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BURMESE SHADOW CALCULATIONS

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Abstract: The methods of calculation of shadow lengths of the Sun and the Moon seem to be a specific for Burmese astronomy and have many original features. The present paper gives a detailed exposition of these methods with an analysis of an example taken from a Burmese manuscript.

Keywords: Burmese astronomy, solar shadow length, lunar shadow length

1 INTRODUCTION

Some of the traditional calculation procedures of South-East Asia appear at first sight to be impenetrable, a complicated jumble of figures accompanied by obscure labels usually of Pali derivation and of no explanatory value. With some application and perseverance, however, and a constant awareness of error in the arithmetic, it is often possible to unravel the reckoning, a process that involves constructing what in modern terms would be the 'right' answers as a means of isolating and being able to replicate what their 'wrong' answers were. In the course of analysis one becomes aware that in a pre-telescopic age number was power and also that the procedures were cast in such a way that their mechanical operation led to results whose theoretical basis went without challenge. But so did whatever anomalies and inconsistencies might creep into the reckoning. A prime example of this is where in the procedure leading to the mean reckoning of the Sun and the Moon it was the practice to subtract 3′ from the Sun's value and 40′ from the Moon's value. This adjustment can clearly be seen to have been a longitudinal correction based upon Ujjain, the ancient Greenwich, and as a correction it was roughly appropriate for Burma; but it was used routinely and hence without comprehension across South-East Asia (see Eade, 1995). An exception is the more modern *Thandeikhta* that only has a lunar correction of –52′ and so deviates from this pattern.

One of the more complex and interesting sets of procedure can be found in various Burmese documents that detail how the Sun's shadow length is to be calculated (for the purpose of casting horoscopes) and by extension how a similar process is applied to the Moon. The Moon's shadow length appears early on (in the fifteenth century) in the ancient Burmese capital of Pagan (Site 479, 21° 9′ 42″ N, 94° 54′ 11″ E, Burmese Era 767 Kason 7 waning (20 April 1405): "Monday, early morning cock crow 3 times, shadow of Moon 6 1/2 feet plus 4 hands, son born."), and at a time well before the revision of the system that displaced the *Makaranta* procedures by the *Thandeikhta*. The evidence we have comes from a period that must post-date this reform since consistently the shadow calculation adjusts for precession, though how far back the procedure stretches beyond the printed form in which some of the data is available cannot be assessed. In what follows we give an account of the correct procedures in modern terminology, together with an indication of what was actually done.

About ten years ago we discovered in a Burmese astronomical text (Mauk, 1971) a strange numerical table that was obviously connected with a shadow calculation. The calculation was, however, badly corrupted and full of errors. Other texts that appeared also were corrupt. One obstacle was also that, as we eventually discovered, the gnomon height was 7 units while the standard length for example in India is 8 or 12 (Pingree, 1978). Finally we obtained a printed text (Thi, 1936) that had a list of intermediate calculation results that were sound and enabled us to recreate the calculation procedure.

Our Burmese informant (Ko Ko Aung, pers. comm., June 2011) indicates that many Burmese villages in older times had a gnomon set up and when a child was born the gnomon would have been consulted for the shadow length, then used as the basis for casting a horoscope. There was a corresponding calculation procedure for the Moon, although given the difficulty of measuring in practice such shadows, one imagines that 'Moon shadow' values were purely notional. The calculation system has the merit that it can be applied at any time irrespective of physical conditions, to say nothing of its assumed superior accuracy because you are then juggling with numbers and not using measurement.

2 FUNDAMENTALS

In order to calculate the relation between time and shadow length you need some fundamental data. First you need to know the date in order to calculate the longitudes of the Sun and the Moon. Since Burmese astronomy, as in all parts of South-East Asian and India, uses sidereal longitudes, you have, in the more 'modern' versions that we have investigated, to correct for precession.

For your particular location you need also to know its latitude and the lengths of the day and night and finally the rising times for the different zodiacal signs at that place. Time in Burmese astronomy, as in Hindu astronomy, is measured in *nadis* and *vinadis* (Burmese *nayi* and *bizana*), 60 *nadis* being a day and night and 60 *vinadis* being a *nadi*.

3 THE LONGITUDES

As the calculation of true longitudes is somewhat complex we have placed an example in the Appendix.

4 PRECESSION

The Burmese used the Hindu system, where the difference between the tropical and sidereal longitudes is represented by a linear zigzag function with an amplitude of 27° and a period of 7,200 years, the zero being in AD 412. For years between AD 412 and AD 2212 it is +54″ per year. The Burmese allowed for precession by using the following algorithm, valid for the time interval above:

1) Convert the Burmese year to the Kaliyuga era by adding 3,739.
2) Add the era constant, 88.
3) Divide the result by 1,800 and save the remainder.
4) Multiply by 9 and divide by 10.
5) Divide the integer part of the result by 60. The integer part of the result is the degrees of the precession; the remainder is the arc minutes.
6) Multiply the remainder of the result in 4) by 6 to get the precession in arc seconds.

For details see the sample calculation below.

5 LENGTH OF DAY AND NIGHT

Using modern mathematical language, the ascensional difference or the difference A (plus or minus) in half a day from 6 hours/15 nadis is given by

$$\sin A = \tan \varphi \tan \delta \qquad (1)$$

where φ is the geographical latitude of the location and δ is the declination of the luminary, the Sun or the Moon. A here is given in degrees with 90° = 6 hours = 15 *nadis*.

The declination, δ, above is given by

$$\sin \delta = \sin \lambda \sin \varepsilon \qquad (2)$$

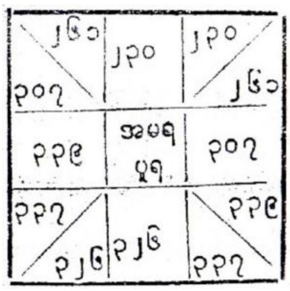
where λ is the true longitude of the luminary, corrected for precession, and ε is the obliquity of the ecliptic, with the Hindu value of 24°. The Moon is treated as though it has zero latitude.

In practice the value of A, in *vinadis*, is given for a number of fixed locations in Burma as the three values for λ = 30°, 60° and 90° and intermediate values are calculated by linear Interpolation. See the Appendix for an example.

Once we know the difference, A, we can calculate the length of one half day, D, by adding to or subtracting A from 6 hours or 15 *nadis*.

6 RISING TIMES

The rising times of the zodiacal signs can be calculated once we know the location, and again this is pre-calculated and displayed in graphical form as a diagram of rising times (see Figure 1 and Table 1). Figure 1 is set up for Amarapura, formerly a capital of Burma, and now in the southern part of the Mandalay conurbation.



261 / 307	230	230 / 261
339	Amara-pura	307
337 / 326	326	339 / 337

Figure 1 (left): Rising time diagram.
Table 1 (right): Translation of numbers in Figure 1.

In modern language these numbers are differences of oblique ascension. The Appendix shows how to calculate these numbers.

The top segment is Aries and the other signs follow in anti-clockwise order. Each sign segment gives the rising time of that sign expressed in *vinadis*.

7 THE SOLAR SHADOW CALCULATION

The Burmese shadow calculation uses a gnomon height, G, of 7 'feet', which usually is further divided into 420 smaller units. The equinoctial noon shadow, S_{eq}, is given by $G \tan \varphi$ for the gnomon and is displayed as a number associated with each listed location (see the Appendix).

The first part of the procedure is to calculate the noon shadow, S_{noon}, for the particular date.

This is given by the expression

$$S_{noon} = \left| G \tan(\varphi - \delta) \right| \quad (3)$$

The vertical bars denote absolute value, skipping the sign.

Figure 2: Table in Mauk (1971: 154).

The Burmese method consists of giving a table of the *phawâ*, the differences between the noon shadow and the equinoctial noon shadow. The table gives the value of this difference of either luminary at longitudes 30°, 60°, 90° and 270°, 300° and 330°, with symmetries 60° = 120°, 30° = 150°, 210° = 330° and 240° = 300°. Intermediate values are interpolated. Once the *phawâ* is calculated, the noon shadow can be computed by subtracting the *phawâ* from the equinoctial noon shadow if the longitude of the luminary is <180°, or else by adding it to the equinoctial noon shadow.

The next step in the procedure is highly interesting. Doing an exact calculation of the relation between time and shadow is very complicated, and remarkably the Burmese resort to a theoretical model. As is usual in South-East Asian reckoning, theory becomes embedded in —effectively buried in—tables. Before the tables are examined it will be useful briefly to consider the model in general terms.

At noon the shadow is shortest and of course equal to the noon shadow. The change in length of the shadow at other times, H, being the time counted from noon, will be zero at noon and infinite at sunset/rise. The change can be modelled by the mathematical expression

$H / (D - H)$

where D is the time from noon to sunrise/set. This expression is clearly zero when $H = 0$ and infinite when $H = D$, i.e., has the correct values as its boundaries. We can multiply this expression by a multiplier, M, without distorting the boundary values.

The Burmese model is now that the total shadow, S, is given by

$$S = S_{noon} + [(M \times H) / (D - H)] \quad (4)$$

with a value of M suitably chosen to approximate the real shadow length for all times from noon to sunrise/set.

Table 2: Translation of the numbers shown in Figure 2.

			Nadis	2	4	6	8	10	12	14	16	18
Pisces	Aries	Virgo	Libra	4	5	6	6	6	5	4:20		
	Taurus	Leo		7	7	7	7	6	6	5	4	
	Gemini	Cancer		8	8	8	7:30	7	7	6	5:30	4
Scorpius	Sagittarius	Capricorn	Aquarius	2	3	4	5	5:40	5:40	5		

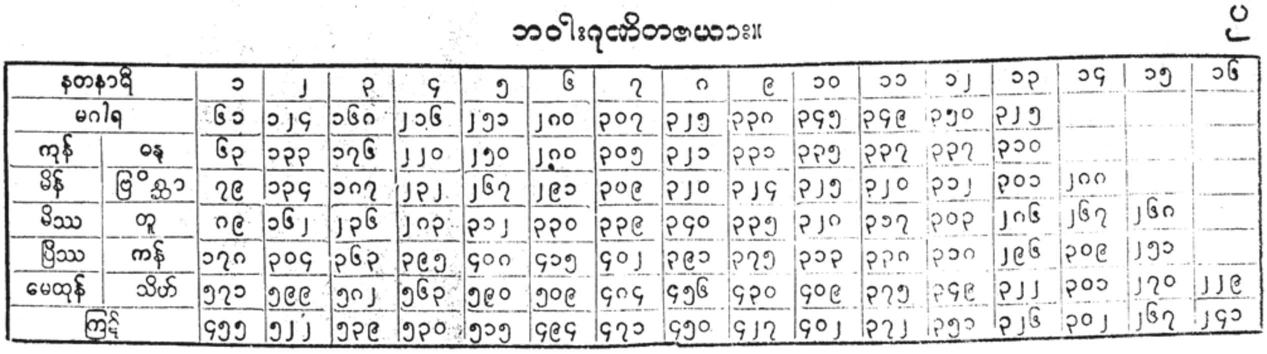


Figure 3: Table in Thi (1936: 12).

Table 3: Translation of numbers in Figure 3.

Nadis		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Capricorn		61	124	168	216	251	280	307	325	338	345	349	350	325			
Aqu	Sag	63	133	176	220	250	280	305	321	331	335	337	337	310			
Pis	Sco	79	134	187	232	267	291	309	320	324	325	320	312	301	288		
Aries	Libra	89	162	236	283	312	330	339	340	335	328	317	303	286	267	268	
Tau	Virgo	178	304	363	395	408	415	402	391	375	313	338	318	296	309	251	
Gem	Leo	571	599	582	563	590	509	484	456	430	409	375	349	322	301	270	229
Cancer		455	522	539	530	515	494	471	450	427	402	372	351	326	302	267	241

The multiplier, M, can be determined empirically or theoretically by inverting this expression where all the quantities in the right hand member can be calculated or measured to give

$$M = [(S - S_{noon}) \times (D - H)] \; / \; H \qquad (5)$$

You would expect, for reasons of scale, that M would be of the order of the gnomon height, 7 or 420, depending on the units used. Unfortunately, it turns out that the multiplier, M, which may be expected to be constant, is in fact a function dependent on the geographical latitude, φ, the time, H, and the longitude, λ, of the luminary. The latitude dependence is rather slow and the Burmese use an M that is a reasonable average, approximately valid for any location in Burma; but they still have to deal with the dependence on time and longitude. The Burmese solve this by having a double-entry multiplier table, the entries being time difference from noon in the horizontal and longitude of the luminary in the vertical.

We had available two printed text variants of this table, one by Mauk (1971: 154) and shown in Figure 2 (see, also, Table 2), and the other by Thi (1936: 12), which is shown in Figure 3 (and see Table 3 as well). The table in Mauk is by comparison crude and condensed, with only four entries for longitude and entries for time only for every second *nadi*. The value for M in this table varies between 2 and 8. The other version of the table uses a unit for M that is 60 times larger (G = 420) and has more entries for both longitude and time.

We do not know the original procedures used to create these tables, but we did a computer calculation of a table using Formula (5), above, and real shadow lengths for geographic latitude 22°. The generated result, Table 4, is quite similar to the Burmese table, the differences being small enough to suggest only minor variation in the original reckoning.

Given the existence of a table for the multiplier, M, the aim of the astrologer is to calculate time from the shadow. If we solve Equation (4) above for the time H we get:

$$H = [(S - S_{noon}) \times D] \; / \; [(S - S_{noon}) + M] \qquad (6)$$

M now appears as an additional term.

There is an obvious problem with the relation above. M is a function of H, so to calculate H we need to know the value of H in order to know M. The Burmese solution is to start with a default value of M = 7 and calculate a preliminary time H and then use this time H to get a better value of M from the table, insert it Formula (6) to get an improved value of H. This is an interesting application of successive approximations.

In practice a mathematically-equivalent expression is used for Formula (6):

$$H = D - \{(D \times M) \; / \; [(S - S_{noon}) + M]\} \qquad (6')$$

This mathematical procedure was turned into a series of steps to be learned by rote and in consequence some of the sources available to us go wildly astray in the elements they select for processing. But even from these confused calculations it was possible to arrive at a good estimate of how the original procedure must have looked and we could use the intermediary calculation values in a printed text (Thi, 1936) to vindicate our estimate.

The resulting time, H, is used to calculate back the shadow, in a reckoning that uses a modified, but mathematically-equivalent, version of Equation (4):

$$S = S_{noon} - [(D \times M) \; / \; (D - H)] - M \qquad (4')$$

8 THE LUNAR SHADOW CALCULATION

To begin with, this calculation is identical to that for the Sun. It tacitly ignores the latitude of the Moon, as was the case also with the planets in astronomical tables. Using the Moon's true longitude, corrected for precession, it is easy to calculate the length of the lunar 'day' and 'night'. Using the formulae above from a measured or notional Moon shadow, the Burmese could calculate the corresponding time from the lunar 'noon', the time of the culmination of the Moon. The problem is now to find the solar clock time of lunar noon.

Knowing the longitude of the Moon will determine the location in the rising time chart of moonrise and moonset. Lunar noon will be located midway in time from these points. As the difference, A, of the solar day is known, the interval in time from sunset to the time instant 45 *nadis* after midnight, or 6pm, is known. Also known is the longitude of the Sun, and thus the locations in the rising time chart of sunrise and sunset. This determines the point in the rising time chart

Table 4: Computer-generated multiplier table

Nadis		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Capricorn		69	128	178	219	254	281	304	321	335	346	354	359	363			
Aqu	Sag	69	127	176	216	249	275	296	312	324	333	339	343	346			
Pis	Sco	72	132	181	220	250	273	290	302	310	315	317	317	316	315		
Aries	Libra	92	165	220	258	284	301	311	316	318	317	313	309	303	296	288	
Tau	Virgo	168	273	329	356	368	371	369	363	355	346	335	325	313	302	290	
Gem	Leo	499	532	524	507	487	467	446	427	407	389	371	354	337	321	306	291
Cancer		449	506	509	498	481	464	445	427	409	391	374	358	342	327	312	298

corresponding to the time instant 6pm. Counting the rising times up to lunar noon will then give the clock-time instant of lunar noon. Knowing this and the value of *H* will finally give the clock time instant corresponding to the given shadow.

9 AN EXAMPLE

The results of a shadow calculation are presented in one of the available texts (Thi, 1936). On its last page (see Figure 4) the text gives a set of intermediary results of a Moon shadow calculation. The date, given in the text, is 1297 Pyatho 3 waning in the Burmese era, 9 January 1936. The location is Amurapura. A shadow

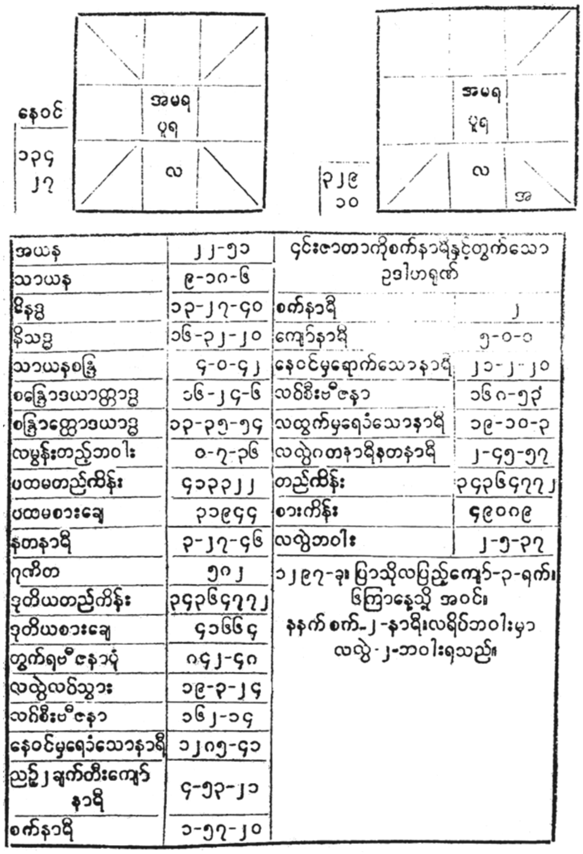


Figure 4: Moon shadow calculation (after Thi, 1936. 25).

length of 2 after lunar noon is also given in the text.

Number format is sexagesimals, a:b:c = a + b/60 + c/3600, except for longitudes where a:b:c = a[zodiacal sign] + b[degrees] + c[arc minutes].

The rising time diagrams at the top have numbers 134:27 and 329:10 to their left. We label the lines in the lower square A, B … T and translate the numbers:

A	22:51	
B	9:18:6	
C	13:27:40	2
D	16:32:20	5:0:0
E	4:0:42	21:2:20
F	16:24:6	168:53
G	13:35:54	19:10:3
H	0:7:36	2:45:57
I	413322	34364772
J	31944	49089
K	3:27:46	2:5:37
L	582	
M	34364772	
N	41664	
O	842:48	
P	19:3:24	
Q	162:14	
R	1285:41	
S	4:53:21	
T	1:57:20	

The number in line A is the correction for precession. Using the algorithm given above we have:

1297 + 3739 + 88 = 5124
5124 / 1800 = 2, remainder 1524
1524 · 9 /10 = 1471, remainder 6
1471 / 60 = <u>22</u>, remainder <u>51</u>
6 · 6 = <u>36</u>

Thus the precession is 22:51:36, where the text has 22:51. The number in line B is the true longitude of the Sun corrected for precession, which is 9:18:6.

The differences of a half-day (in *vinadis*) for Amurapura are 48, 86 and 102, for solar longitudes 30°, 60° and 90° respectively. As the Sun is in sign 9 it will have 30° – 18:6° = 11:54° to reach sign 10. Sign 9 has (by symmetry) a day difference of 102, sign 10 has 86. Interpolation gives 86 + 11:54/30·(102 – 86) = 86 + 6:20:48 = 92:20:48 = 1:32:21.

A half-day is then 15:0:0 – 1:32:21 = 13:27:41, and line C in the text has 13:27:40.
A half-night is 30:0:0 – 13:27:41 = 16:32:19, and line D has 16:32:20.
The number in E is the Moon's true longitude, corrected for precession, 4:0:42.
Calculating the half lunar day and night as for the Sun gives 16:25:7 and 13:34:53, while the text has 16:24:6 and 13:36:54 for lines F and G.

The complete *phawâ* table for Amurapura using the symmetries is shown in Table 5.

The Moon's longitude is 4:0:42. Interpolating in the *phawâ* column gives 157:26. The equinoctial noon shadow for Amurapura is 165, giving a noon shadow of 165 – 157:26 = 7:34, and the text in line H is 0:7:36. The Moon shadow is 2:0:0 and the time is after the Moon's culmination or lunar noon. The Burmese start with a preliminary multiplier, *M* = 7, and they calculate a preliminary time, *H*, using Equation (6′). The half lunar day, *D*, according to the text is 16:24:6.

$D \cdot M$ = 16:24:6 · 7 · 3600 = 413322, line I

$S - S_{noon} + M$ = (2:0:0 – 0:7:36 + 7) · 3600 = 31944, line J

413322 / 31944 = 12:56:20

$H = D$ – 12:56:20 = 16:24:6 – 12:56:20 = 3:27:46, line K

This is the preliminary shadow time. Entering the multiplier table above with sign 4 and time 3 gives a multiplier 582, line L.

Repeating the calculation:

$D \cdot M$ = 16:24:6 · 582 · 3600 = = 34364772, line M

$S - S_{noon} + M$ = ((2:0:0 – 0:7:36) · 60 + 582) · 60 = 41664, line N.

34364772 / 41664 = 824:48

The text has 842:48 in line O but uses 824:48 for the following calculations:

824:48 = 13:44:48

$H = D$ – 13:44:48 = 16:24:6 – 13:44:48 = 2:39:18.

This is the final time in *nadis* after lunar noon. To obtain the time, T, from moonrise we have

$T = D$ + 2:39:18 = 16:24:6 + 2:39:18 = 19:3:24, line P.

The Moon has a longitude 4:0:42 or Leo 0:42. The rising time of the sign Leo is 337 *vinadis*. Thus moonrise is 0:42 / 30 337 = 7:52 *vinadis* into Leo and has 337 – 7:52 = 329: 8 left of that sign. The rising time diagram in the text has 329:10 and thus 7:50 instead of 7:52.

The shadow time 19:3:24 is then 19:3:24 + 0:7:50 = 19:11:14 from the beginning of Leo. Subtracting subsequent rising times: Leo 337, Virgo 326, Libra 326 tells us that the shadow time is 162:14 *vinadis* into Scorpio, line Q.

The Sun has a longitude of 9:18:6, and the opposite point on the ecliptic is 3:18:6, or Gemini 18:6. The rising time for Gemini is 339 *vinadis*, thus sunset is 18:6/30·339 = 204:32 *vinadis* into Gemini and remaining 339 – 204:32 = 134:28 to the beginning of Leo. The rising time diagram in the text has 134:27.

Thus the shadow time interval from sunset is 134:27 + 7:50 + 19:3:24 = 1285:41, line R. The difference from 15 *nadis* of half a solar day is 1:32:20. Thus, sunset occurs at 45:0:0 – 1:32:20 = 43:27:40 *nadis* from solar midnight. Adding the remaining part of Cancer from sunset tells us that the end of Cancer corresponds to 43:27:40 + 134:27 = 45:42:7 *nadis* from solar midnight.

Moonrise corresponds to 45:42:7 + 7:50 = 45:49:57 *nadis* from solar midnight. Adding the half lunar day 16:24:6 and the shadow time interval 2:39:18 after lunar noon gives us 45:49:57 + 16:24:6 + 2:39:18 = 64:53:21 = 4:53:21; the text has 4:53:21 in line S. This is the shadow time in *nadis* after solar midnight. The number in line T in the text is this time converted to hours:minutes:seconds.

The right-hand column of numbers is the back calculation, time to shadow.

D: The shadow time is rounded to 5:0:0, which is 6:39 *vinadis* later than the time in item S. 5:0:0 *nadis* after midnight corresponds to 2 a.m. in line C (right).

E: 21:2:21 is a misprint for 21:32:21 and is R (left) + 6:39.

F: 168:53 = Line Q (left) + 6:39.

G: 19:10:3 = Line P (left) + 6:39.

H: 2:45:57 is the shadow time after lunar noon 2:39:18 increased by 6:39.

Using Equation (4′) we now calculate shadow from time. With H = 2:45:57 ≈ 3 and a lunar longitude 4:0:42 ≈ 4 we get a multiplier M = 582.

D = 16:24:6, $D \cdot M \cdot 3600$ = 354364772, line I.

$(D - H) \cdot 3600$ = (16:24:6 – 2:45:57) · 3600 = 49089, line J.

$D \cdot M / (D - H)$ = 354364772 / 49089 = 700:3:1

$D \cdot M / (D - H) - M$ = 700:3:1 – 582 = 118:3:1 = 1:58:3

$S = S_{noon} + D \cdot M / (D - H) - M$ (Equation 4') = 1:58:3 + 0:7:34 = 2:5:37, line K.

This value is close to the shadow value we started with and gives a check that the calculation is correct.

Table 5: *Phawâ* table (after Thi, 1936: 24).

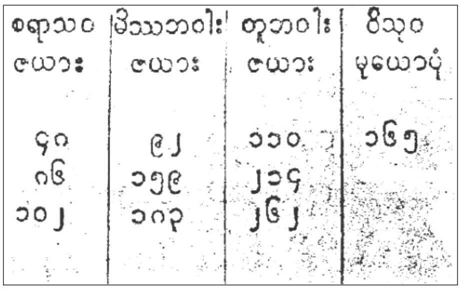
Sign		Phawâ
9		262
10	8	214
11	7	110
0	6	0
1	5	92
2	4	159
3		183

10 DISCUSSION

The original mode of astronomical reckoning in South-East Asia, found in Cambodia as far back as the seventh century AD, was inherited from India, with AD 78 as its epoch (Pingree, 1978). In the late thirteenth century this system was replaced in Burma and Thailand by a canon with an epoch of AD 638, which also was from India. This system, known in Burma as *Makaranta*, was eventually modified into the *Thandeikhta* mode in the mid-eighteenth century. While reference to solar and lunar shadow length occasionally can be found in Burmese records of the fifteenth century, the tradition reflected in our printed texts is from a somewhat later period and is distinctive in its routine use of an adjustment for precession and its adoption of successive approximation for shadow calculation.

This latter technique, although historically of great antiquity elsewhere, is not evident for instance in the reform the Thais made to their calculation of eclipses (assigning an epoch of AD 1142); and the precision and sophistication adopted in shadow reckoning was not, to our knowledge, adopted in other allied procedures. Indeed, it is symptomatic of both the Burmese and the Thai systems that more precise modes of reckoning were adopted only when a particular isolated need for them arose. The Thais continue to use their more approximate method of computation for day-to-day reckoning, and the Burmese use successive approximation for shadow length while at the same time not adjusting for the Moon's considerable motion between rising and setting (see Eade, 1995).

It is also symptomatic of the hold that traditional thinking still retains today in Thailand that there is a market for calendars that use the 638 canon to locate the Sun and the Moon (the annual 'Diary Hon'), while in Burma, as we have found, a complicated text has to posit even in its twentieth century printed form that the shadows of the Sun and of the Moon are more readily accessible than a clock time of day.



၄၈	၉၂	၁၁၀	၁၆၅
၈၆	၁၅၉	၂၁၄	
၁၀၂	၁၈၃	၂၆၂	

Figure 5: Data table for Amurapura (after Thi, 1936: 24).

Table 6: Translation of the numbers in Figure 5.

48	92	110	165
86	159	214	
102	183	262	

According to modern conception, the underlying theory encapsulated in a formula is in general of more importance than the results that it happens to generate. In the tradition to which our texts belong, once an expert has devised a procedure and embodied it in a series of mechanically-implemented steps and in tables, the number eventually arrived at takes on quasi-magical properties. Our concern has been with what it was that the theorist was doing in an ingenious procedure whose rationale lies well below the surface: the purchasers of his text would be concerned, by contrast, with what painfully-acquired and life-controlling number the procedure would generate.

11 ACKNOWLEDGEMENTS

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13 APPENDICES

13.1 Data Table for Amurapura

In Table 6 (see, also Figure 5) the first column displays the difference in day length for longitudes 30˚, 60˚ and 90˚ expressed in *vinadis*. For a translation of the numbers, see the table section. The second column shows the *phawâ* for the same longitudes and by symmetry also for 150˚ and 120˚. The third column shows *phawâs* for longitudes 210˚, 240˚ and 270˚ and by symmetry also for 330˚ and 300˚. The last column shows the equinoctial noon shadow for a gnomon with height 7·60 = 420 units. The value 165 corresponds to geographical latitude 21.45˚.

13.2 Longitude Calculations

A basic quantity that is used for the longitudes is the *ahargaṇa* of date, *a*, the number of expired days since the epoch. We use mainly corresponding Sanskrit terms for the quantities involved in the calculation.

Denoting by *y*, the Burmese year, and by *s*, the *sutin*, the number of elapsed days in that year we have (Irwin, 1909):

$$a = \{[(y - 233) \times 29227] + [(y - 233)/193] + 17742\}/800 + 1 + s \qquad (7)$$

For the Sun we also calculate the *kyamat* of the date:

$$k = \text{remainder}\{[(y - 233) \times 29227] + [(y - 233)/193] + 17742\}/800 \times s \qquad (8)$$

The mean longitude of the sun in arc minutes is then

$$\lambda_{sun} = [(1000 \times k) - (6 \times s)] / 13528 \qquad (9)$$

The Sun's apogee ω_{sun} is assumed to be located at $\omega_{sun} = 2:17:18$.

The anomaly, α_{sun}, of the Sun is

$$\alpha_{sun} = \lambda_{sun} - \omega_{sun} \qquad (10)$$

To calculate the elongation of the Moon from the Sun we first calculate the *avaman*, *A*, and *khaya*, *K*, of the date:

$$\{[(a \times 11) - (y - 233)] / 25 + 175\} / 692$$

$= K{:}A$ (13)

K is the integer part of the division and A the remainder.
The Moon's elongation is then

$\eta = A + (7 \times A) / 173 + 12 \times [(a + K) \bmod 30] \times 60 - 52'$ (14)

and the mean longitude of the Moon is

$\lambda_{moon} = \lambda_{sun} + \eta$ (15)

The Moon's apogee is moving and its location is calculated by first calculating

$u = (a + 316) \bmod 3232$ (16)

and the Moon's apogee then is

$\omega_{moon} = (3 \times u) / 808 - 0:4:24$ (17)

The anomaly is

$\alpha_{moon} = \lambda_{moon} - \omega_{moon}$ (19)

The equation of centre, or the correction in arc minutes to the mean longitude, is given in tabular form as a *chaya*. It is a table with arguments of angle from 0˚ to 90˚ in 24 steps of 3.75˚. If the anomaly is larger than 180˚ we use as argument the 360˚ complement of the anomaly, and if this complement is larger than 90˚ the 180˚ complement of the complement is used.

The *chayas* are given in Table 7 below.

The *chaya* value is added to the mean longitude if the anomaly is larger than 180˚, otherwise subtracted from the mean longitude.

By way of an example, let us examine Burmese year 1297 Pyatho 3 waning, y = 1297, s = 268.

$a = \{[(1297 - 233) \times 29227] + [(1297 - 233) / 193] + 17742\} / 800 + 1 + 268 = 72247$

$k = \text{remainder}\{[(1297 - 233) \times 29227] + [(1297 - 233)/193] + 17742\}/800 \times 268 = 21507$

$\{[(72247 \times 11) - (1297 - 233)] / 25 + 175\} / 692 = 1148:470$

$\lambda_{sun} = [(1000 \times 215078) - (6 \times 268)] / 13528 = 15898' = 6:24:58$

$\eta = 470 + (7 \times 470) / 173 + 12 \times [(72247 + 1148) \bmod 30] \times 60 - 52' = 11237'$

$\lambda_{moon} = 15898 + 11237 = 5535' \quad = 3:2:15$

$u = (72247 + 316) \bmod 3232 - 1459$

$\omega_{moon} = (3 \times 1459) / 808 - 0:4:24 = 5:12:30 - 0:4:24 = 5:8:6$

We get:

$\alpha_{sun} = 11260' = 6:7:40 \qquad \alpha_{moon} = 17649' = 9:24:9$

These anomalies give respectively corrections of +17′ and +276′ and true longitudes are

$\lambda_{sun} = 15915' = 8:25:15 \qquad \lambda_{moon} = 5811' = 3:6:51$

The printed text (Thi, 1936: 25) has 3:7:51. Plus a precession value of 0:22:51 = 4:0:42 for the Moon.

Table 7: *Chayas* for the Sun and Moon (after Mauk, 1971: 85).

Sun		Moon	
Argument	Correction	Argument	Correction
0	0	0	0
1	9	1	20
2	17	2	40
3	26	3	60
4	34	4	79
5	43	5	98
6	51	6	116
7	58	7	134
8	66	8	152
9	73	9	169
10	80	10	185
11	87	11	200
12	93	12	214
13	99	13	228
14	104	14	241
15	109	15	252
16	113	16	262
17	117	17	272
18	121	18	280
19	124	19	287
20	126	20	293
21	128	21	297
22	129	22	300
23	130	23	302
24	131	24	303

12.3 Rising Times and Oblique Ascension

To calculate the rectascension, E, of the Sun given the longitude λ and the obliquity ε = 24˚:

$\tan E = \tan \lambda \cos \varepsilon$ (20)

Subtract the ascensional difference, A, calculated in the day length section above. The oblique ascension is the difference $\Omega = E - A$. The rising times of the zodiacal signs are then the differences between the oblique ascension for sequential signs. As there are 3600 *vinadis* to a solar day and night, a rotation of the Earth by 360˚, the conversion from degrees to *vinadis* is just simply multiplication by 10.

Table 8 shows the result of a calculation of the values in the rising time diagram.

Table 8. Rising times for Amurapura

Longitude (˚)	*E*	*A*	*E–A*	Difference
0	0	0	0	230
30	278	48	230	261
60	577	86	491	307
90	900	102	798	339
120	1223	86	1137	337
150	1522	48	1474	326
180	1800	0	1800	

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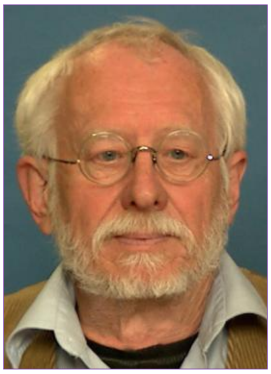

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