ECLIPSES IN THE MIDDLE EAST FROM THE LATE MEDIEVAL ISLAMIC PERIOD TO THE EARLY MODERN PERIOD. PART 1: THE OBSERVATIONS OF SIX LUNAR ECLIPSES FROM THE LATE MEDIEVAL ISLAMIC PERIOD

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Abstract: This paper deals with the analysis of data obtained from observations of two sets of three lunar eclipses in the Late Medieval Islamic Period. The first trio consists of the lunar eclipses of 7 March 1262, 7 April 1270 and 24 January 1274, observed by Muḥyī al-Dīn al-Maghribī from the Maragha Observatory (in north-western Iran), and the second includes those of 2 June and 26 November 1406, and 22 May 1407, observed by Jamshīd Ghiyāth al-Dīn al-Kāshī from Kāshān (in central Iran). The results are that al-Maghribī's values for the magnitudes of these eclipses agree excellently with modern data, and his values for the times when the maximum phases occurred agree to within five minutes with modern values. Al-Kāshī's values for the times of the maximum phases show a rather larger divergence from modern data, varying from about ten minutes to about one hour. The errors in all six values both astronomers computed from their own solar parameters for the longitude of the Sun at the instant of the opposition of the Moon to the Sun in these eclipses remain below ten minutes of arc. The motivation for doing these observations was to measure the lunar epicycle radius r in the Ptolemaic model. Al-Maghribī achieved r = 5;12 and al-Kāshī $r \approx$ 5;17,[1] in terms of the radius of an orbit of R = 60 arbitrary units. It is argued that comparing with modern theory, neither of these two medieval values can be considered an improvement on Ptolemy's value of r = 5;15.

Keywords: lunar eclipses, Middle East, Late Medieval Islamic Period, al-Maghribī, al-Kāshī, Maragha Observatory

1 INTRODUCTION

Over the last few decades around fifty observational reports of solar and lunar eclipses dating to the Early Medieval Islamic Period (ca. AD 801-1000) have been investigated in depth. Alongside reports preserved by other cultures (particularly the Chinese), they have been used to obtain estimates for the rate of the deceleration of the Earth's rotation and the cumulative amount of the change in the length of the day, or ΔT, that is, the difference between Terrestrial Time and Universal Time (e.g., see Morrison and Stephenson, 2004; Steele, 2000; Stephenson, 1997; 2011).

This was perhaps a main characteristic of the rise of astronomy in communities where eclipses were seen as remarkable and frequent celestial events, and were observed so that the data obtained (regardless of how accurately they might be determined) could be compared with those computed on the basis of contemporary tables and theories. This was perhaps the reason why a great deal of energy and effort was employed to make more precise observations of eclipses. For example, Ibn Yūnus (d. 1007) from Cairo gathered reports from some local astronomers and others who had witnessed eclipses, and data they supplied helped him to determine a better estimate for the magnitude of each eclipse and solar-lunar altitudes at specific phases (e.g., in the case of the solar eclipse of 13 December 977, see Ibn Yūnus: 110; and also Caussin de Perceval, 1804: 163; Stephenson, 1997: 473). And the earliest known attempts to reconcile theory with observations in Medieval Islamic astronomy might have been produced in this way. For instance, Ibn Yūnus reported that the Baghdad astronomer Ibn Amājūr (ca. the late ninth to the early tenth century) found that the true longitude of the Moon was 16′ behind that computed from the *Mumtaḥan zīj* composed by Yaḥyā b. Abī Manṣūr at Baghdad in about AD 830 (Ibn Yūnus, 99-100, Caussin de Perceval 1804: 111, 113).[2]

2 SOLAR AND LUNAR ECLIPSES IN THE LATE MEDIEVAL ISLAMIC PERIOD

In the Late Medieval Islamic Period (AD 1001-1450), as large observatories were founded and more astronomical tables were compiled, the number of observational reports of eclipses diminished, and astronomers instead presented the values they had computed for the occurrence times of eclipses in their *zījes*. In Islamic astronomy, the mean motions of the Sun (in longitude) and the Moon (in longitude, in anomaly, and the retrograde motion of its orbital nodes) and their orbital elements (eccentricity, radius of the epicycle) were determined more frequently than the corresponding planetary parameters. Of around twenty-five values that I know for the solar eccentricity and ten values for the lunar orbital elements dating from the Medieval Islamic Period, nearly half were determined in the Early Medieval Islamic Period and the other half in the Late Medieval Islamic Period. Nevertheless, twenty-six of the thirty-four known values for the planetary orbital elements in Islamic astronomy date to the Late Medieval Islamic Period. There can be found an equal number of values (if not more) for the solar, lunar and planetary mean motions both

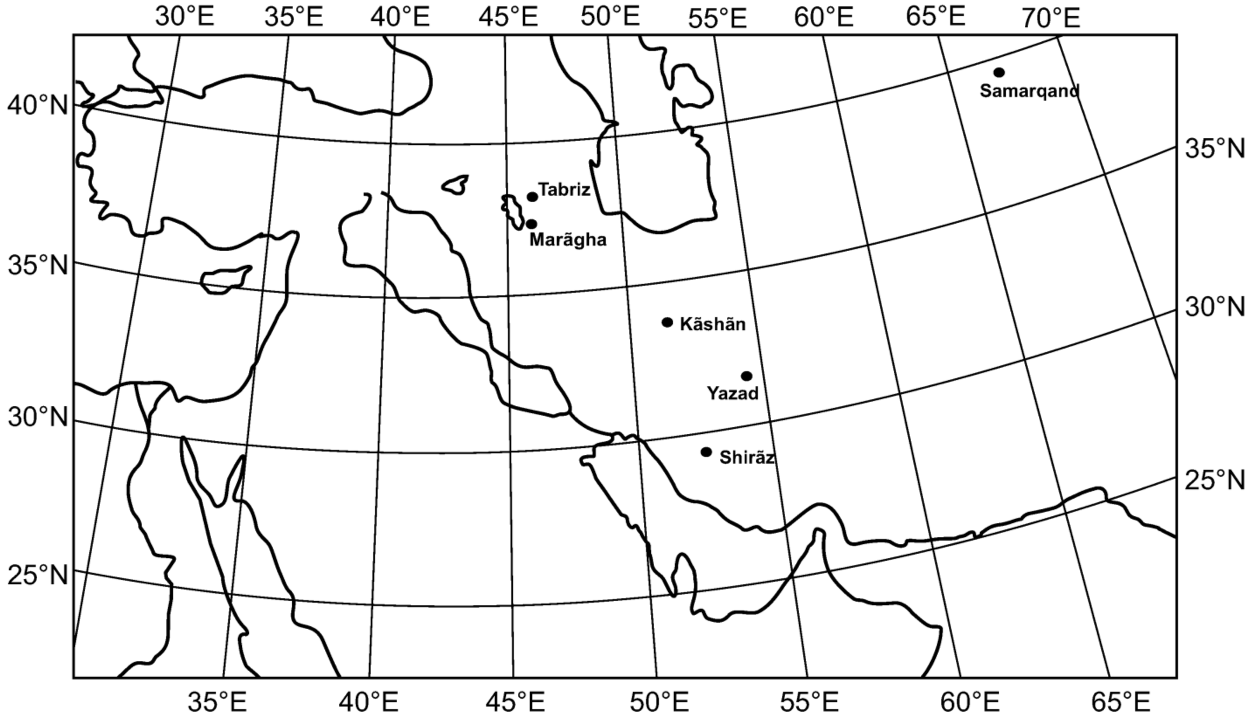


Figure 1: Localities mentioned in the text where eclipses were observed or calculated during the Late Medieval Islamic Period.

in longitude and in anomaly for this Period. An observer could then examine which set of parameter values led to better agreement with the observations. For example, see the case of Ibn al-Fahhād, Bākū or Shirwān, who flourished ca. AD 1172 (van Dalen, 2004: 836).

Figure 1 shows the places in the Middle East for which eclipses were computed or where they were observed in the Late Medieval Islamic Period, and their historical and modern geographical coordinates are listed in Table 1. I know of four worked out examples of solar eclipses that date from the Late Medieval Islamic Period. Three of these, on 30 January 1283, 5 July 1293 and 28 October 1296, were calculated by Shams al-Dīn Muḥamamd al-Wābkanawī al-Bukhārī (who lived at Maragha and Tabriz, between about 1270 and 1320). The first is for the latitude of Mughān (historical: φ = 38°; a green plain in northwestern Iran, modern: φ = 38.5-39.5° N and λ = 45-47° E) and is given in Wābkanawī's *Zīj al-muḥaqqaq al-Sulṭānī*, "Testified zīj for the sultan" (Mozaffari, 2009; 2013a; 2013b). The next two are for the latitude of Tabriz and are recorded in Byzantine Greek sources. These sources (see Pingree, 1985) include the translation of Ibn al-Fahhād's *'Alā'ī zīj* (written around 1172) and some fragments of another *zīj* called the *Revised canon* which was based on the oral instructtions that an Iranian astronomer named Σάμψ Πουχαρής (= Shams al-Bukhārī) gave to Gregory Chioniades when Chioniades was at Tabriz at the turn of the fourteenth century. This astronomer may be identified as Wābkanawī. In the translations, the computation of these two eclipses is embedded within the worked out examples that Wābkanawī provides in order to instruct Chioniades on how the parameters of an eclipse (time, duration, magnitude, etc.) may be computed. The *Revised canon* appears to be the translation of some parts of Wābkanawī's *Zīj* before it was completed around 1320. Information about the 28 October 1296 eclipse for the latitude of Yazd is also found in the anonymous *Sulṭānī zīj* (fols. 138v-140r); some computational examples concerning the various parameters of solar eclipses for the latitude of Shirāz may be found in the *Ashrafī zīj* V.18. While the dates are not indicated, some computations appear to be related to the solar eclipse of 3 April 1307. Critical documentation on this event are a solar longitude of around 20-22°at the beginning and end of the eclipse, and a value of 203° 36′ for the longitude of the lunar ascending node (see Kamālī: fol. 149r).

In the case of lunar eclipses, five worked out examples may be found in the *zījes* of this period: 30 May 1295 for the latitude of Tabriz in Chapter 36 of the Greek translation of the *'Alā'ī zīj* (Pingree, 1985: 352ff.), and for the latitude of Yazd in the *Sulṭānī zīj*; 23 November 1295 in the *Sul-ṭānī zīj* (fols. 137r-138r); 4 January 1303, 9 May 1305, and 14 December 1312 in the *Ashrafī zīj* (Kamālī: fols. 133v-134r, 145v-146r).

These worked out examples were to instruct the reader how the magnitude and times of the phases of eclipses were calculated in a Ptolemaic context using the different procedures passed down to astronomers of this period (either from Greek or Indian sources, or those developed by their Islamic predecessors). In some accounts (e.g. Wābkanawī's computation of the

Table 1: Historical and modern co-ordinates of places shown in Figure 1 where eclipses were observed or computed.

Site	Historical		Modern	
	Latitude	Longitude*	Latitude	Longitude
Marāgha	37;20,30° N	82;00° E	37;24° N	46;12° E
Kāshān	34;00	86;00	33;59	51;27
Tabrīz	38;00	82;00	38;05	46;18
Yazd	32;00(1)–32;15(2)	89;00	31;54	54;22
Shirāz	29;30(3)–29;34(4)	88;00	29;37	52;32

* Measured from the Fortunate Islands (Canary Islands).
(1), (4) *Īlkhānī zīj*, C: p. 197.
(2), (3) *Khāqānī zīj*, IO: fol. 73r.

annular solar eclipse of 30 January 1283), the computed results were compared with the data obtained from observations, or *vice versa*, in order to show whether or not they were in agreement. Surviving computations of eclipses from the Late Medieval Islamic Period show that the theoretical results were acceptable in the Ptolemaic context of medieval astronomy. Wābkanawī's account of the solar eclipse of 30 January 1283 maybe is now a well-known example. The other two examples are the times computed for the middle of the lunar eclipses of 30 May and 23 November 1295 in the *Sulṭānī zīj* for the latitude of Yazd, which are only around –22 and +4 minutes in error (the complete account of the computations of these eclipses will appear in the second part of this paper).[3] Other historical facts reinforce the idea that the accuracy of the eclipse predictions based on the Islamic *zījes* was of interest at this time. For instance, Yelu Chucai (1189–1243), a Chinese astronomer who used some techniques from the Islamic *zījes* to adjust the Chinese calendar during his stay in Samarqand (Transoxania) around 1220, greatly appreciated the accuracy of the eclipse parameters calculated with the aid of Islamic *zījes* (see van Dalen, 2002a: 331).

What is said above maybe explains the reason why the observational reports were no longer given with or accompanied by the computations. The astronomers appear to have found it sufficient to only assert that the solar and lunar parameters they adopted were accurate enough to establish a fair degree of agreement between theory and observation.

Very late in the Medieval Islamic Period and in the Pre-modern Period (ca. 1451-1700), there are some scattered allusions to, and a few surviving accounts of, the computation of eclipses, which show that eclipse parameters were still computed using the same traditional procedures. This Period saw the first attempts to transmit Renaissance astronomy (heliocentricism, the Tychonic system, and so on) to central Iran (see Ben-Zaken, 2009). Nonetheless, the first detailed accounts of the computations of eclipse timings and magnitudes based on modern astronomy appeared (seemingly, for the first time) in the *Muḥammadshāhī Zīj*, a Persian *zīj* completed in Jaipur, India, around 1735 under the patronage of Sawā'ī Jai Singh (1688–1743) (Pingree, 2002), named after Muḥamamdshāh, the Moghal Emperor of India (1702–1748, r. 1719-1748). This work seems to be the first in Islamic-Indian astronomy, in which new astronomy and elliptical orbits were employed practically in order to determine planetary longitudes. The new underlying parameters were adopted from the astronomical tables of Philippe de La Hire (Paris, 1727; see Dalen, 2000 and the references mentioned therein). Included in this *zīj* are some worked examples for computing the longitudes of the Sun, the Moon, Mars and Mercury, as well as the parameters of the partial solar eclipse of Monday 30 Dhu al-Qa'da 1146 H/3 May 1734 and the lunar eclipse of Sunday 15 Dhu al-Hijja 1144 H/8 June 1732 (P1: 206-208, 212-222; P2: 274-276, 280-291; N1: 189-190, 193-201; in P1 and P2 the date of the lunar eclipse is wrongly given as *10* Dhu al-Hijja. Also, note that the dates are not according to the Hijra civil calendar but to its astronomical calendar).

The first computation of the parameters of eclipses on the basis of modern astronomy in the Middle East occurred around the mid-nineteenth century. In the *Nāṣirīd ephemeris* written by Maḥmūd Khān (1866) of Qum (a city in the vicinity of Tehran) for King Nāṣir al-Dīn of the Qājār Dynasty of Iran, the author gives the astronomical ephemeris for the year 788 Jalālī/1282-1283 H/1866-1867 AD for the longitude of Tehran (p. 7), the Iranian capital. It contains lists of the magnitudes and times of two lunar eclipses: 31 March and 24 September 1866 (the times of the phases of these eclipses are given in local mean sidereal time). Maḥmūd Khān also lists the parameters of the partial solar eclipse of 6 March 1867, but doubts that this solar eclipse will actually occur because, in order to compute the eclipse, knowledge of the latitude of the place is necessary, while the latitude of the capital was not known with certainty. The computational accounts of the eclipses mentioned above will be studied in the second part of this paper.

Besides comparing observational and theoretical results and other factors of the same sort, there was another important factor that constituted a principal motive especially for the observation of lunar eclipses: the structural parameters defining the orbit of the Moon in the Ptolemaic model were determined from observations of lunar synodic phenomena. The observation of three lunar eclipses was essential for determining the radius of the epicycle, and the observation of the Moon at quadratures (quarter Moons) for measuring the eccentricity. In *Almagest* IV.6, Ptolemy proposed a mathematical method for determining the size of the lunar epicycle in terms of the radius of its deferent, using data obtained from the observations of a trio of the lunar eclipses (cf. Duke, 2005; Neugebauer, 1975(1): 73-80; Pedersen, 2010: 172-178; Thurston, 1994, Appendix 4: 204ff.; Toomer, 1998: 190-203). To the best of my knowledge, only three Middle Eastern astronomers in the Medieval Islamic Period gave observational data on a trio of lunar eclipses and explained how they determined the lunar epicycle radius from them. They are as follows:

(1) Abū al-Rayḥan al-Bīrūnī (in *al-Qānūn al-mas'ūdī*, *Volume 2*: 742-743): the three lunar eclipses in the period AD 1003-1004, observed from Jurjān (Gurgān, northern Iran), nos. 07224, 07225, and 07227 in NASA's Five Millennium Catalog of Lunar Eclipses (henceforth, referred to as 5MCLE);
(2) Muḥyī al-Dīn al-Maghribī (in *Talkhīṣ al-majisṭī*, fol. 69v): the three lunar eclipses in the period AD 1262-1274, observed from Maragha (northwestern Iran); and
(3) Jamshīd Ghiyāth al-Dīn al-Kāshī (in *Khāqānī zīj*, IO: fols. 4r-v; P: 24-25): the three lunar eclipses in the period AD 1406-1407, observed from Kāshān (central Iran).

Al-Bīrūnī's trio of lunar eclipses have already been analyzed by Said and Stephenson (1997: 45-46) and Stephenson (1997: 491-492), but the reports by the other two astronomers have hitherto remained unnoticed and have not been investigated. These appear to be the only preserved observational reports of lunar eclipses from the Late Medieval Islamic Period, and they are the main focus of the remainder of this paper.

3 THE LUNAR ECLIPSES OBSERVED FROM MARĀGHA BETWEEN AD 1260 AND 1280

Not very much is known about al-Maghribī. His full name is "Abū al-Shukr/Abu al-Karīm/Abu al-Fatḥ Yaḥyā b. Muḥammad b. Abī al-Shukr b. Ḥumīd of the Maghrib (of Tunis, of al-Andalus, or of Cordoba). Al-Maghribī spent some years (after 1237 and until 2 October 1260) in the service of King Nāṣir of Damascus (reign: 1237-1260) in Aleppo before the King was killed by Mongols, and then he was sent to the Maragha Observatory. Other than a short stay in Baghdad in the latter part of the 1270s, he seems to have lived and observed at the Maragha Observatory until his death in June 1283. He taught some students in the Observatory, and appears to have written about 26 works on mathematics, astronomy, and astrology (see Brockelmann, 1937 (Supplement): 868; 1943(1): 626; Rosenfeld and Ihsanoglu, 2003: 226; Sarton, 1953: 1015-1116; Sezgin, 1978: 292; Suter, [1900] 1982: 155; see, also Comes' entry (pages 548-549) in Hockey et al., 2007). Some of al-Maghribī's mathematical works have been studied (e.g. see Hogendijk; 1993; Voux, 1891), and Tekeli's short entry about al-Maghribī in the *Dictionary of Scientific Biography* (Gillispie et al., 1980(9): 555) only covers his mathematical works. Two of his works are the astronomical tables accompanied by explanatory instructions on how to use them, the so-called *zījes*: *Tāj al-azyāj* (written at Aleppo ca. 1257; see Dorce, 2002-2003) and *Adwār al-anwār* (written at Maragha in 1276).

Al-Maghribī's astronomical activities at the Maragha Observatory made him such an outstanding figure that his contemporaries and immediate successors called him by unique honorific titles that denoted his skill in making observations. For instance, Ibn al-Fuwaṭī, the Librarian at the Observatory, called him "... the geometrician of the observations ..." (Ibn al-Fuwaṭī, (5): 117). His observational program is often referred to as 'the new Ilkhānīd observations', to distinguish it from the purported observations conducted at Maragha for the preparation of the *Īlkhānī zīj*. His fame was so widespread that his astrological doctrines were treated with great respect (nine of his treatises are on astrology). An amazing example of this is the interpretation of the appearance of the comet C/1402 D1 based on his astrological dogmas, which led to a very decisive war in the Middle East at the turn of the fifteenth century (see Mozaffari, 2012: 363-364).

In his *Talkhīṣ al-majisṭī*, "Compendium of the *Almagest*", written seemingly after the *Adwār* (i.e., in the latter part of the 1270s), al-Maghribī presented his solar, lunar and planetary observations and computations. The contents of this work have already been introduced by Saliba (1983; 1985; and 1986). Table 2 presents the lunar eclipses observed by him at the Maragha Observatory, arranged chronologically (nos. 07878, 07897, and 07907 in 5MCLE).

Column 1 contains the numbers that al-Maghribī used to refer to each eclipse.

Column 2 presents the dates of the observations given in the text according to the Yazdigird era, and their corresponding dates in the Julian calendar and in Julian Day Numbers. The Yazdigird era originated on 16 June 632, and is used with the Egyptian/Persian year consisting of 12 months of 30 days plus five epago-

Table 2: Lunar eclipses observed by al-Maghribī at the Maragha Observatory.

Nos.	Date	Time	Type	Magnitude	$\lambda_{\odot}$	Stars' Altitudes
1	Night of Wed. 28/2/631 Y 7 March 1262 JDN 2182069	630 y 1 m 27d 8;18 h	TD	total	354;22,50	*At the start of totality:* Regulus (α Leo): 51° West *At the end of totality:* Spica (α Vir): 17° East
2	Night of Tue. 1/4/639 Y 7 April 1270 JDN 2185022	638 y 3 m 0d 10;13 h	P	≈ (1/2)+(1/3) from south	24;53, 1	*At the beginning of the eclipse:* Arcturus (α Boo): 42° East *At the end of the eclipse:* Regulus (α Leo): 35° West
3	Night of Wed. 18/1/643 Y 24Jan. 1274 JDN 2186410	642 y 0 m 17d 14;0 h	P	≈ 4/5 from north	311;41,28	*At the beginning of the eclipse:* Arcturus (α Boo): 35° East *At the end of the eclipse:* Arcturus (α Boo): 68° East

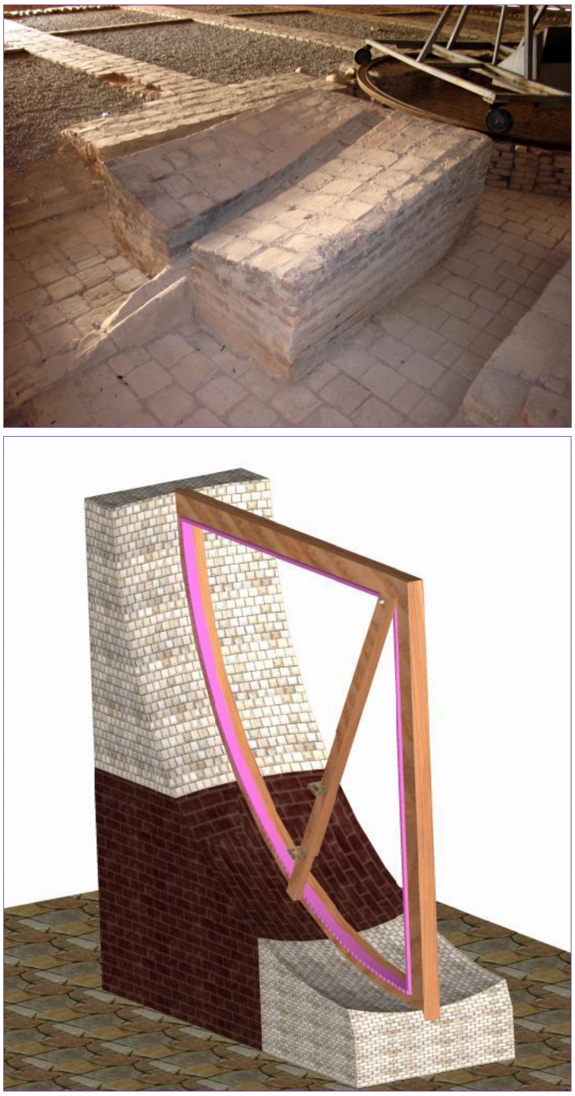


Figure 2: The central quadrant of the Maragha Observatory. (a): The remnant of the instrument's base (picture taken by the author), (b): a virtual reconstruction of it based on the dimensions given by al-ʿUrḍī, the instrument-maker at the Observatory (drawn by Dr. Elkhan N. Sabziev).

Table 3: Al-Maghribī's measured times of the maximum phases of the three lunar eclipses observed from Maragha in comparison with modern data.

Nos.	al-Maghribī	Modern	Error
1	08:18 h	08:13 h	+5 m
2	10:13 h	10:08 h	+5 m
3	14:00 h	13:57 h	+3 m

menal days which, in the Early Medieval Islamic Period, were put after the eighth month. But in the Late Medieval Islamic Period, they were transferred to the end of the year. In order to convert the dates from the Yazdigird era to the Julian one, it needs to be kept in mind that in Islamic chronology the day is traditionally reckoned from sunset, and hence 'night' precedes 'day'. As a result, for example, the night of Wednesday, 28/2/631 Yazdigird, is the time interval between sunset on Tuesday, the 27th and sunrise on the 28th. This confusion cannot occur when we use the equivalent Julian dates. Since al-Maghribī has made the precise time of the maximum phase of each eclipse available (see Column 3), the dates can be converted conveniently.

Column 3 presents the times of the eclipses, that is, the instants when the maximum phases occurred, counted from the beginning of the Yazdigird era. Al-Maghribī counted the hours using a clepsydra, from the instant of the meridian transit of the Sun (true noon), which was observed using the central quadrant of the Observatory erected on the local meridian line (see Figures 2a and 2b).

The instants of true noon for the days of the eclipses are, respectively, 12:10, 12:00 and 12:15 (-1 day), according to the mean local time (MLT) of Maragha (≈ UT + 3hr 4m). The true times of the maximum phase of the three eclipses are 20:23, 22:08, and 02:12, respectively, calculated from 5MCLE for the longitude of Maragha. Thus, the times of the eclipses after true noon, measured in hours, are as listed in Table 3.

The central quadrant had been engraved for each 0.5′, and the majority of al-Maghribī's meridian altitude measurements were performed with the aid of it. Al-Maghribī appears to have been so interested in the instrument that he composed a poem during his observations of AD 1265-1266 to praise it, and an astrologer named Majd al-Dīn Abū Muḥammad al-Ḥasan b. Ibrāhīm b. Yūsūf al-Baʿalbakī engraved the poem on the quadrant (Ibn al-Fuwaṭī, Vol. 4, 413-414).

Al-Maghribī frequently referred to the application of a clepsydra, to which the Persian name *pangān* (Arabicized as *bankām;* Pl. *bankāmāt*) was assigned, in his systematic observations (however, he wrongly mentions its name as *man-kām*). We only know the general shape of the instrument: it was in the form of a floating bowl (*ṭās*), with a hole in its apex and two graduated scales (usually drawn with the aid of an astrolabe) for both the equal and unequal hours on its peripheral surface. When this bowl was placed in a vessel containing water, as the water drained into it the level of the water determined the time at each moment. The first description of this type of clepsydra in the Medieval Islamic Period appears in al-Ṣūfī's *Book on the Astrolabe* (1995: Chapters 354-357: 299-302). This type of clepsydra can be traced back to Babylonian and Indian texts from the first millennium BC (Pingree, 1973: 3-4). Archaeological excavations have un-earthed its earliest models in India, apparently be-longing to the same period (Rao, 2005: 205-206).

Based on the information given by al-Maghribī, one can only speculate about its calibrations, as nothing more is known about its structure. The clepsydra used, of course, appears to have been of a good accuracy, so that it could establish time intervals to within a few minutes (and in the case of the lunar eclipses, the errors were within ±5 minutes). Thus, it does not seem that it was a simple drainage clepsydra. It should be mentioned here that in medieval Chinese astronomy the use of clepsydras having compound mechanical components had been established since at least the eleventh century (Needham, 1981: 136). Due to the verified cultural relations between Iran and China, in the Mongolian Period, and especially considering the fact that some Chinese astronomers (e.g., at least Fu Mengchi, also referred to as Fu Muzhai) worked at the Maragha Observatory (van Dalen, 2002a: 334; 2002b; 2004), perhaps there was a connection between the clepsydra of the Maragha Observatory and the elaborate Chinese time-measuring devices.

Column 4 in Table 2 indicates the type of the eclipse; TD denotes 'Total eclipse with a perceptible duration (lit. 'staying', *makth*)', while P stands for 'Partial'.

Column 5 in Table 2 presents the magnitude of the eclipse. Modern values are listed in Table 4 (from 5MCLE). These might be a naked eye estimate; however, two different types of optical devices used for directly measuring eclipse magnitudes had by this time been invented, and examples of both were constructed at the Maragha Observatory. Ptolemy (*Almagest*, V.14) used a dioptra, originally described by Hipparchus, that was four cubits, or about 185.28 cm, in length (Toomer, 1998: 56). This had a fixed lower pinnula on which there was a hole for sighting, and a movable outer pinnula, which was placed in front of the Sun. The solar/lunar angular diameter meter was calculated based on the movable pinnula's width and the distance between the two pinnulae. In his *Fī kayfīyya al-arṣād*, "How to make the observations", Mu'ayyad al-Dīn al-'Urḍī (d. 1266) modified the dioptra so that it could be used to determine the eclipsed area/ diameter of the Sun or the Moon (Seemann, 1929, 61-71). Thus, al-Maghribī had a specific instrument for measuring the magnitude of eclipses at his disposal, which he may have applied to these three lunar eclipses. In the *Risāla al-Ghāzāniyya fi 'l-ālāt al-raṣadiyya,* "Ghāzān's treatise on observational instruments", and in Wābkanawī's *Zīj* (IV.15, 8: Y: fols. 159r-159v, T: fols. 92r-92v), an instrument used as a pinhole image device is introduced that can measure the magnitude of solar eclipses. This treatise contains physical descriptions and applications of twelve new observational instruments that date from the second period of the Maragha Observatory, and these are presumed to have been the inventions of Ghāzān Khān, the seventh ruler of the Ilkhanid Dynasty of Iran (reign: 21 October 1295-17 May 1304) (see Mozaffari and Zoitti, 2012: 419-422; 2013; Zotti and Mozaffari, 2010: 165, 167).

Table 4: Al-Maghribī's values for the magnitudes of the three lunar eclipses observed from Maragha in comparison with modern data.

Nos.	al-Maghribī	Modern
1	total	1.77
2	0.833	0.823
3	0.8	0.77

Column 6 in Table 2 gives the true longitude of the Sun, $\lambda_{\odot}$, at the time of the maximum phase of each eclipse. Al-Maghribī has indeed calculated these values based on his solar tables; in other words, they are not observational data. In order to measure the lunar epicycle's radius, it is necessary as the first step to obtain the Moon's longitudes at the instants of the maximum phases of a trio of lunar eclipses, i.e., when it was in true opposition to the Sun. Then they can readily be calculated as $\lambda_{☽} = \lambda_{\odot} + 180°$. A comparison with the modern values is shown in Table 5.

Column 7 in Table 2 shows the observed altitudes of some bright stars which were generally used in order to determine the duration and the

Table 5: Al-Maghribī's computed values of the longitude of the Sun at the time of the maximum phases of the three lunar eclipses observed from Maragha in comparison with modern data.

Nos.	$\lambda_{\odot}$	
	al-Maghribī	Modern
1	354;22,50°	354;20,04°
2	24;53,01	24;52,17
3	311;41,28	311;36,54

Table 6: Lunar eclipses observed by al-Kāshī in Kāshān: the times of the maximum phases after midnight.

Nos.	Date	al-Kāshī's local times		Modern local times		Error	
		Apparent	Mean	Apparent	Mean		
1	30/6/775 Y 2 June 1406 JDN 2234752	3;14,30 h	2;56,29 h	4;08,59 h	4;07,01 h	−54.5 m	−70.5 m
2	27/12/775 Y 26 Nov. 1406 JDN 2234929	1;13,05	0;48,46	1;06,07	0;57,27	+7.0	−8.7
3	18/6/776 Y 22 May 1407 JDN 2235106	4;18,30	3;58,46	4;43,52	4;40,05	−25.4	−41.3

time of the phases of each eclipse. The position with respect to the horizon of a particular celestial body may be given by means of its altitude plus its direction with respect to the meridian line; e.g., '51° East' means an altitude of 51° at a given instant, while located east of the meridian. An important note here is that in the case of eclipse No. 1, the directions al-Maghribī cites for the measured altitudes do not express the direction of the star with respect to the meridian, but with reference to the lunar disk. Otherwise, the altitudes should have been expressed as 51° East for Regulus and 17° East for Spica at, respecttively, the start and end of totality.

Based on what al-Maghribī says (*Talkhīṣ*, fol. 67v), these were the eclipses that he "... dealt with observing them with extreme accuracy ...", and thus he could rely on his observations and be confident about the correctness of the data obtained from them. While he was based at the Maragha Observatory, nine other lunar eclipses were observable at their maximum phases from Maragha, and al-Maghribī may have witnessed these as well.

4 THE LUNAR ECLIPSES OBSERVED FROM KĀSHĀN IN AD 1406 AND 1407

Jamshīd Ghiyāth al-Dīn al-Kāshī (ca. 1380–1429) was an Iranian mathematician and astron--omer, who is maybe better known for his computation of sin 1° (Aaboe, 1954; Rosenfeld and Hogendijk, 2002/2003). He flourished in his native city, Kāshān (in central Iran), where he observed the three lunar eclipses considered here, but later he moved to the Samarqand Observatory established by Ulugh Beg (1394–1449) (see Kennedy, 1983: 722-744). Al-Kāshī revised the *Īlkhanī zīj*, which was written at the Maragha Observatory around one and a half centuries earlier. The results were apparently incorporated into his *Khāqānī zīj* (for a brief survey of it see Kennedy, 1998a; for its parts on spherical astronomy see Kennedy, 1998b: Part XVIII, and for the account of planetary latitudes in it see van Brummelen, 2006). Al-Kāshī invented some instruments that served as mechanical computers for doing astronomical calculations, and one of these was a lunar eclipse computer (Kennedy, 1983: 448-480).

Al-Kāshī's three lunar eclipses are summarized in Table 6 (nos. 08220, 08221, and 08222 in 5MCLE). Column 2 presents the dates given by al-Kāshī according to the Yazdigird era and their corresponding dates in the Julian calendar and in Julian Day numbers. It is worth mentioning that for eclipses Nos. 1 and 2 the civil date is given, but for eclipse No. 3 it is the astronomical date (from noon to noon); the civil date of the eclipse No. 3 is 19/6/776 Y. Columns 3 and 4 contain al-Kāshī's time of the maximum phase of each eclipse, respectively, in apparent and mean local times. Columns 5 and 6 show the modern times, obtained from 5MCLE, for the longitude of Kāshān. Columns 7 and 8 contain the difference between al-Kāshī's times and modern times.

The values that al-Kāshī took for the equation of time (the difference between the apparent and mean local times in Columns 3 and 4 in Table 6) can be obtained from his table for the equation of time (al-Kāshī, IO: fols. 126v-127r; cf. Kennedy, 1998b: Part VII), in which the equation of time is tabulated as a function of the true solar longitude. For example, for eclipse No. 2, the Table gives E(252°) = 0;24,17^h and E(253°) = 0;23,52^h for the beginning of the year 712 Y and thus, by means of linear interpolation between the two, the figure for the equation of time at the time of eclipse No. 2, for which al-Kāshī gave $\lambda_\odot$ = 252;13,53,38°, is calculated as E(252;13,53,38°) ≈ 0;24,11^h. There are also changes in the equation of time over long time intervals (per century, and over seven centuries) included in the Table; for $\lambda_\odot$ from 250° to 257° the Table gives the amount of the correction as 11 seconds per century counted from the year 712 Y, and thus it is 11·(775–712)/100 ≈ 7 seconds for the year 775 Y. As a result, E(252;13,53,38°) ≈ 0;24,18^h for that year.

As we have already seen, al-Maghribī simply took the point diametrically opposite the Sun as the position of the Moon on the ecliptic at the times of the maximum phases of the lunar eclipses, but al-Kāshī considered the difference in the positions of the Moon on the ecliptic and in its orbit due to the ~5° inclination of the latter to the former (Figure 3), as shall be explained presently. Al-Kāshī tabulated some other parameters

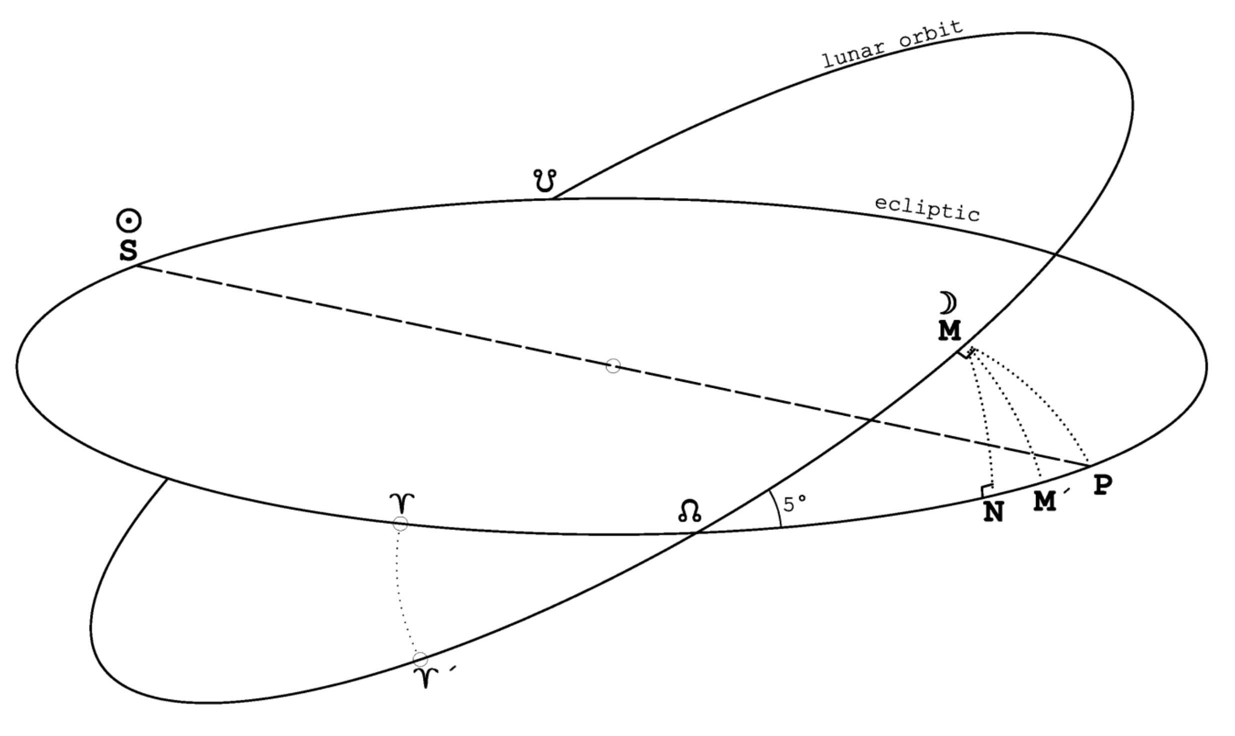


Figure 3: The inclined lunar orbit and the ecliptic.

for the instants of the maximum phases of these eclipses, which are presented in Table 7. They include the solar longitude, $\lambda_{\odot}$ (Column 2); the longitude of the lunar ascending node counted in the direction of decreasing longitude, i.e., 360°– $\lambda_{☊}$ (Column 3); and the difference between $\lambda_{\odot}$ and the longitude of the lunar orbital node that was close to the Sun at the time, i.e., the ascending node for the eclipses Nos. 1 and 3 and the descending node for the eclipse No. 2 (Column 4). In fact, it is the sum of Columns 2 and 3 minus 360° for eclipses Nos. 1 and 3, and minus 540° for the eclipse No. 2. (but of course al-Kāshī did not use the positive and negative signs as shown in Table 7; they are used here to show when the Sun was ahead of the node in longitude, i.e., +, or behind it, i.e., –). That is equal to the difference in longitude between the centre of the Earth's shadow and the other lunar node, which is near the Moon at the time of the maximum phase of the eclipse, i.e., the descending node in the case of eclipses Nos. 1 and 3 and the descending node in the case of eclipse No. 2.

With regards to Figure 3, at the time of the maximum phase of a lunar eclipse, the Sun is at S, close to the lunar descending node, and P is the centre of the Earth's shadow near the lunar ascending node. The distance S☋ = $\lambda_{\odot}$– $\lambda_{☋}$ is equal to P☊ = $\lambda_{☽}$– $\lambda_{☊}$. The intersection of the lunar inclined orbit with the great circle passing through P and S that is perpendicular to the lunar inclined orbit defines the position of the Moon on its inclined orbit at the time of the observation, i.e., M. M′☊ is taken as equal to M☊, and N is the projection of M onto the ecliptic. From the values of the distance between the centre of the Earth's shadow and a node, i.e. the values of $\lambda_{\odot}$ – $\lambda_{☊ \text{ (or ☋)}}$ listed in Column 4, al-Kāshī computed its difference from the distance of the Moon from that node (Column 5), which is called the 'equation of shift' (*ta'dīl-i naql*; or 'reduction to the ecliptic' in the modern terminology). The values of $\lambda_{\odot}$+180° were then adjusted by this equation to produce the lunar longitudes, $\lambda_{☽}$, with reference to its inclined orbit, i.e., M♈′ in Figure 3 (Column 6).

As al-Kāshī points out, Ptolemy already noticed the difference between the positions of the Moon in its orbit and on the ecliptic (*Almagest* IV.6; Toomer, 1998: 191; Pedersen, 2010: 199-200; note that al-Kāshī wrongly referred to *Almagest* VI.6), but since its effect is small enough to be ignored he did not consider it in the determination of the lunar parameters or in the computation of the eclipses. From the earliest steps in the rise of astronomy in medieval Islam, astronomers took this equation into account; a table showing it first appeared in Yaḥyā b. Abī Manṣūr's *Zīj al-mumtaḥan* (Kennedy and Pingree, 1981: 168 and 310). Al-Maghribī, in *Talkhīṣ al-majisṭī* V.11, calls it 'the equation of the inclined sphere of the Moon' (*ta'dīl al-falak al-mā'il*) or 'the equation of shift', just like al-Kāshī, but in the other *zījes*, it is called the 'third equation' after the 'first equation', which is the 'equation of centre', and the 'second equation', which is the 'equation of anomaly'. The 5° inclination of

Table 7: Al-Kāshī's longitudes of the Sun and lunar ascending node for the times of the maximum phases of the lunar eclipses.

No.	$\lambda_{\odot}$	$360° - \lambda_{☊}$	$\lambda_{\odot} - \lambda_{☊}$ (or ☋)	Equation of shift	$\lambda_{☽}$ (inc. orb.)
1	78;55,10,41°	274;32,46°	−6;32,03°	+0;1,29, 2°	258;56,39,43°
2	252;13,53,38	283;54,51	−3;51,15	+0;0,52,40	72;14,46,18
3	68;14,18,43	293;17,40	+1;31,59	−0;0,20,58	248;13,57,45

the Moon's orbit to the ecliptic also means that the maximum phase of a lunar eclipse does not always occur exactly at the time when the Moon is in opposition to the Sun, except in the case of central lunar eclipses when the lunar latitude is exactly zero. Jābir b. Aflaḥ (Spain, the first half of the twelfth century) noticed this difference (see Bellver, 2008: 63). Some medieval astronomers (e.g., see Wābkanawī, III.11.4: T: fol. 63r, Y: fol. 114r, P: fol. 96r) believed that the inclination of the lunar orbit should be taken into account in order to compute a more accurate value for the duration of the eclipse's phases.

The equation of shift, *c*, may simply be calculated by

$$c = \tan^{-1}(\tan(\lambda_{☽} - \lambda_{☊}) \cos 5°) - (\lambda_{☽} - \lambda_{☊}) \qquad (1)$$

It does not matter whether $\lambda_{☽}$ is the lunar longitude with reference to the ecliptic or to its inclined orbit. The equation is subtractive in the first and third quadrants ($0 < \lambda_{☽} - \lambda_{☊} < 90°$, $180° < \lambda_{☽} - \lambda_{☊} < 270°$) and additive in the second and fourth quadrants ($90° < \lambda_{☽} - \lambda_{☊} < 180°$, $270° < \lambda_{☽} - \lambda_{☊} < 360°$). The maximum value of the equation is 0;6,33° for $\lambda_{☽} - \lambda_{☊} \approx 45°$. In the majority of the tables found in the Islamic *zījes*, the maximum value is 0;6,40° (e.g., Khāzinī, fol. 135r, *Īlkhānī zīj*, C: 84; al-Maghribī, *Talkhīṣ*, fol. 83v; Kamālī, fols. 67r and 243v; Wābkanawī, T: fol. 156r). Al-Bīrūnī (*al-Qānūn*, 2: 810), Kāshī (IO: fol. 133v; P: fol. 51v), and Ulugh Beg (P1: fol. 126v; P2: fol. 145r) accurately gave 0;6,33°.

It is noteworthy that al-Kāshī employed this equation in the inverse manner: from the *zījes*, the longitude of the Moon with reference to its inclined orbit was first computed and the resultant was then adjusted by the equation of shift to produce the ecliptical longitude of the Moon, while al-Kāshī had the latter and wished to compute the former. The recomputed values for the equation of shift in the three eclipses are, respectively, +0;1,28,45°, +0;0,52,38°, and −0;0,20,59°.

The longitude of the Moon at the maximum phase of the eclipse was obtained from the solar longitude which was computed from the solar parameters (eccentricity, mean motion, longitude of the apogee) which were already determined or adopted by other astronomers. Al-Maghribī determined a set of the solar parameters for the Maragha Observatory (see Saliba, 1985), and based upon these he computed the lunar longitudes for the maximum phases of his three lunar eclipses (Table 5). Through his project of revising the *Īlkhānī zīj*, al-Kāshī adopted the values employed in it for the solar eccentricity and mean motion, which are those that Ibn Yūnus applied in his *Zīj al-kabīr al-ḥākimī*. Unlike al-Maghribī, al-Kāshī does not supply us with an account of his solar observations (if any), which would have been very useful as we could have calculated the longitude of the Sun and of the lunar ascending node at the times of the maximum phases of these eclipses taken from the *Īlkhānī zīj* and compared them with the longitudes given by al-Kāshī (Table 7, Columns 2 and 3) and with modern data. It may then have been possible to determine to what extent they were dependent upon each other and/or if al-Kāshī's revision of the *Īlkhānī zīj* might have improved the quantities computed from it in comparison to modern data. In the tables of the geographical coordinates of the cities in the Islamic *zījes*, there is a 4° difference between the longitudes of Maragha and Kāshān (*Īlkhānī zīj*, C: 197; al-Kāshī, IO: fols. 73v-74r), corresponding to a time difference of 16 minutes between the two sites. Thus, 16 minutes were subtracted from al-Kāshī's mean local times (cf. Table 6, Column 4 and Table 8, Column 2), and the values of $\lambda_{\odot}$ and $\lambda_{☊}$ were calculated from the *Īlkhānī zīj* for the resulting times (Table 8, Columns 3 and 4). The modern values are given in Columns 5 and 6. The difference between the modern longitude values and those of al-Kāshī and the *Īlkhānī zīj* are presented in Columns 7-10. As we can see, al-Kāshī's values for the solar longitude are more exact than those computed from the *Īlkhānī zīj* by around 10′. Considering the longitude of the lunar ascending node, there are systematic errors of +14′ and +18′, respectively, in the values given by the *Īlkhānī zīj* and al-Kāshī.

5 DISCUSSION

5.1 Accuracy of the Medieval Values for the Lunar Epicycle Radius

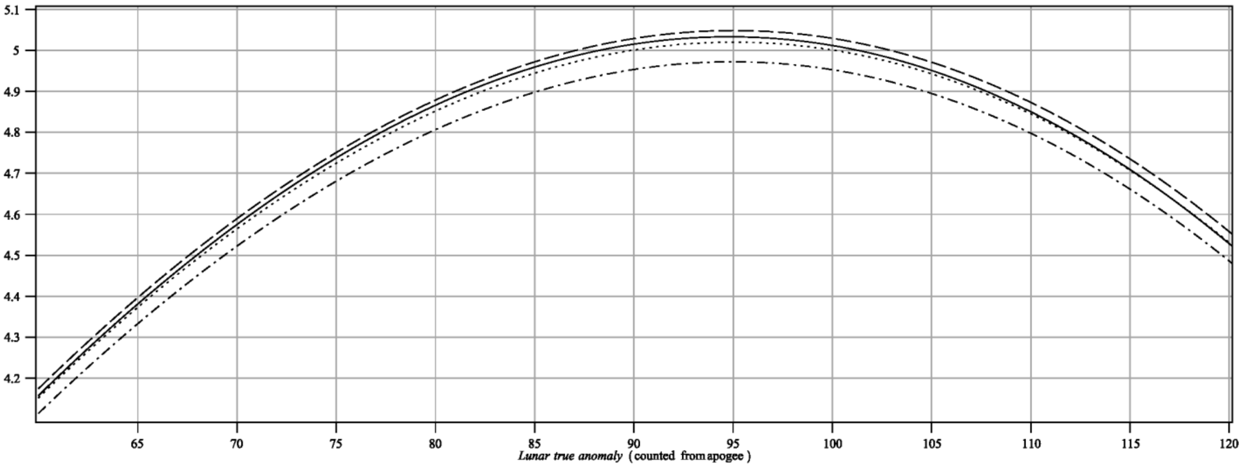
From *Almagest* IV.11 (Toomer 1998: 212-213), it is clear that Hipparchus had already established the principle that it was necessary to use a trio of lunar eclipses close in time, so that any long-term error in the mean motions would have a minimal effect on the determination of the size of the lunar epicycle. This was strictly followed by al-Bīrūnī and al-Kāshī while al-Maghribī apparently selected his three lunar eclipses from those observed during a 12-year period.

Based on their observations, al-Bīrūnī and al-Maghribī calculated the lunar epicycle radius as r = 5;12 and al-Kāshī as r = 5;16,46,36, while

Table 8: Longitudes of the Sun and lunar ascending node according to the *Īlkhānī zīj* for al-Kāshī's times of the maximum phases of the lunar eclipses.

No.	Mean local time of Maragha	*Īlkhānī zīj*		Modern		*Īlkhānī zīj*–Modern		al-Kāshī(*)–Modern	
		$\lambda_\odot$	$\lambda_☊$	$\lambda_\odot$	$\lambda_☊$	$\Delta\lambda_\odot$	$\Delta\lambda_☊$	$\Delta\lambda_\odot$	$\Delta\lambda_☊$
1	2;40,29	78;46,10°	85;23,24°	79;01,46°	85;08,52°	–0;15,36°	+0;14,32°	–0;6,35°	+0;18,22°
2	0;32,46	252;23,18	76;01,17	252;12,57	75;46,54	+0;10,21	+0;14,23	+0;0,57	+0;18,15
3	3;42,46	68;05,42	66;38,28	68;18,49	66;24,02	–0;13,07	+0;14,26	–0;4,30	+0;18,18

(*) See Table 7, columns 2 and 3.



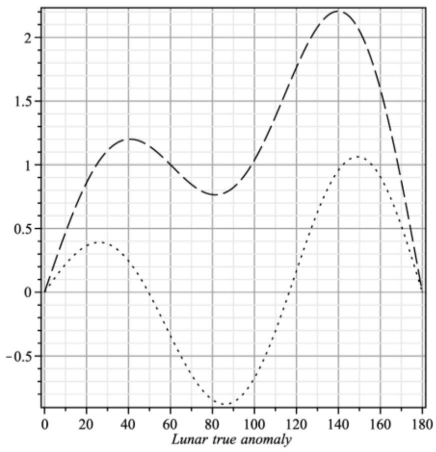


Figure 4(a) top: The two major terms in the formula for finding the difference between the lunar true and mean longitudes in modern astronomy are 22640″ · sin *α* – 769″ · sin 2*α* (called 'major inequality') and 4586″ · sin (2$\overline{\eta}$ – *α*) (called 'evection'), where *α* is the mean anomaly of the Moon, measured from apogee and $\overline{\eta}$, its mean elongation from the Sun. In the Ptolemaic sense, the 'first inequality of the Moon' is relevant to the syzygies, i.e., when $\overline{\eta}$ = 0 or 180°, and is given by $\tan^{-1}$ (*r* ·sin *α*/(*R* + cos *α*)). According to modern theory, its size is then computed from ~ 5.015° · sin *α* – 0.214° · sin 2*α*. The graphs shows the values of the first inequality of the Moon (measured in degrees) based on Ptolemy's *r* = 5;15 (the dotted graph), al-Kāshī's *r* = 5;16,46,36 (the dashed graph), and al-Maghribī's and Bīrūnī's *r* = 5;12 (the dotted-dashed graph) (*R* = 60). The continuous graph is based on the above-mentioned modern formula. Figure 4(b) right: The difference between the values computed for the first inequality of the Moon from the modern theory and from Ptolemy's and al-Kāshī values for *r*, shown respectively, as the dotted and dashed graphs.

Ptolemy reported *r* = 5;15 in terms of the radius of an orbit of *R* = 60 arbitrary units. The accounts of the determination of the lunar parameters by these medieval astronomers will be studied in three separate papers by the present author. It may, nonetheless, be appropriate to take a look at their achievements in comparison with modern theory. In the Hipparchan and Ptolemaic lunar models, the epicycle is to account for the first inequality of the Moon in the Ptolemaic sense (see Neugebauer 1975(3): 1106-1108). Figure 4a depicts the graphs of this inequality based on the three above-mentioned values for *r* (Ptolemy: the dotted graph, al-Bīrūnī and al-Maghribī: the dotted-dashed graph, and al-Kāshī: the dashed graph) and on the modern formula (the continuous graph). Figure 4b shows the difference between the values computed for this inequality from modern theory and from Ptolemy's and al-Kāshī's values for *r* (respectively, the dotted and dashed graphs). It is then evident that Ptolemy's value of 5;15 for *r* keeps the values of the first inequality in closer agreement with modern theory than do the two medieval values for *r*.

5.2 Accuracy of the Lunar Eclipse Observations in the Late Medieval Islamic Period

As mentioned earlier, making use of bowl-shape clepsydras may be traced back to Babylonian and ancient Indian astronomy. It is probable that the Babylonians utilized clepsydras in order to determine eclipse timings (Stephenson, 1997: 59). In Chinese astronomy there was a long-term intention to measure the times of the phases of eclipses directly with the aid of clepsydras (Stephenson, 1997: Chapter 9). The accuracy attained is around 15 minutes. However, there is evidence to verify that Chinese astronomers could measure the times of sunrise and sunset with an accuracy of around 5 minutes (Stephenson, 1997: 278). In medieval Islamic astronomy eclipse timings were usually measured directly from the altitude of the Sun (in the case of solar eclipses) or reference stars (in the case of lunar eclipses). In mid-latitudes and for mid-altitudes, such measurements might be accurate to within 5-6 minutes (Stephenson, 1997: 466). Although time-measuring instruments were in common use in medieval Islamic society, no details of the application of any device for measuring the times of the phases of eclipses may be found in medieval Islamic astronomy prior to al-Maghribī. As suggested earlier, the clepsydra he used may have been a Chinese model that was brought to the Maragha Observatory by Chinese astronomers. Since then, the use of the two methods (altitude-clepsydra) together appears to have been established as the standard in the second period of the Maragha Observatory (1283-1320). The times measured by *Pangān* were called *sā'āt al-bankām*, 'Pangān's time', in order to distinguish them from the times computed from altitude readings, *sā'āt al-irtifā'*, or 'altitude time'. For instance, Wābkanawī (IV. 15.8–9: T: fols. 92r-v, Y: fols. 159r-160r, P: fols. 139r-140r) emphasized that this might reduce the probable errors in time measurement. The use of the two methods simultaneously is also proposed in the already-mentioned "Ghāzān's treatise on observational instruments", written in the same period (the translation of the relevant passage in Mozaffari and Zotti, 2012: 419-421). It is noteworthy that during the seventeenth century, European astronomers still preferred to time eclipses by measuring altitudes rather than relying on mechanical clocks (Stephenson and Said, 1991: 207, note 26).

Unlike al-Bīrūnī and al-Maghribī, al-Kāshī did not explain how he measured the times during his observations of lunar eclipses. Neither was an instrument mentioned, nor did his account include any stellar altitudes. With regards to the times reported (Tables 2 and 8, Stephenson 1997: 491-492), it is obvious that al-Bīrūnī and al-Maghribī were better observers than al-Kāshī.

6 NOTES

1. Throughout this paper I use a sexagesimal notation, where a semi-colon always follows the number or primary unit (usually degrees or hours), after which comas are used. For example, in this Abstract 5;17 means 5 and 17/60 units, while in Table 1 on page 314 37;20,30° = 37° 20′ 30″. On page 318 in the second paragraph in the right hand column $0;24,17^h$ = 0h 24m 17s and 252;13,53,38° = 252 degrees 13 minutes 53 38/60 seconds. On page 320, immediately after Equation (1), 0;6,33° = 0 degrees 6 minutes 33 seconds, and later, in the second paragraph in Section 5.1, 5;16,46,36 means 5 + 16/60 + 46/3600 + 36/21600 (where 3600 = 60 × 60 and 21600 = 60 × 60 × 60).
2. For Islamic *zījes* considered in this paper see Kennedy (1956) and King and Samsó (2001). A new survey of Islamic astronomical handbooks has been prepared by Dr Benno van Dalen, and his *Islamic Astronomical Tables. Mathematical Analysis and Historical Investigation* will be published by Ashgate/Variorum in February 2014. Biographical sketches of astronomers mentioned in this paper can be found in Gillispie (1970-1980) and Hockey et al. (2007).
3. Note that I consider errors of around half an hour or so as tolerable because, as we will see later in the paper (in Section 5.1), the elements incorporated in the Ptolemaic model are to account only for the two lunar anomalies. Furthermore, the times given in purely observational reports from the Late Medieval Islamic Period differ in accuracy from about +5 minutes to about one hour, which is nearly equal to the errors in the theoretical values computed from the astronomical tables.

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