

THE HYDROGEN ABUNDANCE IN STARS: A FIRST MAJOR STEP FOR QUANTITATIVE ASTROPHYSICS

Daive Cenadelli

Istituto di Fisica Generale Applicata, Università degli Studi di Milano, via Brera 28,
20121 Milano, Italy
davide.cenadelli@unimi.it

Abstract: Historiography has recognized that Saha's work in the early 1920s was the beginning of a quantitative era in astrophysics, and the deduction of the large hydrogen abundance in stars around 1930 was a major outcome of Saha's theory. In this paper, the development of stellar physics in these years is analysed, and the recognition of the hydrogen abundance is pointed out as the first major achievement of the quantitative era. This idea is sustained from two different points of view. First, there exists a tight scientific continuity from Saha's investigative papers up to Russell's 1929 paper where the hydrogen abundance was clearly worked out: the whole of the 1920s should therefore be considered as a scientific discontinuity that paved the way for modern stellar spectroscopy. Second, in 1932 the same conclusion was reached by Strömberg and Eddington, who were working on the problem of internal stellar structure. Thus, the hydrogen abundance can be viewed as the first major step of the quantitative era, as it led to the first sound theory of stellar structure, both for the inner and the surface regions of stars.

Keywords: stellar spectroscopy, stellar composition, Saha-Fowler equation, Eddington 'standard model'

1 INTRODUCTION

In the year 1835 the French philosopher, Auguste Comte, speaking of celestial bodies, wrote:

We conceive the possibility to determine their shapes, distances, sizes and movements; whereas we shall never be able to study by any means their chemical composition, or their mineralogical structure ... our positive knowledge about the celestial bodies is necessarily limited to their geometric and mechanical phenomena alone, without being able to pursue the other physical and chemical researches ... which require them to be accessible to all our different observation methods. (Comte, 1835: 8-9; our English translation).



Figure 1: Megh Nad Saha, 1893–1956 (courtesy Wikipedia).

This often-cited quotation is very popular among astrophysicists as it points out what the state of the art in

astronomy was at that time. Comte's opinion, which was readily shared by contemporary astronomers, was to be thoroughly dismissed within a few decades as astrophysics emerged and the physical structure of stars began to be investigated. Nevertheless, it took a long time before any firm knowledge about the chemical composition of stars—that Comte had explicitly cited—could be arrived at.

In fact, no sound knowledge in that respect could be reached before a theory of matter at the atomic level was available. The transition between the so-called 'qualitative' and 'quantitative' eras had to occur. The division into these two eras was suggested in an important paper by DeVorkin and Kenat (1983a). As these authors note, this idea was taken from a paper D.H. Menzel published in 1972 in the *Annals of the New York Academy of Sciences*. That paper was split in two parts, each of which dealt with one of the two eras: "The history of astronomical spectroscopy I - qualitative chemical analysis and radial velocities" and "The history of astronomical spectroscopy II - quantitative chemical analysis and the structure of the solar atmosphere". The watershed between them is the first application, starting from 1920 on, of atomic physics to the spectroscopic observations of stars. It was performed through the identification of the dependence upon temperature and gas pressure of the ionization and excitation of atoms, and the subsequent physical interpretation of the Harvard spectral sequence. That happened to be an event of the highest scientific importance, since such an interpretation had been awaited for a long time, implicitly ever since the first formulations of spectral classification some sixty years earlier.

The important achievements we are dealing with were gained by an entire community of astrophysicists, but we can recognise two special names among them: Megh Nad Saha (Figure 1) and Henry Norris Russell (Figure 2). The former was the man who first described the ionization of atoms in terms of gas temperature and pressure. The latter, in his turn, was greatly concerned with the problem of the physical interpretation of stellar spectra, after he had worked out the colour-magnitude diagram that in the 1930s was given his name (along with that of Hertzsprung). If we had to

identify the beginning of the new era in one exact moment, we could do no better than to cite Saha's 1920 paper "Ionisation in the solar chromosphere", that contains his famous formula. At the same time that Saha, Russell and others were investigating the fruitful outcomes of the application of atomic theory to spectroscopy, Arthur Stanley Eddington was attacking the problem of stellar internal structure. Introducing into his stellar structure model such fundamental concepts as radiation pressure, the absorption coefficient and the mean molecular weight, he worked out his 'standard model'.

In this paper I want to discuss in detail the historical and scientific aspects of the contributions given by these scholars, pointing out in which way the two fields of investigation—Saha and Russell and their stellar surfaces, and Eddington and his stellar interiors—were due to meet in the years around 1930 when the prevalence of hydrogen in stellar composition was figured out.

2 THE SEARCH FOR A PHYSICAL INTERPRETATION OF THE HARVARD SPECTRAL SEQUENCE

The famous spectral sequence devised at Harvard University during five decades was worked out as an empirical task (Hearnshaw, 1986: 104-142). By the time the Harvard astronomers began to work on it, and in particular from 1901 onwards when Annie J. Cannon devised the familiar sequence O, B, A, F, G, K and M, nobody knew how to interpret physically the occurrence of a one-dimensional sequence, in which colour was strictly related to the visible spectral features.

Some light on that matter was cast at the beginning of the twentieth century, when stellar temperatures began to be measured on the basis of Planck's Law (Hearnshaw, 1986: 219-222). In the years 1905-1909, J. Wilsing and J. Scheiner at Potsdam visually measured colour temperatures of 109 stars, establishing in that way that colour was indeed a temperature-related parameter. The hottest star in their sample turned out to be λ Ori at 12,800 K, the coldest ones μ Gem and κ Ser at 2,800 K. The values were affected by large systematic errors, especially for hot stars.¹ As C.G. Abbot noticed, these errors were allegedly due to the fact that the contribution of dark lines had not been taken into account. In fact, line blocking and subsequent deviations from the black body curve are very severe in the blue region of stellar spectra, where hot stars mainly radiate. Further work was carried out at Potsdam by Wilsing and W.H.J. Münch upon another sample of 90 stars, but they still underestimated values for hot stars. By the same time, at Paris Observatory C. Nordmann visually assessed color indices for fourteen stars, observing them through red and blue filters. Then he derived temperatures from comparisons with Planck's curves.

Another way to tackle the problem was through photographic photometry. K. Schwarzschild was the leading pioneer in establishing these techniques. Essentially, in the years around 1900 he laid down the basic concepts and paved the way for the determination of colour indices that was performed by A. Hnatek in 1911. Hnatek tried to avoid the problems due to the greater sensitivity in the blue and the non-linear

response of photographic plates by exposing calibration plates as well, and using these to reduce the stellar spectra that he recorded. He measured the temperatures of seven stars relative to Altair for which he adopted Wilsing and Scheiner's value of 7,100 K (500 K lower than the correct figure, which today is estimated to be around 7,600 K). H. Rosenberg exploited Schwarzschild's techniques, too. He took images of spectra for a wide sample of stars and derived their temperatures. He obtained reliable values for colder stars but too high ones for hot stars, especially in comparison to those of Wilsing and Scheiner, which, in their turn, were underestimated. The differences in some cases were astonishing: up to 10,000 K! The fact that the temperatures of hot stars are very difficult to deal with was not clear in those early days. In conclusion, we may say that by the mid-1910s the spectral sequence was generally thought to be a temperature sequence, although temperature measures were subject to large systematic errors. Nevertheless, the way in which spectral features were related to colour, and thus to temperature, was poorly understood. In other words, although everybody in the astrophysical community thought that temperature had to play a major role in the production of spectral lines, nobody knew exactly how this should happen.



Figure 2: Henry Norris Russell, 1877–1957 (courtesy Yerkes Observatory).

It is noteworthy that in the years around 1890, J. Norman Lockyer (Figure 3), an English amateur astronomer who devoted his spare time to astronomical spectroscopy, had a remarkable intuition on that matter. He observed that the spectrum of a given element shows different lines if it is heated up at different temperatures. He then surmised that as the temperature increases elements are split into smaller components that he called 'proto-elements', which were responsible for the so-called 'enhanced lines'. He wrote:

I call the latter [lines obtained at higher temperature that Lockyer had previously referred to] "*proto-metallic*" lines, and consider the substances which produce them, obtained at the highest available laboratory temperatures, "*proto-metals*", that is, a finer form of the metal ... (Lockyer, 1900: 57).

He further noticed:

We have then to face the fact that on the dissociation hypothesis ... the metals which exist at the temperature of the arc [i.e. at lower temperatures] are broken up into finer forms, which I have termed protometals, [that are responsible for the] enhanced spectrum ... (Lockyer, 1900: 81).



Figure 3: J. Norman Lockyer, 1836–1920 (after *Proceedings of the Royal Society*, 1909).

It is easy to see in Lockyer's proto-elements an anticipation of the concept of ionized elements. Lockyer's work on stellar spectra was the basis upon which he devised a theory of stellar evolution. This theory was based upon the so-called 'meteoritic hypothesis' (Lockyer, 1887; 1888), according to which all heavenly bodies were formed by meteor swarms that collided and then grouped together, driven by gravity. They first formed nebulae, then young, low-temperature stars that afterwards contracted and heated up. Finally the contraction stopped and the stars cooled down again. Lockyer thought he could describe this process by means of a colour-changing pattern, of the kind red → yellow → blue → yellow → red. This scheme in which a star passes twice throughout the spectral sequence was devised almost twenty years before Russell worked out a similar evolutionary process based upon the H-R Diagram (although today such evolutionary schemes are totally discarded). Lockyer's work involved remarkable insight, but it was quite odd at the same time. Russell's contemporary, H. MacPherson (1920: 226), noticed that in Lockyer's theory "... truth and error seem to have been strangely intermixed." while Hearnshaw (1986: 93) points out that "... his work on the enhanced lines illustrated Lockyer's unusual scientific insight, in spite of his unorthodoxy." In any case, apart from such intuitions, towards 1920 knowledge was still lacking, and the occurrence of spectral lines throughout the spectral sequence remained unexplained. Astronomers began to get frustrated about this. As E. Arthur Milne

(1924: 95) observed some years later: "There appeared to be a definite relation between effective temperature and type of spectrum ... but the connection was empirical. There was a gap in the logical argument."

If there was but one scholar longing for a theoretical explanation, that person was Russell. He had made major contributions to the field of stellar spectroscopy devising the colour-brightness (absolute magnitude) diagram, and his interests extended from stellar evolution to the determination of stellar masses in multiple systems. In the papers that he published in the years before 1920, that lack of knowledge is rarely stressed (and sometimes even a slight sense of defeat seems to emerge from Russell's words). For example, in 1919, just before learning of Saha's work, Russell wrote:

There is now good reason to believe that the differences between the main classes of spectra arise from differences in the effective surface temperature of the stars, and that differences in their other physical characteristics play only a minor rôle in the spectra, but reveal themselves in differences in detail, formerly described as "peculiarities" when they were noticed at all. The investigation of these finer differences is to-day the most promising field in stellar spectroscopy. (Russell, 1919: 395).

In 1921, after Saha's work had been published and Russell had immediately realised its importance, he published in the *Publications of the Astronomical Society of the Pacific* the paper "The properties of matter as illustrated by the stars", that consisted of an historical synopsis of stellar physics up to that time. In it, the development of spectroscopy from its beginning was surveyed and great emphasis was placed on recent achievements: about 6 pages (of the 16) were devoted to a detailed discussion of atomic properties and their relation to spectra. An acknowledgment to Saha was explicitly given.

In 1922 Russell stressed once more the importance of Saha's contribution:

The principles of ionization theory will evidently be of great importance throughout the whole world of astrophysics, and Dr. Saha has made an application of the highest interest to the question of the physical meaning of the sequence of stellar spectra ...

The possibilities of the new method appear to be very great. To utilize it fully, years of work will be required to study the behavior of the elements ... in the stars, in laboratory spectra, and by the direct measurement of ionisation; but the prospect of increase of our knowledge, both of atoms and of stars, as a result of such researches, makes it urgently desirable that they should be carried out. (Russell, 1922: 143-144).

3 THERMAL IONIZATION AND EXCITATION

As we have seen, the long-awaited explanation came from 1920 onwards as the newborn Bohr-Sommerfeld theory of the atom met astrophysics, and that happened at first thanks to the work of a mathematically-skilled, Indian physicist, who was deeply interested in the developments that the theory of the atom was undergoing in Europe. He was Megh Nad Saha.

In 1920, Saha (1920a: 479) devised his famous formula

$$\log\left(\frac{x^2}{1-x^2}P\right) = -\frac{U}{KT} + \frac{5}{2}\log T + \log\frac{(2\pi m)^{3/2}K^{5/2}}{h^3} \quad (1)$$

where x is ratio of ionized to total number of atoms, P the gas pressure, U the ionization potential, T the absolute temperature, m the mass of the electron, K is Boltzmann's constant and h is Planck's constant.

The formula had been obtained to describe ionization as a function of T and P in a gas constituted of only one element in local thermodynamic equilibrium. In devising this formula, Saha drew on the process of chemical dissociation presented by J. Eggert (1919), and extended this idea to the atomic realm (meaning ionization being analogous to dissociation). In fact, there was not really a sound basis that it could rely on. In 1923 Ralph H. Fowler, a mathematician who came to astrophysics after studying Emden's equation and who was greatly interested in mathematical physics, devised it on the ground of considerations in statistical mechanics. Fowler (1923: 21) found that

$$\log\left(\frac{x^2}{1-x^2}P\right) = -\frac{U}{KT} + \frac{5}{2}\log T + \log\frac{(2\pi m)^{3/2}K^{5/2}}{h^3} - \log b(T) \quad (2)$$

This is the very same equation derived by Saha, the only difference being that a term $-\log b(T)$ appears, where $b(T)$ is the partition function and is usually of the order of unity (thus $\log b(T) \approx 0$).

But Fowler's contribution was not the first reference to Saha's work, as Russell had already mentioned it in 1922, when the American astrophysicist noticed that:

If atoms of several different kinds, all capable of ionization, are present, the situation is somewhat more complicated. To use [Saha's] equation, introducing for P the value of the partial pressure of the vapor of each element separately, is inadmissible, since one of the products of the reaction - free electrons - is produced by all the ionizations. (Russell, 1922: 121).

Russell (ibid.) then went on to generalize Saha's equation in the case where different elements were simultaneously present. If we call a_1, a_2, \dots the numbers of atoms of different kinds; x_1, x_2, \dots the ratios of ionized to total atoms for the different elements; x^* the ratio of ionized to total number of atoms of all kinds ($x^* = \sum a_i x_i / \sum a_i$); then for the generic element Saha's equation becomes:

$$\frac{x_i}{1-x_i} \frac{x^*}{1+x^*} = \frac{\Omega_i}{P} \quad (3)$$

where

$$\log \Omega_i = -\frac{5036U_i}{T} + 2.5\log T - 6.5 \quad (4)$$

where U_i is the ionization potential of the considered element (and in Equation (4) the numerical values of the constants have been inserted).

Russell also studied the occurrence of further ionization states such as doubly-ionized atoms. He concluded that usually for any element "... there will not simultaneously be any noticeable proportion of atoms in all three states of ionization." (Russell, 1922: 125). A comparison with relative intensities of lines in the solar spectrum and in the spectra of sunspots confirmed the theory.

In 1923 Fowler, after his aforementioned contribution, returned to the topic and with E. Arthur Milne published the paper "The intensities of absorption lines

in stellar spectra, and the temperatures and pressures in the reversing layers of the stars". The two scholars realized that they had to consider also the thermal dependence of atomic excitation, which relies on Boltzmann's distribution and which Saha had not taken into account. It is a fundamental feature if we consider that absorption lines of the optical series of elements such as H and He originate from excited levels:

It is easy to calculate from Saha's theory as it stands the condition for the maximum intensity of lines like the H and K lines of calcium ... As the temperature of Ca vapour increases the concentration of Ca^+ atoms steadily increases until (at a point where the proportion of neutral atoms is very small) second-stage ionisation sets in and the concentration of Ca^+ atoms diminishes.

But Saha's theory has not hitherto accounted quantitatively for the maxima of such lines as the Balmer lines of hydrogen ... Before an H atom can absorb a Balmer line the electron must be lifted into a 2-quantum orbit ... Saha pointed out that as the temperature increases the fraction of atoms in the higher quantum states will increase, but stated that he could give no definite calculation. [We want to] point out that with the aid of the general theory of assemblies of atoms, electrons and radiation in statistical equilibrium, it is possible to determine the fraction of excited atoms under given conditions of temperature and pressure and to use this to discuss the intensities of lines such as those of the Balmer lines. (Fowler and Milne, 1923: 404-405).

Fowler and Milne started from an equation similar to Equation (2):

$$\log\left(\frac{x^2}{1-x^2}P_e\right) = -\frac{U}{KT} + \frac{5}{2}\log T + \log\frac{(2\pi m)^{3/2}K^{5/2}}{h^3} \sigma - \log b(T) \quad (5)$$

This differs from Fowler's earlier equation in two respects:

- 1) P is replaced by the pressure of the electrons alone P_e , and Fowler and Milne (1923: 407n) acknowledge Russell's 1922 paper for this; and
- 2) The term σ is introduced, which is "... the number of valency electrons in the atom in equivalent orbits." (Fowler and Milne, 1923: 407). It was inserted as "... any one of the σ equivalent electrons may be removed in ionisation. In our applications $\sigma = 1$ or 2." (ibid, footnote).

From that starting point, they succeeded in incorporating the excited levels of an atom into an equation, describing them as a function of T . They called n_r the number of neutral atoms of a given element in the excitation state r , q_r the statistical weight, χ_r the energy of that state and χ_1 the energy of the ground state, and devised the formula:

$$n_r = \frac{q_r e^{-(\chi_1 - \chi_r)/KT}}{b(T) + aT^{5/2} e^{-\chi_1/KT}} \quad (6)$$

where the term a is given by:

$$a = \frac{(2\pi m)^{3/2} K^{5/2} \sigma}{h^3 P_e} \quad (7)$$

Fowler and Milne were able to calculate as a function of temperature and electronic pressure the percentage of ionized and excited atoms, i.e. the percentage of atoms in the proper conditions to absorb any set of spectral lines. The two scholars succeeded in estimating the electronic pressure to be about 10^{-4} atm and could thus plot the number of atoms

capable of absorbing any set of spectral lines as a function of temperature (see Figure 4). This pressure estimate was made supposing that the Balmer lines should reach a maximum at 10,000 K and calculating for which pressure it actually happened. They further observed that such a value for pressure was suitable also for other elements. In the case of lines absorbed from excited states (like the Balmer series) the relation plotted in Figure 4 is Equation (6); for other series, formulae derived from Saha are used. In Figure 4, a scale of the temperature for the different spectral classes is deduced. It is obtained by assigning to the class where a certain set of lines shows a maximum of intensity, the temperature for which the absorption of those lines turns out to be the largest (for the given pressure). This method was called the method of *intensity maxima*. It can be applied to all lines, except those absorbed from the ground state of a neutral atom (that do not show an intensity maximum at any temperature). It allowed Fowler and Milne to calibrate the absolute temperature scale for spectral classes, and compare it with that devised from spectrophotometric measures. The two scholars noticed that a good agreement was reached, although they thought their temperature scale was only provisional, mainly because of uncertainty surrounding the pressure value: "... indeed, if a value $P_e = 10^{-4}$ atmos. can be assumed on other grounds, the temperature scale to which we are led is independent of any adjustment." (Fowler and Milne, 1923: 421).

In conclusion, by exploiting Saha's earlier work and utilizing the Boltzmann distribution, Fowler and Milne succeeded in describing the ionization and excitation states of atoms as a function of temperature and electron pressure. Their achievements are best summarised by the following formula:

$$N_{j,k}/N = f(T, P_e) \tag{8}$$

where $N_{j,k}$ is the number of atoms in the generic state

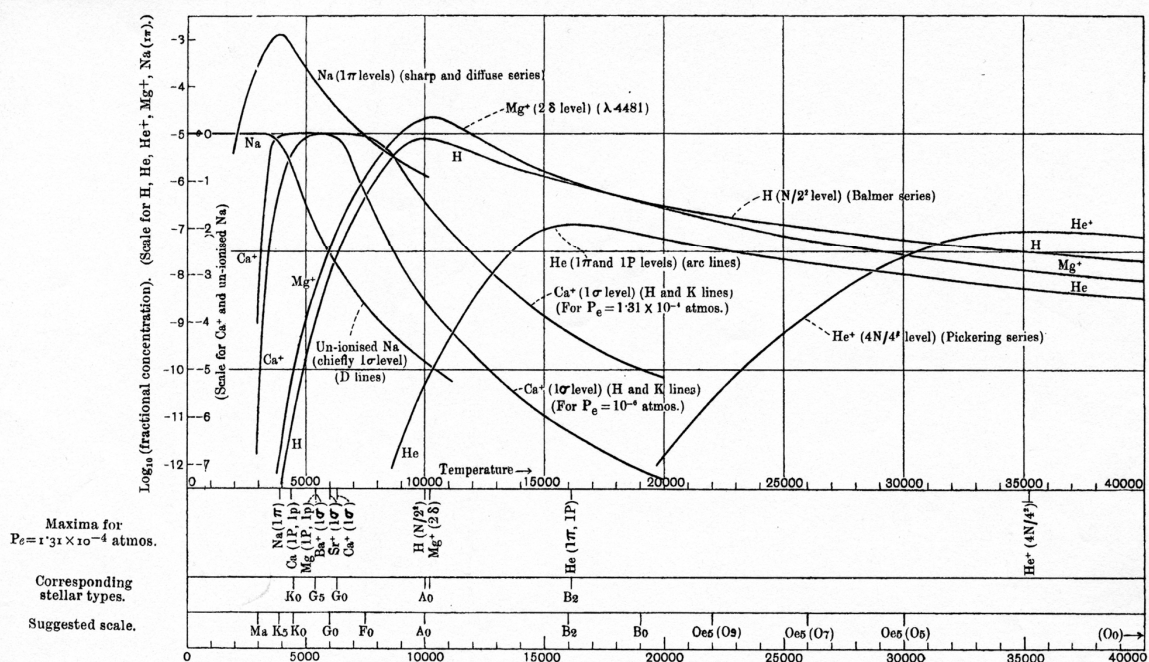
of ionization j and excitation k , N the total number of atoms of that kind and $f(T, P_e)$ a proper function of T and P_e .

4 THE FIRST DETERMINATION OF THE CHEMICAL COMPOSITION OF STELLAR ATMOSPHERES

Fowler and Milne (1923) realized how their work could pave the way for the determination of the chemical composition of stellar atmospheres. In fact, from Equation (8) we have:

$$N_{j,k} = N \cdot (N_{j,k}/N) = N \cdot f(T, P_e) \tag{9}$$

So, if we can estimate the absolute number of absorbing atoms $N_{j,k}$ that contribute to create a certain line, we can work out the relative abundance of the relevant element through their (and Saha's) function $f(T, P_e)$. In other words, well-pronounced lines far from the optimum T and P_e conditions mean high abundance, and conversely, on the contrary, weak lines near the optimum conditions mean low abundance. Notice that the transition rates should be taken into consideration, as Fowler and Milne explicitly stated. If we try to estimate $N_{j,k}$ at the intensity maximum of a certain set of lines, we can encounter problems due to line saturation, and thus it may be very difficult to evaluate $N_{j,k}$ without a theory of line-formation and line-broadening. In other words, we cannot calibrate the line intensity against the absolute number of absorbing atoms. It was some years later, after further progress, that this problem would be faced (see below in this Section). Fowler and Milne used a more feasible way, by trying to determine at which point in the spectral sequence a set of lines makes its first appearance (i.e. a so-called marginal appearance). They argued that if the absolute number of atoms needed to create a weak, just detectable, line could be determined, then formula (9) could be used to work out the abundance, N , of that element.



(Note.—The partial electron pressure $P_e = 1.31 \times 10^{-4}$ is chosen so as to give a maximum for the Balmer lines at $T_{max} = 10,000^\circ$. Curves for all elements are then calculated for this pressure. An alternative curve is given for Ca^+ .)

Figure 4: Plot of the number of atoms capable of absorbing several sets of spectral lines as a function of temperature; a temperature scale is deduced (after Fowler and Milne, 1923: 420B; courtesy: Blackwell Publishing).

It was a couple of years later that Cecilia H. Payne (Figure 5) took up the suggestion (for Payne's contributions see Hearnshaw, 1986: 229-231, and DeVorkin and Kenat, 1983a: 124-127). In her Ph.D. dissertation, *Stellar Atmospheres*, she worked out the chemical composition of stellar atmospheres, starting from the standpoint that the chemical composition was the same for all stars and differences in spectra were only due to T and P_e . She had at her disposal the huge number of spectra analysed by Pickering and his collaborators at Harvard, and she defined a scale of intensity for different lines. Then she worked out a temperature scale from intensity maxima, fixing $P_e \approx 10^{-4}$ in much in the same way that Fowler and Milne had done, and finally she could exploit the marginal appearances method and find abundances for eighteen elements. Payne's results showed that a very good agreement could be found with the composition of the Earth's crust, except for hydrogen and helium, which turned out to be several orders of magnitude more abundant in stars. According to Russell, these discrepancies were supposed to be spurious; he guessed that a similarity of chemical composition between the solar atmosphere, and perhaps the entire Sun and the Earth, as a consequence of the common birth of the two celestial bodies, was supposed by the 'Nebular Hypothesis'.

Payne's determination of the chemical composition of stellar atmospheres, the first to be performed, was a major achievement; as DeVorkin and Kenat (1983a: 126) underline: "Payne's thesis *Stellar Atmospheres* brought to maturity that which Saha's theory had first made possible." In other words, we may interpret Saha's work as a fundamental moment in history, not so much for the novelty of his formula—that indeed was not completely new as it was derived from a very similar one that Eggert had found to describe chemical dissociation—but rather as it paved the way, building a bridge between observational data and quantum theory, to the physical interpretation of the spectral sequence (as first performed by Saha himself and outlined in his 1920b and 1921 papers), and to quantitative spectral analysis (temperatures and chemical composition). Although it was not Saha himself who was the main actor in all these steps, à la Eddington, we can see that he played a major role.

As DeVorkin and Kenat (1983a: 126) underline, Payne could relate the fundamental theoretical refinements made by Russell, Fowler and Milne to the huge body of spectroscopic data available at Harvard while developing her doctorate thesis, and she also had a much greater knowledge of ionisation potentials, something which Saha lacked and explicitly complained about (see Saha, 1921: 153).

The problem of hydrogen abundance had been revealed for the first time, but much progress was still required. In fact, Payne's work still contained some 'stumbling blocks'. Apart from not considering transition rates, upon which very little was known at the time, it was bound to the subjective concept of marginal appearance: it was necessary to know precisely how many atoms contributed to the absorption of a faintly visible line, an approximate estimate being not enough. For this reason, from the second half of the 1920s the attention of astrophysicists was more and more drawn towards the problem of the mechanism involved in line formation—e.g. in their broadening,

which influenced their visibility—and to the practical calibration of line intensity on the number of absorbing atoms.

As early as 1924 John Q. Stewart had dealt with this problem, in that year publishing a paper on "The width of absorption lines in a rarefied gas" in the *Astrophysical Journal*. Using a semi-classical theory of photon scattering in the solar atmosphere, he showed that the width of a line may be due to high abundance as well as to high pressure. He then proposed a relationship between line width and the number of absorbing atoms, also inferring what the minimum number of atoms was in order to give marginal appearance. Soon after, Stewart returned to the problem with Russell (Russell and Stewart, 1924), and evidence for a high hydrogen abundance emerged from their work, but this was not believed. In the following years astrophysicists tried in many ways to explain these strange results (e.g. advocating departures from thermodynamical equilibrium), until the high hydrogen abundance was finally accepted (as we shall see in the next Section).

Another major problem was that the line intensity was not by any means defined in a quantitative manner. Rowland's scale for the solar spectrum was still in use, and this assigned a number to each line that expressed its intensity. This was an arbitrary scale and was not physically defined, with all the inherent errors this could introduce.

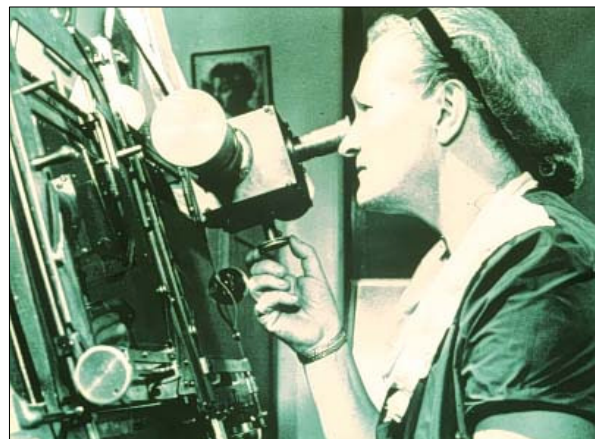


Figure 5: Cecilia H. Payne, 1900–1979 (courtesy Astronomical Society of the Pacific).

5 FINAL ACCEPTANCE OF THE ABUNDANCE OF HYDROGEN

The final rush that finished off the exploitation of Saha's theory saw once again Russell as a main character.² In 1928 Walter S. Adams, Charlotte E. Moore and Russell published a paper titled "A calibration of Rowland's scale of intensities for solar lines" in the *Astrophysical Journal*. By that time the theoretical intensities of lines within the same multiplet had become available, and the three scholars observed the different intensity of lines of some multiplets and compared them to the expected ones. In this way they succeeded in calibrating Rowland's scale to the number of atoms, although only approximately. Their most noteworthy conclusion was:

The most obvious result of the present investigation is to emphasize the enormous differences in the number of atoms which are involved in the formation of the

stronger and weaker Fraunhofer lines. From the weakest perceptible lines ... to such lines as H α or the great iron lines in the violet ($\lambda\lambda$, 3720, 3735), this number increases by a factor of approximately a million. For the H and K lines, which are too strong to be calibrated, the factor must be much greater. (Adams, Russell and Moore, 1928: 8).

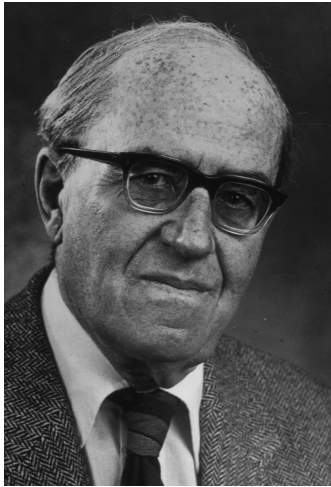


Figure 6: Albrecht Unsöld, 1905–1995 (courtesy www.phys-astro.sonoma.edu/.../unsoldSml.jpg).

In the same issue of the *Astrophysical Journal*, immediately following the above paper, there was another paper by Adams and Russell titled “Preliminary results of a new method for the analysis of stellar spectra”. In it they sought to extend to the stars what they had just deduced from the solar spectrum, comparing the different line intensities.

Table 1: Comparison of the elemental abundances found by Payne and Russell.

Element	Payne (log n of atoms)	Russell (log n of atoms)
H	12.9	11.5
He	10.2	
C	6.4	7.4
O	8.0	9.0
Na	7.1	7.2
Mg	7.5	7.8
Al	6.9	6.4
Si	7.5	7.3
K	5.3	6.8
Ca	6.7	6.7
Fe	6.7	7.2

However, the conclusive link between theory and observation was offered in that same period by Albrecht Unsöld (Figure 6). Utilizing a semi-classical treatment of the scattering process, and considering only this to be effective in line formation, he exploited the developments that had occurred in quantum theory by this time, and established a relationship between the number of atoms and the widths of several strong lines (Unsöld, 1927; 1928). Observing lines of the same element (e.g. Ca and Sr) both in the neutral and in the ionized state, and assuming a proper value for the temperature of the solar surface, he could deduce the pressure using Saha’s equation (it happened to be $P_e = 10^{-6}$ atm). With this value he could derive the abundances of elements present in only one ionisation state. The hydrogen abundance inferred from the Balmer lines resulted in an enormous value. In comparing his work with Payne’s results, Unsöld commented:

These [Payne’s values] postulate that the mean chemical composition of all stars is the same ... The agreement of the results is fine and constitutes a strong support to the opinion here yet in different occasions advocated, that the abundance of the chemical elements in the whole universe is constant. Hydrogen is thus for example ca. $5 \cdot 10^6$ times more frequent than Ca. C.H. Payne also arrives at $\sim 10^6$. (Unsöld, 1927: 777 and 781; our translation).

Although Unsöld remained skeptical about the hydrogen abundance, he had found the keystone that had been lacking. By linking quantum theory with the formation of spectral lines, he gave Russell just what was needed: the zero point of Rowland’s scale necessary to pass from relative to absolute abundances.

Consequently, in his fundamental paper, “On the composition of the Sun’s atmosphere”, Russell (1929) could estimate the electron pressure in the very same way Unsöld had done (and he found a similar value), then derive total abundances, starting from those of the different ionisation and excitation states, through Saha’s equation. Russell summarized his results in a table containing 56 elements and 6 molecular compounds, in which a comparison was performed with Payne’s values. Their respective values for some elements are listed in Table 1. Russell (1929: 65) commented on the agreement with Payne’s values, observing that

... Miss Payne’s results were determined by a different theoretical method ... About the only common features are the observations of spectral lines and the use of the ionization theory.

The scenario Russell had to face was so very different from what he had foreseen, although no more unexpected: too many indications pointed towards a high hydrogen abundance. Nor could they be neglected any longer, or attributed to unknown explanations. Russell (1929: 79) was ready to admit it, and he called this new awareness “... reconnaissance of new territory.” To conclude, we must observe that Russell had actually applied in a very crude way what was to be formalized as the curve of growth technique. In particular, Russell lacked the concept of equivalent width and was still tied to the old Rowland scale that was soon to be abandoned. Between 1927 and 1931, H. von Klüber, M. Minnaert, G.F.W. Mulders and B. van Assenbergh all introduced the idea of equivalent width as well as the curve of growth technique, thereby placing the determination of the chemical composition of stellar atmospheres on a firm physically base.

6 EDDINGTON AND THE OPACITY DISCREPANCY

In the years 1916-1924 Arthur Eddington (Figure 7) published a series of papers in the *Monthly Notices of the Royal Astronomical Society* in which we worked out his ‘standard model’ of stellar structure. Eddington applied the concept of radiative equilibrium, and pointed out the importance of radiation pressure in addition to gas pressure. He started from the idea that the stars were gaseous spheres in hydrostatic equilibrium and that the perfect gas law held. He took into consideration only giant stars, since he thought the ideal gas condition to be certainly fulfilled for their more rarefied gases (Eddington, 1916: 16). On the other hand, he thought that dwarf stars should not be in a condition of perfect gas, at least in the inner and denser regions. Eddington allegedly was influenced by

the theory of stellar evolution that Russell had worked out to interpret the occurrence of giant and dwarf stars in the H-R Diagram. Russell had developed that theory in the very same paper in which he had presented the first graphical representation of the Diagram (Russell, 1914). According to Russell, stars begin their life as contracting giant stars. As the radius decreases the surface temperature increases, until contraction stops when the density becomes so high that the perfect gas condition ceases to hold. From that point on gas becomes highly incompressible and the star begins to cool, descending the Main Sequence.

Starting from the aforementioned hypotheses, Eddington succeeded in demonstrating that under these assumptions a star in radiative equilibrium can be described by a polytropic model of index 3 (Eddington, 1916: 21). About the assumptions made by Eddington and their acceptability see Mestel (2004).

After having succeeded in determining the values of the status parameter in the interior of a star, Eddington wished to obtain an expression for the brightness L . This led him to introduce the absorption coefficient Γ , defined as the radiation amount absorbed per unit mass and cross-section. This was dependent upon the distance from the star's centre, as radiation absorption depends upon the physical conditions of matter. This is a natural step in a radiative model. As for Γ , Eddington came upon the work of H.A. Kramers (1923) where a dependence of the following kind was deduced for the photoionization processes:

$$\Gamma \propto \frac{\rho}{\mu T^{3.5}} \quad (10)$$

Starting from this, Eddington (1924b: 310) succeeded in deducing a relevant formula, known as the 'mass-luminosity relation':

$$L \propto M^{7/5} (1 - \beta)^{3/2} \mu^{4/5} T_e^{4/5} \quad (11)$$

where L is the star's luminosity, M its mass, T_e the effective temperature, μ is the mean molecular weight (i.e. the mean mass per particle expressed in units of hydrogen mass) and β is the ratio of the gaseous to total pressure and is tied to the stellar mass and to μ itself via the famous 'quartic' relation (Eddington, 1918: 210; 1924a:109):³

$$1 - \beta = \text{const} \times M^2 \mu^4 \beta^4 \quad (12)$$

Hence Equation (11) is more properly a luminosity – mass – mean molecular weight – effective temperature relation. It is possible to introduce in the formula the radius in place of the effective temperature, using the relationship $L = 4\pi\sigma R^2 T^4$. In this way we have a luminosity – mass – mean molecular weight – radius relation.

The major role played by μ is evident from Equations (11) and (12). Eddington was to be involved with this parameter for a long time, but where could he turn to in order to estimate its value? This value depends upon two factors: (1) the chemical composition of stellar gases that determines which elements are present and in what amounts; and (2) the physical conditions of temperature and pressure that determine the ionization of different elements. In 1916, when he attacked this problem, Eddington could count on neither a theory of ionization as a consequence of T and P nor any trustworthy estimate of chemical composition. As we have seen, the theory

would be developed by Saha in 1920, and only then could the chemical composition of stellar gases be deduced from spectra.

Relying upon hazy estimates of composition, Eddington tentatively supposed that stellar gases could be composed of monoatomic iron vapor, from which he thought a value $\mu = 54$ to be reliable (Eddington, 1916: 22). He then realized, however, that the high temperature of the stellar interiors should produce a high ionization degree, in agreement with atomic theory, in order to lessen considerably the value of μ . Eventually he embraced a position that was argued by other colleagues:

The suggestion that at these high temperatures we are concerned with particles smaller than the atom was made to me independently by Newall, Jeans and Lindemann. ... I had supposed that atomic disintegration [ionization], though undoubtedly occurring, could not have proceeded very far; but Jeans has convinced me that a rather extreme state of disintegration is possible, and indeed seems more plausible. (Eddington, 1917: 596-597).



Figure 7: Sir Arthur Eddington, 1882–1944 (courtesy Wikipedia).

From that standpoint, it followed that at sufficiently high temperatures μ had to assume a value around 2 irrespectively of chemical composition, as for most elements (other than hydrogen, of course!) atomic weight is about half of the mass number. Eddington (1917: 596) then opted for a value of $\mu = 2$, but he went on to often use the value 2.11, maintaining the hypothesis of ferrous material. Meanwhile, in his Bakerian Lecture delivered on 17 May 1917 (and published in Jeans 1919: 209-210) James Jeans expressed the view that the value $\mu = 54$ was much too high and that $\mu = 2$ was more reliable. By the way, the idea that Jeans and Eddington had to apply the hypothesis of ionization to high temperatures constituted the very first astrophysical application of the quantum theory of the atom.

It should be noted that the high degree of ionization undergone by matter in a stellar interior has a remarkable outcome: due to the much smaller dimensions of particles, the ideal gas law holds even at the very high densities of dwarf stars. Initially Eddington was not prepared to admit this, as he relied upon Russell's evolutionary interpretation of the giant-dwarf duality. The British astrophysicist had to change his mind when he realized that the mass-luminosity relation (i.e. Equation (11)) also fitted the data for dwarf stars (Eddington, 1924b: 308-309). Thus, he came to the conclusion that dwarf stars are also made of perfect gas. Eddington (1924b: 320) also realized that

... in the interior of a star the atoms of moderate atomic weight are stripped down to the K level, and have radii of the order 10^{-10} cm; lighter elements, such as carbon and oxygen, are reduced to the bare nucleus. The maximum density, corresponding to contact of these reduced atomic spheres, must be at least 100,000, and any star with mean density below 1000 ought to behave as a perfect gas.

In that same paper, Eddington explicitly discarded Russell's theory of stellar evolution, interpreting the Main Sequence as a sequence of quasi-equilibrium points corresponding to different masses.

Another surprise was awaiting Eddington. The constant of proportionality in Equation (11)—that is undetermined so long as the constant in Equation (10) is also undetermined, from which Equation (11) follows—was deduced by Eddington from the observed values of M , L and surface temperature for Capella (a double system of giant stars of well-known dynamical features and parallax). But if the constant in Equation (10) is worked out purely from quantum theory rather than from observed values, it turns out to be about 10 times higher (and this is called the 'opacity discrepancy'). Owing to the major role played by μ in the mass-luminosity relation, it is clear that a change in its value could reduce or even eliminate the difference. Eddington soon realized that the μ value could be the key for solving the problem. In *The Internal Constitution of the Stars*, published in 1926, he noticed that if the percentage of hydrogen is around one third in mass, the lowered value of μ helps to remove the discrepancy:

There is one way in which [the two values for the absorption coefficient, worked out from theory and from observational data] can be reconciled by an assumed chemical composition of the star, namely, by mixing a considerable proportion of hydrogen with a heavier element, say, iron ... Hydrogen is the only element which can make these changes ... I was formerly attracted by the view that stars, especially in the giant stage, contain a large proportion of hydrogen – the idea being that the stars are the main, if not the only, seat of the manufacture of the higher elements from protons and electrons, the star's heat being incidentally provided by the process.⁴ But the low molecular weight involved is out of keeping with the general trend of astronomical evidence ... I would much prefer to find some other explanation [for the discrepancy]. (Eddington, 1926: 244-245).

At that time almost nothing was known about the chemical composition of stars, as spectroscopy was just beginning to address the problem. Thus, Eddington did not feel confident to change the value of μ , as he thought such a hydrogen percentage to be much too high.

But in the following years, as we have seen, a greater concern arose among scholars who were dealing with spectroscopic problems. Eddington undoubtedly followed with interest the debate that finally led to the acceptance of the greater hydrogen abundances at the surfaces of stars. But what about their interiors? Was it possible that a different composition was to be found there? The idea was not odd, as it had been considered by S. Rosseland in 1925, who, on the basis of electrostatic considerations, was led to believe that a high surface abundance did not automatically mean a similar abundance in the interior of stars. However, the idea was later discarded.

In 1932 Bengt Strömberg published a paper titled "The opacity of stellar matter and the hydrogen content of the stars" that finally led Eddington to take part in the quest. Strömberg (1932: 122-123) referred explicitly to the doubts that Eddington had put forward, but noted that in light of the work on stellar spectra by Unsöld, Russell and others, they could be put aside:

The main argument against the hypothesis [of the great hydrogen abundance] is that the high abundance of hydrogen required seems rather improbable at first sight. It is however now an established fact, after the work of W.H. Mc Crea, H.N. Russell and A. Unsöld ... that in the atmospheres of the Sun and the stars in general hydrogen occurs in the proportion of about one half by weight ... We shall *trust* the theoretical value of the coefficient of opacity and *deduce* the molecular weight and hence the hydrogen abundance for the stars with known M , R and L , where Eddington trusted the molecular weight (no hydrogen) and deduced the coefficient of absorption.

Strömberg then calculated theoretically the opacity coefficient, and worked out the Emden-Eddington solutions (as he called them) for stars of known M , R and L . This led him to deduce a hydrogen abundance around one third of the total mass.

Eddington's classic paper on the subject, "The hydrogen content of the stars", was published in that same year. Its starting point is the observation of a strong μ -dependence in the mass-luminosity relation: even a small change in its value will change significantly the relation between M and L . Consequently, it was necessary to lower it to a value close to 1. Eddington (1932: 471) then observed:

When the luminosity of a star is computed from its mass and radius with the value of the absorption coefficient obtained from pure physical theory, the result comes out too bright. This well-known discrepancy amounts to a factor 10 for diffuse or massive stars, and is still larger for the Sun and smaller stars in a less highly ionised condition. This result is subject to the reservation that the stars do not contain a large proportion of hydrogen.

This last statement was evidently the hypothesis he needed to reject, and to convince himself, Eddington calculated for the Sun—whose mass and radius were well-known— L as a function of μ , assuming a composition of hydrogen plus ferrous materials in variable proportions. The resulting curve was plotted in a diagram, where the difference between calculated and real luminosity for different hydrogen percentages was shown (see Figure 8).

Eddington noted that two values could be accepted for the abundance of this element: around 33% and almost 100%. Similar results were obtained for other stars, so that "... there must be some cause which

makes the hydrogen content of the stars nearly uniform.” (Eddington, 1932: 476). He thought the lower abundance to be more trustworthy:

For each star there are two solutions - two possible proportions of hydrogen consistent with the observed luminosity. In one solution the star is chiefly hydrogen (about 99½ per cent.) with only a trace of other elements. The other solution, which rightly or wrongly I have assumed to be the more probable, gives approximately 33 per cent. hydrogen in the Sun, Capella, Algol and Krüger 60. These stars were selected as having first-class observational data and covering a wide variety of mass and density ... The surprising thing is the steadiness of the hydrogen content, shown not only in the four stars above mentioned, but in the general adherence of the star to a mass - luminosity curve. (Eddington, 1932: 472).

In fact, if the hydrogen percentages were very different from star to star, there should be a strong scattering in brightness for stars of similar mass and there would not be any mass-luminosity relation at all.

What other indications did Eddington have to induce him to believe in the unexpectedly-high presence of hydrogen? Was what he had worked out from the aforementioned deduction just a strong indication, or was it real proof? Eddington had adopted some questionable assumptions when developing his model, so the occurrence of any indication of a different kind was certainly welcome. As he pointed out:

Partly by elimination of alternative explanations, and partly by the recent evidence of great abundance of hydrogen in stellar atmospheres coupled with our theoretical knowledge that hydrogen will not escape to the outside but will be kept stirred by currents set up in rotating stars, the hydrogen hypothesis has now come into prominence. If the proportion of hydrogen is given, the composition of the rest of the material makes very little difference to the luminosity. Thus if we are convinced that there is no other serious uncertainty in the problem, it is theoretically possible to determine the hydrogen content of a star of known L, M and R. We have simply to find what proportion of hydrogen mixed with other elements will give a luminosity agreeing with the observed luminosity. This is no longer a matter of speculative curiosity; such determinations are needed to compare with and check the determinations of abundance of hydrogen in stellar atmospheres made by H.N. Russell and others. (Eddington (1932: 472).

Eddington’s choice for a percentage around 33% is evidently based on the fact that he thought it to be less extreme, much in the same way as Strömgren had done. In the decades that followed, further research would demonstrate that the extreme percentage is in fact much closer to reality.⁵

By the way, Eddington argued that even with a hydrogen abundance of around 33%, the mass-luminosity relation still deviated from observational data for the most massive stars. He came to that conclusion after analyzing the data for V Puppis, a star of 19 solar masses (which today is known to comprise two components of spectral types B1 and B3). The calculated luminosity turned out to be ~1.5 magnitudes too large, and the hydrogen proportion had to be increased to yield the expected one. Although Eddington (1932: 479) thought that “... there is some ground to think that the proportion of hydrogen in the most massive stars is greater than 33% ...”, other factors could be at work. For example, he was aware of the

uncertainty in the surface temperature for hot stars. Furthermore, by that time bolometric corrections were calculated in a semi-empirical way that had not undergone substantial improvement for many years. Scholars still tended to refer to the work carried out by Hertzsprung in 1906 (reference to this work, as well as ‘state of the art’ knowledge in this respect, is given in Eddington, 1926: 138-139). This could affect significantly the luminosity values of very hot stars that mainly radiate outside the visible region.

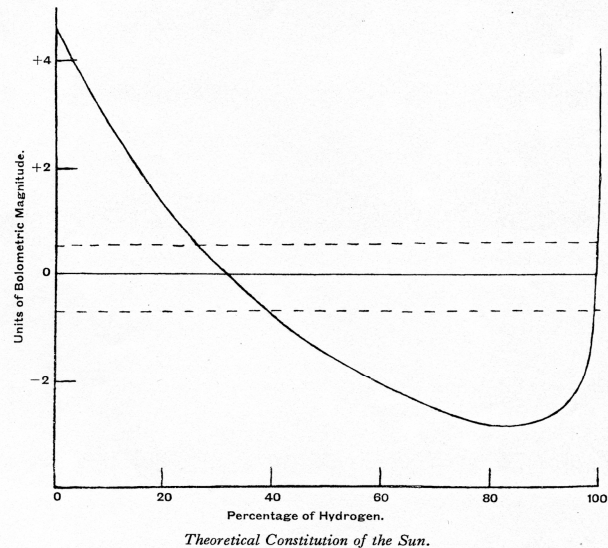


Figure 8: Plot of the brightness of the Sun versus the mass percentage of hydrogen. The value 0 corresponds to the real brightness and identifies two possible values for hydrogen (33% and 99.5%). The dotted lines correspond to other possible structures of the Sun obtained by hypothesizing on the distribution of energy sources other than those Eddington actually used (after Eddington, 1932: 476; courtesy: Blackwell Publishing).

7 CONCLUSIONS

During the 1920s and the beginning of the 1930s important developments took place in stellar astronomy. Previously, knowledge about the nature of the stars was essentially limited to the following achievements:

- Stars were thought to be hot gaseous bodies, and this was confirmed by spectra that highlighted the absorption by gaseous material nearby the surface. No quantitative chemical analyses had been carried out and in this regard not much more was known than Kirchhoff’s qualitative analyses of the 1860s.
- Surface temperatures were assumed to be between 3,000 and 15,000 degrees. There were, however, no reasonable guesses for what the temperature might reach in the interiors of the stars. As regards gaseous pressure, precise calculations had not been made, although reasonable estimates for the inner regions could be worked out from the condition of hydrostatic equilibrium.
- A series of empirical regularities had been found—such as the Hertzsprung-Russell Diagram—made possible by the gathering of observational evidence and progress in detection equipment and techniques.

In the fifteen years from 1920, a transition took place from this set of empirical results to a new series of achievements which were based upon the sound

theoretical background provided by the quantum theory of the atom. We have seen how this led to the merger of two fields of investigation, stellar surfaces and stellar interiors, and the progress astrophysics underwent is epitomized by the passage from a fragmentary set of results to a unitary and complete *corpus* of knowledge that included values for specific stellar parameters and the chemical composition of stellar interiors and surfaces. In this process we see the convergence of quantum theory, thermodynamics, spectroscopy and issues regarding the chemical composition of the Universe. The acknowledgement of the prevalence of hydrogen in stellar composition should be thought of as a meaningful moment in the history of astrophysics, as it was the first application of quantum theory to the stars that led scholars to depict a whole new reliable picture of stellar structure.

8 NOTES

1. More recent measures for λ Ori indicate a temperature of around 30,000 K (see <http://webviz.u-strasbg.fr/viz-bin/VizieR>).
2. A fine discussion of the reasons that kept scholars from initially admitting the high hydrogen abundance and later induced them to accept it can be found in DeVorkin and Kenat (1983b: 204-208). In this paper I do not intend to go into such detail; my main interest is in drawing a picture of the principal advances that were made possible by physical theory.
3. The constant in Equation (12) was estimated by Eddington (1924b: 309) to be 0.00309 if M is expressed in solar units.
4. Eddington long speculated about the nature of the process capable of supplying a star's energetic output. Here it is enough to say that in 1926 his ideas were merely speculative, but only a few years later light began to be cast upon this matter. Nevertheless, Eddington showed great insight when he guessed that hydrogen could play a key role. In this respect, giant stars should contain more hydrogen, so long as they were thought to be younger.
5. In historical perspective, it should be noticed that a mass-luminosity relation actually holds only if a law of energy release is known. Yet in the 1920s, Milne, Jeans and Vogt had pointed out how Eddington's relation was based upon an incomplete system of equations (see Cowling, 1966: 126). In any case, this should not cast a shadow upon the scientific and historical importance of Eddington's model or belittle his deep physical insight.

9 REFERENCES

Adams, W.S., and Russell, H.N., 1928. Preliminary results of a new method for the analysis of stellar spectra. *Astrophysical Journal*, 68, 9-36.

Adams, W.S., Russell, H.N., and Moore, C.E., 1928. A calibration of Rowland's scale of intensities for solar lines. *Astrophysical Journal*, 68, 1-8.

Comte, A., 1835. *Course de Philosophie Positive*, X. Paris, Bachelier.

Cowling, T.G., 1966. The development of the theory of stellar structure. *Quarterly Journal of the Royal Astronomical Society*, 7, 121-137.

DeVorkin, D.H., and Kenat, R., 1983a. Quantum physics and the stars (I): the establishment of a stellar temperature scale. *Journal for the History of Astronomy*, 14, 102-132.

DeVorkin, D.H., and Kenat, R., 1983b. Quantum physics and the stars (II): Henry Norris Russell and the abundances of the elements in the atmospheres of the Sun and stars. *Journal for the History of Astronomy*, 14, 180-222.

Eddington, A.S., 1916. On the radiative equilibrium of the stars. *Monthly Notices of the Royal Astronomical Society*, 77, 16-35.

Eddington, A.S., 1917. Further notes on the radiative equilibrium of the stars. *Monthly Notices of the Royal Astronomical Society*, 77, 596-612.

Eddington, A.S., 1918. On the conditions in the interior of a star. *The Astrophysical Journal*, 48, 205-213.

Eddington, A.S., 1924a. The absorption of radiation inside a star (second paper). *Monthly Notices of the Royal Astronomical Society*, 84, 104-123.

Eddington, A.S., 1924b. On the relation between the masses and luminosities of the stars. *Monthly Notices of the Royal Astronomical Society*, 84, 308-332.

Eddington, A.S., 1926. *The Internal Constitution of the Stars*. Cambridge, Cambridge University Press.

Eddington, A.S., 1932. The hydrogen content of the stars. *Monthly Notices of the Royal Astronomical Society*, 92, 471-481.

Eggert, J., 1919. Über den Dissoziationszustand der Fixsterngase. *Physikalische Zeitschrift*, 20, 570-574.

Fowler, R.H., 1923. Dissociation-equilibria by the method of partitions. *Philosophical Magazine*, 45, 1-33.

Fowler, R.H., and Milne, E.A., 1923. The intensities of absorption lines in stellar spectra, and the temperatures and pressures in the reversing layers of the stars. *Monthly Notices of the Royal Astronomical Society*, 83, 403-424.

Hearnshaw, J.B., 1986. *The Analysis of Starlight: One Hundred and Fifty Years of Astronomical Spectroscopy*. Cambridge, Cambridge University Press.

Jeans, J.H., 1919. Bakerian Lecture, 1917: The configurations of rotating compressible masses. *Philosophical Transactions of the Royal Society of London. Series A*, 218, 157-210.

Kramers, H.A., 1923. On the theory of X-ray absorption and of the continuous X-ray spectrum. *Philosophical Magazine*, 46, 836-871.

Lockyer, J.N., 1887. Researches on the spectra of meteorites. *Proceedings of the Royal Society*, 43, 117-156.

Lockyer J.N., 1888. Suggestions on the classification of the various species of heavenly bodies. *Proceedings of the Royal Society* 44, 1-95.

Lockyer J.N., 1900. *Inorganic Evolution as Studied by Spectrum Analysis*. London, Macmillan.

MacPherson, H., 1920. Recent theories of stellar evolution. *Popular Astronomy*, 28, 225-231.

Menzel, D.H., 1972. The history of astronomical spectroscopy. I: qualitative chemical analysis and radial velocities. II: quantitative chemical analysis and the structure of the solar atmosphere. *Annals of the New York Academy of Sciences*, 198, 225-244.

Mestel, L., 2004. Arthur Stanley Eddington: pioneer of stellar structure theory. *Journal of Astronomical History and Heritage*, 7, 65-73.

Milne, E.A., 1924. Recent work in stellar physics. *Proceedings of the Physical Society, London*, 36, 94-113.

Rosseland, S., 1925. On the distribution of hydrogen in a star. *Monthly Notices of the Royal Astronomical Society*, 85, 541-546.

Russell, H.N., 1914. Relations between the spectra and other characteristics of the stars. *Nature*, 93, 227-231, 252-258 and 281-286.

Russell, H.N., 1919. Some problems of sidereal astronomy. *Proceedings of the National Academy of Sciences of the United States of America*, 5(10), 391-416.

Russell, H.N., 1921. The properties of matter as illustrated by the stars. *Publications of the Astronomical Society of the Pacific*, 33, 275-290.

Russell, H.N., 1922. The theory of ionization and the sun-spot spectrum. *Astrophysical Journal*, 55, 119-144.

- Russell, H.N., 1929. On the composition of the Sun's atmosphere. *Astrophysical Journal*, 70, 11-82.
- Russell, H.N., and Stewart J., 1924. Pressures at the Sun's surface. *Astrophysical Journal*, 59, 197-209.
- Saha, M.N., 1920a. Ionisation in the solar chromosphere. *Philosophical Magazine*, 40, 472-488.
- Saha, M.N., 1920b. Elements in the Sun. *Philosophical Magazine*, 40, 809-824.
- Saha, M.N., 1921. On a physical theory of stellar spectra. *Proceedings of the Royal Society*, 99A, 135-153.
- Stewart, J.Q., 1924. The width of absorption lines in a rarefied gas. *Astrophysical Journal*, 59, 30-36.
- Strömngren, B., 1932. The opacity of stellar matter and the hydrogen content of the stars. *Zeitschrift für Astrophysik*, 4, 118-152.
- Unsöld, A., 1927. Über die Struktur der Fraunhoferschen Linien und die Dynamik der Sonnenchromosphäre. *Zeitschrift für Physik*, 44, 793-809.
- Unsöld, A., 1928. Über die Struktur der Fraunhoferschen Linien und die quantitative Spektralanalyse der Sonnenatmosphäre. *Zeitschrift für Physik*, 46, 765-781.

Davide Cenadelli is a physicist and historian of astronomy at Milan University. His research work focuses upon the history of stellar astrophysics in the twentieth century. He is mainly interested in stellar spectroscopy, in the relationship between atomic physics and astrophysics, and in stellar structure and evolution theories. He is deeply involved in scientific education programmes both at high school and university level, and he is the author of several papers on this subject.