when comparing observation with computation it seems most appropriate to take the midpoint of the relevant interval. However, when an observation was said to occur either 'at the entrance to' (*ru*) or 'after' (*hou*) a particular mark I have assumed the beginning or end of that interval.

The report in AD 586 evidently contains a mistake in citing either the time of first contact or maximal phase since the two times are almost identical (respectively "at 2 marks in the hour of *chen*" and "after 2 marks in the hour of *chen*"). Computation indicates that sunrise at Chang'an on December 16 (assuming a level horizon) would occur at a local time of 7.07 h. near the start of the (double) hour of *chen*. Hence, since the angular equivalent of the linear unit *zhang* can only be crudely estimated, the recorded time for first contact of "2 marks in the hour of *chen*" would seem reasonable. This suggests that the recorded time of maximal phase may be significantly in error. However, it seems best to retain both measurements in subsequent analysis.

Table 2:  $\Delta T$  values derived from Chinese solar eclipse times recorded in a primary source.

Year	Contact	Double Hour	Mark	Meas LT (h)	ΔT (sec)
493	1	wei	(begin)	13.3	5,200
585 585 585	1 M 4	wu wei wei	6+ 1+ 5	12.68 13.48 14.32	10,100 10,400 10,400
586 586 586	1 M 4	chen chen si	2 2+ 3–	7.60 7.72 9.72	3,700 6,900 4,900
594	1	wei	3+	13.96	6,300
761 761 761	1 M 4	chen si wu	6+ 1+ –1	8.68 9.48 10.96	2,300 3,600 3,300

Although the eclipse of AD 761 was recorded as total, the computed semi-duration of the total phase on the central line was only about 2 minutes. Hence in the determination of  $\Delta T$  from the timing of the onset of totality I have neglected this very short interval. Two of the measurements for this eclipse use the term *zheng* ("precisely"), implying that they were regarded as particularly accurate.

In addition to the above measurements, a solar eclipse timing noted in astronomical treatise (Chapter 12) of the *Nanqishu* also merits investigation. The account of the eclipse of AD 493 January 4 in this treatise relates that "... it was not until the start of the hour *wei* that it was seen that the Sun began to be eclipsed". The capital of the time was Jiankang (now Nanjing: latitude =  $32.03^{\circ}$  N, longitude =  $118.78^{\circ}$  E). The double hour *wei* extended from 13 h to 15 h. Assuming that the observation was made midway through the first third of the double hour *wei*, and thus at approximately 13.3 h, a value for  $\Delta T$  of around 5,200 sec may be deduced.

Table 3:  $\Delta T$  values derived from Chinese solar eclipse times recorded in a secondary source: the Yuanshi.

Year	Contact	Double Hour	Mark	Meas LT (h)	ΔT (sec)
690	М	oi	1	10.08	2.500
680	IVI	SÍ	4	10.08	2,500
700	M	shen	(begin)	15.3	3,500
702	M	shen	3	15.84	2,300
721	M	wu	3+	11.96	2,300

The significant details of the selected timed observations between AD 493 and 761, together with the results of analysis, are summarised in Table 2. In this table I have listed column by column: the year of observation; contact (1 = beginning, M = maximal

phase, 4 = end); double hour; number of marks within the double hour; equivalent measured local time; and the computed value of  $\Delta T$ . In column 4, a minus sign or plus sign following the appropriate mark indicates an observation at the start or end of that mark. For 4th contact in AD 761, the minus sign *before* the mark number indicates that the observation was said to be made 1 mark before the hour cited.

Additional observations are to be found in the Shoushili, the calendar treatise (Chapter 53) of the Yuanshi—the official history of the Yuan Dynasty (AD 1261 to 1367). In this treatise, many solar and lunar eclipse times are compiled—some from earlier dynasties; the objective was to test the accuracy of existing eclipse tables. Seven observations of solar eclipses prior to AD 1000 are recorded in this work. These are all from the Tang Dynasty. The Julian dates are: AD 680 November 27, 681 November 16, 691 May 4, 700 May 23, 702 September 26, 707 July 4 and 721 September 26. Presumably, each observation was made at the Tang capital of Chang'an. However, although each eclipse is also reported in the astronomical treatises of both the Jiu Tangshu and the Xin Tangshu, these works make no mention of any measurements of eclipse times on these dates. Hence corroborative information is lacking.

The *Yuanshi* treatise notes only the time of maximal phase in each case. For instance, the report of the solar eclipse of AD 680 November 27 may be translated as follows:

Tang Dynasty. Yonglong reign period, first year, gengchen, 11th month, day renshen, the first day of the month. (The Sun) was eclipsed. At 4 marks in the hour of si, it reached its maximum. (Yuanshi: 53).

For the eclipses of AD 681 and 707, only very approximate times—to the nearest double hour—are quoted in the Yuanshi. Hence these two records will not be considered further. The report in AD 700 notes the time as the beginning of the hour shen; in this case (as for the eclipse of AD 493 discussed above) I have estimated the time as 0.3 h after the start of the double hour. On the four remaining dates AD 680, 691, 702 and 721, times are given to the nearest mark. Preliminary computations using equation (1) indicate that in AD 691 the eclipse would reach its maximum nearly half an hour before the Sun rose at the Chinese capital of Chang'an. Hence it would be impossible to estimate the time of maximal phase; the recorded time presumably relates to the greatest visible phase—an indeterminate moment. I have thus rejected the obser-

Investigation of the four selected eclipses (AD 680, 700, 702 and 721) is summarised in Table 3. In each case the Tang capital was Chang'an. Data tabulated are as follows: year of observation; contact ( $M = \max$  phase); double hour; number of marks within the double hour; equivalent measured local time; and the result derived for  $\Delta T$ .

# 3.2 Chinese Timings of Lunar Eclipses

Reports in Chinese history of fairly precise timings of lunar eclipses—to the nearest 20 or 30 minutes—commence in AD 434. Between this date and AD 596 several times are measured to the nearest fifth of a night watch. Unfortunately, after AD 596, no further lunar eclipse timings are preserved until after AD 900.

By then it had become customary to quote measurements in double hours and marks, as for solar eclipses.

Unlike marks, night watches were not of fixed length but varied with the seasons. The night from dusk to dawn—specifically lasting from 2.5 marks (or 0.6 hours) after sunset to 2.5 marks before sunrise—was subdivided into five equal watches (geng). Each of these units was in turn divided into five equal intervals, variously termed chang (calls), chou (rods), or tian (points) in different dynasties. The length of one of these subdivisions varied from about 20 minutes in summer to 30 minutes in winter. As in the case of marks, "calls" (or their equivalent units) were time-intervals. Hence I have assumed that each observation was made at the mid-point of the appropriate time-interval.

The precise Julian dates of the various eclipses are as follows: AD 434 September 5, 437 January 8, 437 July 3, 440 October 26 (all reported in Chapter 12 of the *Songshu*); 543 May 5 (*Yuanshi*, Chapter 53); and 585 January 21, 592 August 28, 593 August 18, 595 December 22 and 596 December 11 (all cited in Chapter 17 of the *Suishu*). The place of observation for the earlier observations—up to and including AD 543—was Jiankang, the capital of the Song and Liang Dynasties. However, the later observations were all made at the Sui capital of Daxing.

As an example, I have selected the account of the eclipse of AD 434 September 5. This was predicted to occur around sunrise (when the new astronomical day began), but was actually observed towards the end of the previous calendar day:

Yuanjia reign period, 11th year, 7th month, 16th day, full Moon. The Moon was eclipsed. The calculated time was the (double) hour of *mao* (5–7 a.m.). On the 15th day, at the second call of the fourth watch, at the start of the hour of *chou* (soon after 1 a.m.) the eclipse began. At the 4th call the eclipse was total ... (*Songshu*: 12).

Two of the lunar observations require special comment. In AD 595, it was reported that the eclipse began after (hou) the fourth division of the first watch. On this occasion, I have assumed the end of the fourth division, rather than the middle. Further, in AD 596 it is stated that in the first division of the third watch the Moon was seen through clouds to be already  $\frac{3}{15}$  eclipsed. Since the computed interval between 1st and 2nd contact was 1.08 h, and the Moon passed almost centrally through the Earth's shadow, I have assumed that true first contact would have occurred 0.22 h before the observed LT of 22.97 h and thus around 22.75 h.

In Table 4, the following details are listed, column by column, for each individual measurement: year of occurrence; contact (1 = beginning, 2 = start of totality, 4 = end of eclipse, M = maximal phase of a partial eclipse); number of watch; number of division; measured local time (LT) and result for  $\Delta T$  in seconds.

# 4 UNTIMED CHINESE RECORDS OF LARGE SOLAR ECLIPSES

Several total or near-total eclipses are recorded in Chinese history during the interval from AD 300 to 800. An observation of a *total* eclipse can be used to establish limits to the value of  $\Delta T$  at that date without the need for measurements of time. Alternatively, a

careful observation of a large *partial* eclipse may rule out a discrete range of  $\Delta T$  values. The standard term to describe a total eclipse is ji. This is an archaic expression, use of which can be traced back as far as the 8th century BC in China. Some large partial eclipses are carefully described using the term *bujin rugou* ("not complete and like a hook"). A less definitive expressions is "almost complete" (ji jin).

Table 4:  $\Delta T$  values derived from Chinese timings of lunar eclipses.

Year	СТ	Watch	Div	LT (h)	ΔT (sec)
434	1	IV	2	1.61	1,700
434	2	IV	4	2.42	2,800
437a	2	1	3	18.95	7,500
437b	1	II	4	21.97	6,000
437b	2	III	1	22.99	6,500
440	1	II	1	20.71	6,200
440	M	II	3	21.65	8,100
543	1	III	3	0.00	4,600
585	1		1	18.01	5,900
585	M		4	19.51	5,800
585	4		1	20.51	7,100
592	1	1	3	20.07	5,800
593	1	V	1	3.03	4,700
595	1		4+	19.60	5,400
595	M		3	21.41	4,000
595	4		1	22.96	3,500
596	1	III	1–	22.75	4,600
596	4	IV	3	2.58	4,800

## 4.1 Total Solar Eclipses

In the various official histories of China, accounts of eclipses of the Sun which allege totality come into two main categories. Most accounts merely note that an eclipse was total (ji), without supplementary description. However, occasional reports provide additional supporting evidence such as the visibility of several stars. Before proceeding further, it seems important to consider the reliability of the very brief observations in the first of these categories.

Between AD 300 and 800, as many as seven eclipses are recorded simply as total (ii), without any qualifying details. Computation reveals that two of these events, occurring on AD 522 June 10 and 756 October 28, were indeed total on the Earth's surface. However, the remaining five were only annular. The Julian dates of these latter eclipses, together with the computed magnitudes in the zone of annularity (figures which are independent of  $\Delta T$ ) are as follows: AD 306 July 27 (0.94), 360 August 28 (0.93), 516 April 16 (0.94), 562 October 14 (0.98) and AD 616 May 21 (0.97). Central annular eclipses, in which the Sun is reduced to a bright ring, are rare at any one place, on average occurring once every 220 years or so (Stephenson, 1997: 54). Hence the probability that five eclipses were annular at the appropriate Chinese capital in a period of about 300 years is rather low.

Annular eclipses are by no means as spectacular as their total counterparts. Hence although accounts of total solar eclipses occur frequently in early literature, descriptions of the ring phase are extremely rare. Throughout Chinese history there appears to be no evidence to suggest that *ji* ("total") was ever used as a technical term to indicate a central annular eclipse as well as one that was fully total. On the contrary, an account of the annular eclipse of AD 1292 January 21 in the astronomical treatise (Chapter 48) of the *Yuanshi* clearly describes the ring phase ("the Sun's body was like a golden ring"). Furthermore, this same text specifically asserts that the eclipse "was not able to be total (*bu neng ji*)". Investigation of two of the seven allegedly total eclipses listed above (the *annular* eclipses of AD 360 and 562) is also helpful in assessing the reliability of brief reports of totality.

In all, three separate Chinese accounts of the eclipse of AD 360 are preserved. The imperial annals (Chapter 8) of the *Jinshu* allege totality (using the term *ji*), but make no further comment. However, the corresponding account in the five element treatise of the *Songshu* (Chapter 34: section on Jin eclipse records) declares that the eclipse was partial—i.e. "not complete and like a hook". The astronomical treatise (Chapter 12) of the *Jinshu* confirms that the eclipse was "almost complete". The capital of the time, where the observation evidently took place, was Luoyang.

Referring now to the annular eclipse of AD 562, computation shows that the magnitude cannot have attained more than 0.50 at the capital of Jiankang (latitude 32.0° N) for any value of  $\Delta T$ . This eclipse would only be central far to the south; the track of annularity could never have reached further north than latitude 16° N. It seems plausible to infer that this report represents a prediction rather than an observation. Serious attempts at eclipse prediction in China can be traced at least to the Later Han Dynasty (AD 23–220).

The above considerations suggest that the reliability of eclipse records which merely mention the term ji ("total") without confirmatory details is questionable. I shall therefore disregard such reports in the determination of  $\Delta T$ . This remark will also apply to the only Japanese report of a total solar eclipse during the selected period. Occurring on a Julian date equivalent to AD 628 April 10, this is recorded in the Nihon Shoki simply as "complete" (jin), without any further description. Further, the place of observation is doubtful: possibly Asakusa (latitude = 34.50° N, longitude = 135.83° E) or perhaps Dazaifu (latitude = 33.52° N, longitude =  $130.53^{\circ}$  E). In this context, it should be stressed that accounts of celestial phenomena in the various East Asian histories are often no more than brief summaries of the original reports, possibly with the loss of important information. It seems plausible that if an editor lacked the necessary astronomical knowledge he may simply have recorded any unusually large eclipse mentioned in his sources as total.

Fortunately, detailed descriptions of two total solar eclipses observed in China are preserved in the period under discussion: AD 454 August 10 and 761 August 5. In both instances, because of the reported visibility of several stars, we can be much more confident that the extant record of totality is reliable. In this section I have thus restricted my attention to deriving  $\Delta T$  limits from these two observations.

### 4.1.1 The Total Solar Eclipse of AD 454 August 10

The five element treatise in the *Songshu* notes the visibility of various stars by day at this total eclipse:

Xiaojian reign period, 1st year, 7th month, day *bingxu*, the first day of the month. The Sun was eclipsed; it was total. The constellations (*xiu*) were brightly lit. (*Song-shu*: 34).

Although the year and lunar month are both correct in the above record, the cyclical day is in error; in place of bingxu it is necessary to read bingshen, which was, in fact, the first day of the 7th lunar month. Evidently a minor scribal error is responsible. Brief reports of this same eclipse are contained in the imperial annals of both the Songshu (Chapter 6) and Nanshi (Chapter 2). Each source gives the correct date as "7th month, day bingshen, the first day of the month." When this date is converted to the Julian calendar (i.e. AD 454 August 10), it agrees exactly with the computed date of a total solar eclipse visible in China.

The astronomical treatise of the *Songshu* also describes a total solar eclipse which it assigns to the previous year:

Yuanjia reign period, 30th year, 7th month, day *xinchou*, the Sun was eclipsed; it was total and all the stars were seen. (*Songshu*: 34).

This entry is not found in the annals of either the *Songshu* or *Nanshi*. The above date corresponds to AD 453 August 20: precisely twelve lunar months (i.e. one lunar year) before the eclipse of AD 454 August 10. However, there was no eclipse on (or near) that day. Since the only total solar eclipse visible in China for several years around this time occurred in AD 454, it would appear that the compilers of the *Songshi* treatise must have mistakenly filed two separate reports of the same event one year apart.

The assertion of totality in both entries in the astronomical treatise is affirmed by the visibility of many stars. It may further be presumed that in AD 454 totality was witnessed at the capital of the time: Jiankang. Although this is not directly stated in the records, no other place of observation is specified. By comparison, in reporting the solar eclipse of AD 429 December 12, the same treatise of the *Songshu* (Chapter 34), although noting that the eclipse was "not complete and like a hook", added that "... in Hebei province (in northern China) the Earth was in darkness". Evidently the description of the partial phase in AD 429 represents the observation at Jiankang.

Somewhat unusually, the configuration of the eclipse track in AD 454 over the Earth's surface was such that two discrete ranges of  $\Delta T$  are indicated by the one observation of totality at Jiankang: values of  $\Delta T$  either between 6,150 and 7,900 sec or between 50 and 1,800 sec would satisfy the observation. The former limits would apply if the eclipse occurred in the early morning (around 8.00 a.m.) the latter in the late morning (around 10.30 a.m.). Unfortunately, the time of day is not recorded, so that both alternatives need to considered further (see Section 5).

### 4.1.2 The Total Solar Eclipse of AD 761 August 5

The measured times of this eclipse, as reported in the *tianwen zhi* of the *Jiu Tangshu*, have already been discussed in Section 3.1. However, it seems appropriate to repeat the translation here:

Shangyuan reign period, 2nd year, 7th month, day guiwei, the first day of the month. The Sun was

eclipsed; the large stars were all seen. The Astronomer Royal, Chu Dan, reported: "On day *guiwei* the Sun diminished. Precisely after 6 marks in the hour of *chen*, the loss began. Precisely after 1 mark in the hour of *si* it was total. At 1 mark before the hour of *wu* it was restored to fullness." (*Jiu Tangshu*: 36).

A further account of the same eclipse is to be found in the *tianwen zhi* of the *Xin Tangshu*:

Shangyuan reign period, 2nd year, 7th month, day *guiwei*, the first day of the month. The Sun was eclipsed; it was total. The large stars were all seen. It was 4 deg in *Zhang* (lunar lodge). (*Xin Tangshu*: 32).

Both the incidence of totality and the visibility of "all the large stars" are also briefly reported in the annals (Chapter 10) of the *Jiu Tangshu*. There can thus be no reasonable doubt that the total phase was witnessed at the Tang capital of Chang'an. Computation shows that the eclipse would only be total at Chang'an for values of  $\Delta T$  between 1,700 and 3,250 sec.

# 4.2 Very Large Partial Solar Eclipses

Between AD 350 and 800, five eclipses were clearly described as partial, in each case using the expression "not complete and like a hook". The Julian dates of these events are as follows: AD 360 August 28; 429 December 12; 702 September 26; 729 October 27; and 754 June 25. On the first two dates, the Chinese capital was Jiankang; on the remaining dates the seat of government was Chang'an.

As noted previously (Section 4.1), the first of these eclipses was annular; all the rest were total on the Earth's surface. The statement in the *Jin* annals that the eclipse of AD 360 was "total" is contradicted in the treatises of both the *Songshu* and *Jinshu*. However, there are no conflicting accounts of the partial phase eclipse of AD 429 (also reported in the *Songshu*) or the three later eclipses of AD 702, 729 and 754 (all recorded in Tang history).

Prior to the Tang Dynasty (which commenced in AD 618), the various imperial annals never describe partial solar eclipses; if the phase was not said to be total, only the mere occurrence of an eclipse is mentioned. However, the Tang annals give detailed descriptions of three partial eclipses: occurring in AD 702, 729 and 754.

The annals (Chapter 6) of the *Jiu Tangshu* record the eclipse of AD 702 in unusual detail: "It was not complete and like a hook; it was seen at the capital and in the four directions." Similar accounts of a partial phase (i.e. "not complete and like a hook") are to be found in the *Jiu Tangshu* annals (Chapters 6 and 9) in both AD 729 and 754, but without any comment on the place of observation. The astronomical treatise of the *Xin Tangshu* (Chapter 32) confirms that each of the three eclipses was partial: "almost total" in both AD 702 and 754; and "not complete and like a hook" in AD 729. In each case an estimate of the right ascension of the Sun is quoted to the nearest degree, indicative of an observation by the official astronomers at Chang'an.

According to the *Songshu* treatise (Chapter 34), a star (or stars) was observed during the partial eclipse in AD 429. Computation reveals that on this occasion the track of totality on the terrestrial surface did not extend

further south than latitude  $35.0^\circ$  N. Thus on any value of  $\Delta T$  the umbral shadow would pass to the north of the Song capital of Jiankang (latitude =  $32.0^\circ$ ). Similarly, in AD 754, the zone of totality did not extend further north than  $30.0^\circ$  N, and thus significantly to the south of Chang'an (latitude =  $34.3^\circ$ ). Therefore both eclipses would be no more than partial at the respective Chinese capitals, regardless of the value of  $\Delta T$ . All that can be deduced here is that the report of a partial eclipse in the appropriate official history is confirmed in each case. Computation indicates that the magnitude at Jiankang in AD 429 could never have exceeded 0.94; that at Chang'an in AD 754 would not be greater than 0.86.

Hence there now remain only three partial eclipses which can be used to set viable limits to  $\Delta T$ : AD 360, 702 and 729. For the first of these events, any value of  $\Delta T$  less than 7,100 sec or greater than 9,400 sec would produce a partial eclipse at Jiankang. However, all intermediate results (i.e. between 7,100 and 9,400 sec) would render the eclipse annular there, and are thus invalid. Similarly, in AD 702  $\Delta T$  could be either less than 1,450 sec or greater than 2,750 sec; results between 1,450 and 2,750 sec are excluded since they would lead to a total eclipse at Chang'an. Finally, in AD 729, only values of  $\Delta T$  less than 400 sec or greater than 1,200 sec can achieve a partial eclipse at Chang'an.

It is unfortunate that none of the records indicates whether the upper or lower part of the Sun was covered during the eclipse. Such details would have led to only a single range of acceptable  $\Delta T$  values; however, information of this type tends to be extremely rare throughout ancient or medieval history.

# **5 DISCUSSION OF RESULTS**

The  $\Delta T$  values and limits derived above are depicted in Figure 1. In this diagram, results obtained from solar eclipse timings are shown by open circles, those from lunar eclipse timings by shaded circles. Heavy vertical lines terminated at each end by short horizontal bars denote limits to  $\Delta T$  determined from total solar eclipses. Broken vertical lines, each terminated at one end by a horizontal bar and at the other end by an arrow head (the latter implying that they extend beyond the upper or lower edge of the diagram), indicate the set of values of  $\Delta T$  which ensure a partial eclipse. The mean long-term parabola,  $\Delta T = 32\,t^2$  sec, is also shown for comparison.

In Figure 1, the various  $\Delta T$  values include more than ten results derived from previously unused timings—mainly of solar eclipses. Over the brief period from AD 585 to AD 596, seventeen separate results obtained from solar and lunar timings (as tabulated in Tables 2 and 4) are displayed in the diagram. These observations are all derived from the same source: the calendar treatise (Chapter 17) of the *Suishu*. Over the relatively brief interval of 12 years which the observations cover, it is probable that the timings would be made by much the same group of court astronomers at the Sui capital of Daxing.

In general, both the solar and lunar timings between AD 585 and 596 provide a reasonably self-consistent set of  $\Delta T$  results. However—as reference to Table 2 shows—there are three obvious exceptions: in each case observations of the solar eclipse of AD 585. The

three reported times of this event all lead to very high figures for  $\Delta T$  of between 10,100 and 10,400 sec; these are well above the region covered by Figure 1. Clearly the scatter of the remaining fourteen  $\Delta T$  values is rather large (emphasizing the low precision of measurement). However, applying equal weights, these data yield a useful mean result for  $\Delta T$  of 5,200  $\pm$  300 sec at the epoch AD 590. This solution, shown by a short dotted line in Figure 1, is in close accord with the figure deduced from Equation (1) at this date: i.e. 4,850 sec.

A plausible explanation of the three anomalous observations of the solar eclipse of AD 585 is a series of scribal errors. Making the simple assumption that each recorded measurement is exactly 1 double hour too early leads to results for  $\Delta T$  in AD 585 of 4,400, 5,000 and 5,300 sec. These figures are in close accord with those derived from the remaining solar and lunar data in the *Suishu* calendar treatise.

The dates of the nineteen eclipse timings preserved from other periods between AD 300 and 800 (including the three European measurements in AD 364) are much more widespread. Hence the observations can give no more than a general indication of the variation of  $\Delta T$  over this interval. As is apparent from Figure 1, most of the  $\Delta T$  values derived from these data roughly follow the long-term parabolic trend. However, the two results from AD 434 are particularly discordant: these are based on measurements of separate stages of the same lunar eclipse (first and second contact). Presumably scribal errors are responsible. It is curious that the most discordant measurements in each of the two Chinese groups of data discussed above are the very earliest: respective dates of AD 585 and 434.

Limits to  $\Delta T$  as derived from observations of six solar eclipses (three total and three partial) are displayed in Figure 1. For each of the total eclipses of AD 484 and 761, only one set of values of  $\Delta T$  can satisfy the observation. However, for the total solar eclipse of AD 454 and the partial eclipses of AD 360, 702 and 729, there are two separate regions of solution space. Denoted (a) and (b) for each date, these are separated by wide exclusion zones of width between about 1,000 and 4,000 sec. It is thus necessary to decide between each of the four pairs of alternatives (a) and (b). For AD 360, only solution (a) is shown in Figure 1; the alternative—AD 360(b) (greater than 9,400 sec)—lies well above the region covered by the diagram.

Selection between AD 454(a) and (b) and between AD 729(a) and (b) presents few problems. In each case, effective comparison can be made with a unique set of limits obtained from an observation of totality only a few decades later (respectively in AD 484 and 761). Choice of the alternative limits AD 454(a) and AD 729(a) would necessitate two separate sharp rises in  $\Delta T$ —each by at least 1,500 sec in only about 30 years. Comparison with the results obtained from several hundred eclipse observations between 700 BC and 50 BC and from AD 800 to 1600 reveals that such rapid changes are unprecedented. As noted earlier (Section 1), at no time during these two periods do the extensive sets of data reveal any evidence of departure from the mean long-term parabola by more than about 500 sec. Further, in these same two lengthy intervals there is no suggestion of any rise in  $\Delta T$ , only slow variations in the rate of decline (Morrison and Stephenson, 2001; Stephenson and Morrison, 1995).

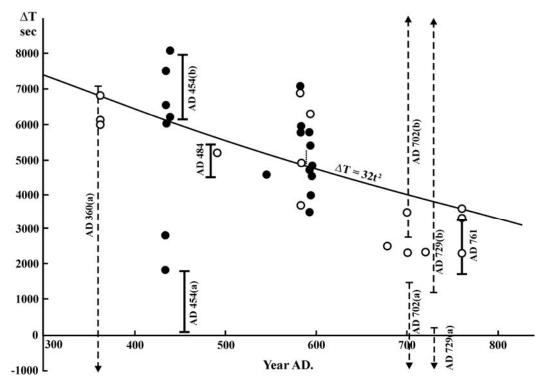


Figure 1: ΔT values and limits derived from solar and lunar eclipse observations between AD 300 and 800.

Hence, rather than invoking major variations in  $\Delta T$  between AD 300 and 800—which would require complex geophysical explanations—there are sound reasons for rejecting AD 454(a) and AD 729(a) and selecting the alternatives AD 454(b) and AD 729(b). The solution AD 454(b) is in slight discord with that for AD 484, but if a magnitude of 0.99 (rather than full totality) at Athens in AD 484 is accepted—as discussed in Section 3.1—the upper limit to  $\Delta T$  at that date would be 5,800 sec. This revised limit would be in very close agreement with the AD 454 lower limit.

Acceptance of AD 702(a) rather than AD 702 (b) would only require a minor rise in  $\Delta T$  between this date and AD 761. Nevertheless, the fact that AD 702(a) lies about 2500 sec below the long-term parabola is probably sufficient reason for rejecting this solution and preferring AD 702(b) instead. Similarly, solution AD 360(b)—which is beyond the upper edge of Figure 1—lies roughly 2,500 sec above the mean parabola and hence may also be rejected.

Figure 2 is copied from Figure 1 but with the discarded (a) solutions—for AD 454, 702 and 729—removed for clarity. It is apparent that the limits set by the five eclipses of AD 360 (partial), 454 (total), 484 (total), 702 (partial) and 761 (total)—together with the mean solution based on most of the timings at Jiankang around AD 590—define a fairly narrow region of  $\Delta T$  deviating by no more than about 500 sec from the mean long-term parabola. This result is in accord with the variations noted in the centuries prior to AD 300 and after AD 800.

In summary, the various eclipse observations between AD 300 and 800—both timed and untimed—indicate a gradual decline in  $\Delta T$ , fairly close to the long-term trend.

Kawabata *et al.* (2004) focused their attention on reports of only three solar eclipses—AD 616 (an annular eclipse, but recorded as total in China), AD 628 (total in Japan), and AD 702 (nearly total in China). From their analysis of these observations (together with a reported occultation of Mars by the almost full Moon in AD 681), Kawabata *et al.* proposed a sharp reduction in ΔT in the 7th century AD to between about 2,700 and 3,000 sec. This would require a marked increase in the moment of inertia of the Earth. According to Kawabata *et al.*, a possible explanation is a general rise in sea-level due to an increase in global temperature.

However, as noted above (Section 4.1), the records in both AD 616 and 628 are of questionable reliability; in particular, they merely mention that the eclipse was total without giving any further details. Further it would be difficult for the unaided eye to decide whether a close approach of the bright lunar disc to Mars was an occultation or merely an appulse. The present analysis of a much larger body of data does not support the conclusions of Kawabata *et al.* It is surely important to consider all of the available eclipse data, rather than selecting a few specific examples.

#### 6 CONCLUSION

Over the five centuries covered by the present investigation, eclipse observations reveal that variations in  $\Delta T$  about the mean long-term parabola  $32t^2$  were fairly small, probably at no time exceeding 500 sec. In particular, there is no need to assume any major variations in the Earth's rate of rotation throughout the period covered by the diagram. Hence for historical researches on eclipses and other celestial phenomena throughout the whole of the period from 700 BC to AD 1600, use of Equation (1) is probably satisfactory for most purposes.

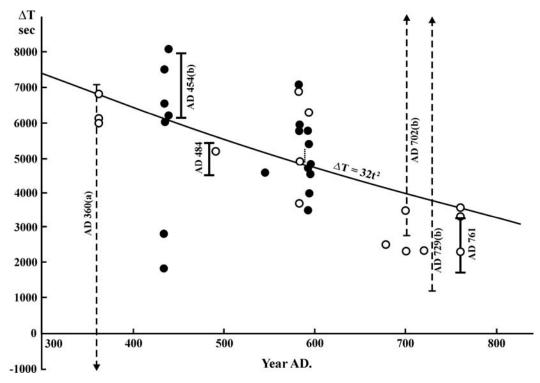


Figure 2: Edited version of Figure 1, with highly discordant sets of limits removed.

#### 7 NOTES

1. It should be mentioned that there are two official histories of the Tang Dynasty. These are the *Jiu Tangshu* (*Old History of the Tang*), completed in AD 945, and the *Xin Tangshu* (*New History of the Tang*), completed in AD 1060.

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  - F. Richard Stephenson is Emeritus Professor at Durham University, a Leverhulme Emeritus Fellow, and an Adjunct Professor in the Centre for Astronomy at James Cook University, Townsville, Australia. His research interests relate primarily to Applied Historical Astronomy, and he has published numerous papers and books, including Historical Eclipses and Earth's Rotation (1997) and Historical Supernovae and their Remnants (2002, co-authored by David Green). Richard is a former President of IAU Commission 41 and he is an Advisory Editor for the Journal for the History of Astronomy and is on the Editorial Board of the Journal of Astronomical History and Heritage.