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DECLINATIONS IN THE *ALMAGEST*: ACCURACY, EPOCH, AND OBSERVERS

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Abstract: *Almagest* declinations attributed to Timocharis, Aristyllos, Hipparchus, and Ptolemy are investigated through comparisons of the reported declinations with the declinations computed from modern positions translated to the earlier epochs. Consistent results indicate an observational accuracy of ≈ 0.1° and epochs of: Timocharis, c. 298 BC; Aristyllos, c. 256 BC, and Hipparchus, c. 128 BC. The ≈ 42-year difference between Aristyllos and Timocharis is confirmed to be statistically significant. The declinations attributed to Ptolemy were likely two distinct groups—observations taken c. AD 57 and observations taken c. AD 128. The later observations could have been taken by Ptolemy himself.

Keywords: The *Almagest*, Aristyllos, Hipparchus, Ptolemy, Timocharis, stellar declinations

1 INTRODUCTION

The declinations of stars in the *Almagest* are given in Book VII, Chapter 3. In some translations (e.g., Taliaferro, 1952) they are given in the text following the practice of earlier Greek language versions (Heiberg, 1903; Ptolemy, 1538), whereas our principal source (Toomer, 1998) lists the declinations in a table. There are three values for each of 18 stars for a total of 54 observations. The observers are listed as Timocharis, Aristyllos, Hipparchus, and in Ptolemy's words (Toomer, 1998), "As found by us." We refer to the latter as Ptolemy(?) because of uncertainty in his participation. See Section 4 for discussion. These declinations were given without their right ascensions. Declinations are relatively easy to observe. An observer with knowledge of the observing site's latitude can determine a star's declination by measuring its altitude at meridian crossing. Right ascension measurements are more difficult and involve determining a star's angular distance from the right ascension zero point or from a star of presumed known right ascension (e.g., van de Kamp, 1967: Chapter 2, Section 2). The actual observers (besides possible problems with Ptolemy) are somewhat in doubt. We discuss this in Sections 2, 3, 4 and 5.

We note that the declinations that are the subject of this paper are only those recorded in the *Almagest* (Book VII, Chapter 3) and are not augmented from other sources. Specifically, Manitius (1894) gave some 44 declinations from Hipparchus in his *Commentary on Aratos.* These are not close to the *Almagest* declinations in accuracy (Maeyama, 1984) and we do not consider them here. Also note that the declinations under discussion in this paper are distinct from Ptolemy's extensive star catalogue in which the positions are recorded in ecliptic latitude and longitude (Toomer, 1998: 341–399).

Ancient astronomical data can assist modern astronomy by providing specific information such as the date and circumstances of an eclipse (Eddy, 1987) or by providing insight into the origins of modern astronomy. Understanding the *Almagest* declinations is significant because they are an important facet of the beginnings of modern astronomy, via astrometry. Evans (1998: 259) notes that the observations by Timocharis are the oldest observations of position in Greek astronomy and that he "… may be considered the founder of careful and systematic observations among the Greeks." These observations were recorded, survived through history, and were accurate (as we will demonstrate). They were seriously used by Tycho Brahe (e.g., Brahe; 1648; Moesgaard, 1989) and Edmond Halley (e.g., Halley, 1717; cf. Brandt, 2010) many centuries later. Of course, there were other ancient astronomers actively observing the sky, particularly in Babylonia and China. They had lists or catalogues of star positions, but these did not have accurate star positions as their primary goal or did not survive. Babylonian astronomers determined the positions of approximately 31 stars in the zodiacal belt, the so-called Normal Stars, for use in their astronomical diaries as reference points for keeping track of the movements of the Moon and planets (Sachs, 1974; Sachs and Hunger, 1988). This list is probably the first catalogue of star positions. In discussing them, Sachs (1952) would write, that the "… catalogue of Normal Stars … Despite its grossness in rounding-off to integer degrees and its other inaccuracies of as much as 1° or 2°, is nonetheless a real catalogue …" China had a tradition of observing celestial phenomena and keeping records. These have been used, for example, to help determine the past motion of Halley's Comet as far back as 240 BC (Yeomans and Kiang, 1981). Some stellar positions were determined circa 300 BC and they were used to make a map of the heavens (Ronan, 1996; Thurston, 1994). Unfortunately, the original positions were lost. The *Almagest* declinations are special and we know of no other comparable stellar positions from this period in antiquity.

In this paper, we describe, review, and update several earlier approaches for evaluation and draw conclusions from the results. All involve a comparison of the recorded position to a modern calculated position that allows for precession and proper motion. These include the earlier work of Pannekoek (1955), Maeyama (1984), and Rawlins (unpublished manuscript, 1982a). Our preliminary reports were given in Brandt, Zimmer, and Jones (2011; 2013) and in Zimmer, Brandt, and Jones (2013). We also report on a new approach to the data (Section 3). All approaches are consistent with a remarkable accuracy of ≈ 0.1°.

Using translated modern positions to determine the epochs of historical observations has appeared in a recent paper (Barron et al., 2008). Their method yields epochs of images by running proper motions backward in time to produce the configuration that best matches the image. Thus, they fit an entire image rather than individual stars as done by the methods applied to the *Almagest* declinations.

2 THE DATA AND LISTED OBSERVERS

The *Almagest* declinations are listed in Table 1 following Toomer (1998) together with modern names and designations.

Table 1: Declinations in the *Almagest*[1]

Star		Declination(°)			
Almagest Description	Designation	Timocharis	Aristyllos	Hipparchus	Ptolemy(?)
The bright star in Aquila	Altair-α Aql	+5 $^{4}/_{5}$	—	+5 $^{4}/_{5}$	+5 $^{5}/_{6}$
The middle of the Pleiades	Alcyone-η Tau[2,4]	+14 ½	—	+15 $^{1}/_{6}$	+16 ¼
The bright star in the Hyades	Aldebaran-α Tau	+8 ¼	—	+9 ¾	+11
The brightest star in Auriga, called Capella	Capella-α Aur[4]	—	+40	+40 $^{2}/_{5}$	+41 $^{1}/_{6}$
The star in the advance shoulder of Orion	Bellatrix-γ Ori[4]	+1 $^{1}/_{5}$	—	+1 $^{4}/_{5}$	+2 ½
The star in the rear shoulder of Orion	Betelgeuse-α Ori	+3 $^{5}/_{6}$	—	+4 $^{1}/_{3}$	+5 ¼
The bright star in the mouth of Canis Major	Sirius-α CMa	–16 $^{1}/_{3}$	—	–16	–15 ¾
The more advanced of the [two] bright stars in the heads of Gemini	Pollux-β Gem	—	+33	+33 $^{1}/_{6}$	+33 $^{2}/_{5}$
The rearmost [of the bright stars in the heads of Gemini]	Castor-α Gem	—	+30	+30	+30 $^{1}/_{6}$
The star in the heart of Leo	Regulus-α Leo	+21 $^{1}/_{3}$	—	+20 $^{2}/_{3}$	+19 $^{5}/_{6}$
The star called Spica	Spica-α Vir[4]	+1 $^{2}/_{5}$	—	+ $^{3}/_{5}$	– ½
Of the three stars in the tail of Ursa Major -the one at the top	Alcaid-η UMa[4]	—	+61 ½	+60 ¾	+59 $^{2}/_{3}$
-the second from the end, in the middle of the tail	Mizar-ζ UMa	—	+67 ¼	+66 ½	+65
-the third from the end, about where the tail joins [the body]	Alioth-ε UMa	—	+68 ½	+67 $^{19}/_{30}$[3]	+66 ¼
Arcturus	Arcturus-α Boo[4]	+31 ½	—	+31	+29 $^{5}/_{6}$
Of the bright stars in the claws of Scorpius [i.e., in Libra] -the one in the tip of the southern claw	Zubenelgenubi-α Lib	–5	—	–5 $^{3}/_{5}$	–7 $^{1}/_{6}$
-the one in the tip of the northern claw	Zubeneschamali-β Lib	+1 $^{1}/_{5}$	—	+ $^{2}/_{5}$	–1
The bright star in the chest of Scorpius, called Antares	Antares-α Sco	–18 $^{1}/_{3}$	—	–19	–20 ¼

Notes:

1 This Table follows Toomer (1998), with additions.

2 The star η Tau is used as a surrogate for the middle of the Pleiades.

3 Toomer (1998) gives +67 $^{3}/_{5}$° in his table, but lists in a footnote a value of +67 $^{2}/_{3}$°, which he notes "… may be correct". We have adopted the mean or +67$^{19}/_{30}$°.

4 Denotes the six stars selected by Ptolemy for his precession discussion.

Declinations for these stars at a given epoch are readily calculated from modern observations allowing for precession and proper motion. We use position and proper motion values from *Hipparco*s (the European Space Agency's astrometry mission) (Perryman et al., 1997) and precession/geodesy based on NOVAS 3.0 (Kaplan et al., 2009). These modern data should be an improvement on input data used by Maeyama and Rawlins.

We can compute plots of residuals, the *O* (observed) – *C* (computed) declinations, versus epoch (given here by calendar year) for each observer. These are given in Figures 1, 2, 3, and 4. These plots are the basic data for analysis. They can be approached in different ways, but these plots plus some historical information are all that we have.

Uncertainty over the actual observers of the declinations starts with the *Almagest* itself. In an introductory paragraph to the declinations (Toomer, 1998: 330), they are described "… as recorded by the school of Timocharis, as recorded by Hipparchus, and also as determined in the same fashion by ourselves." Thus, the text implies that the declinations attributed to Timocharis might be part of a group effort. While the observations by Timocharis and Aristyllos are noted separately in the Almagest Table, Toomer lists their observations in the same column. To add to the general uncertainty, the translation by Taliaferro (1952) does not say "… school of Timocharis." For matters involving the Greek text and translation, we have consulted Dr Lorenzo Garcia, Classics Program, Department of Foreign Languages and Literature, University of New Mexico. The accurate translation

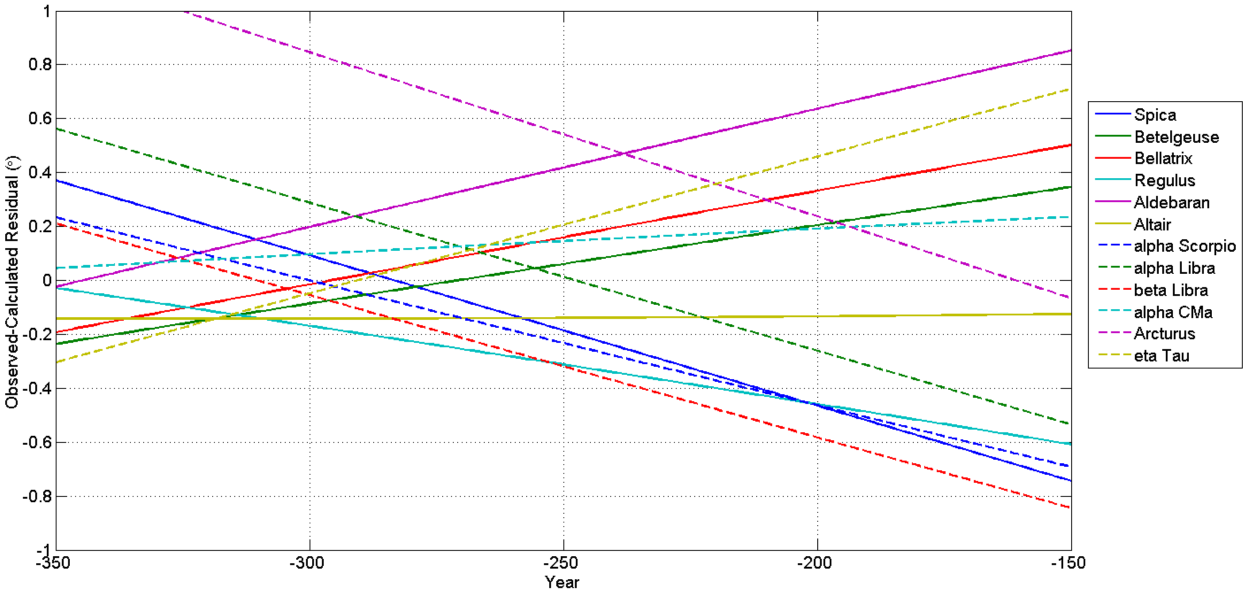

Figure 1: Residuals (Observed minus calculated, *O* – *C*, declinations) for Timocharis as a function of year.

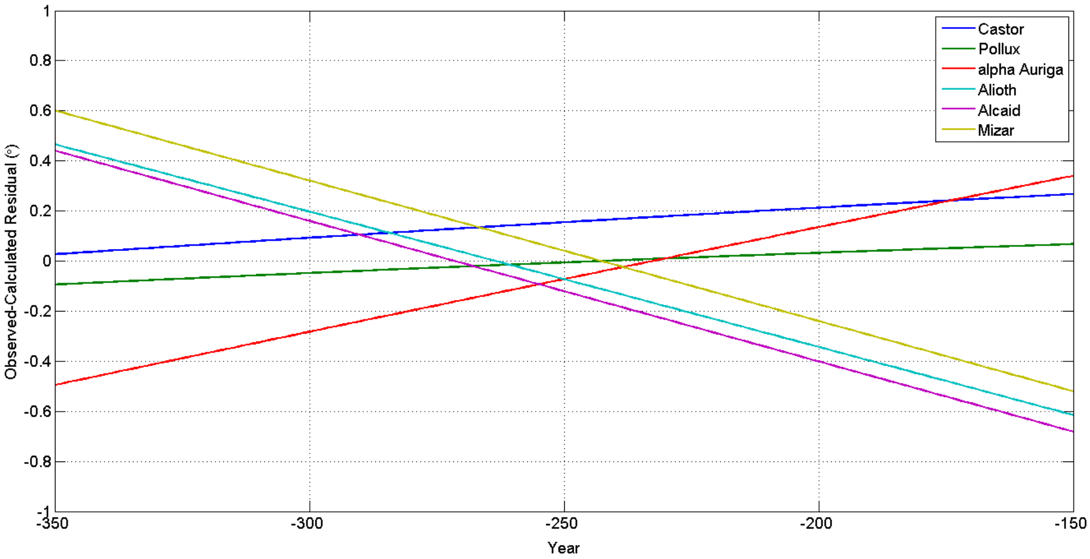

Figure 2: Residuals for Aristyllos as a function of year.

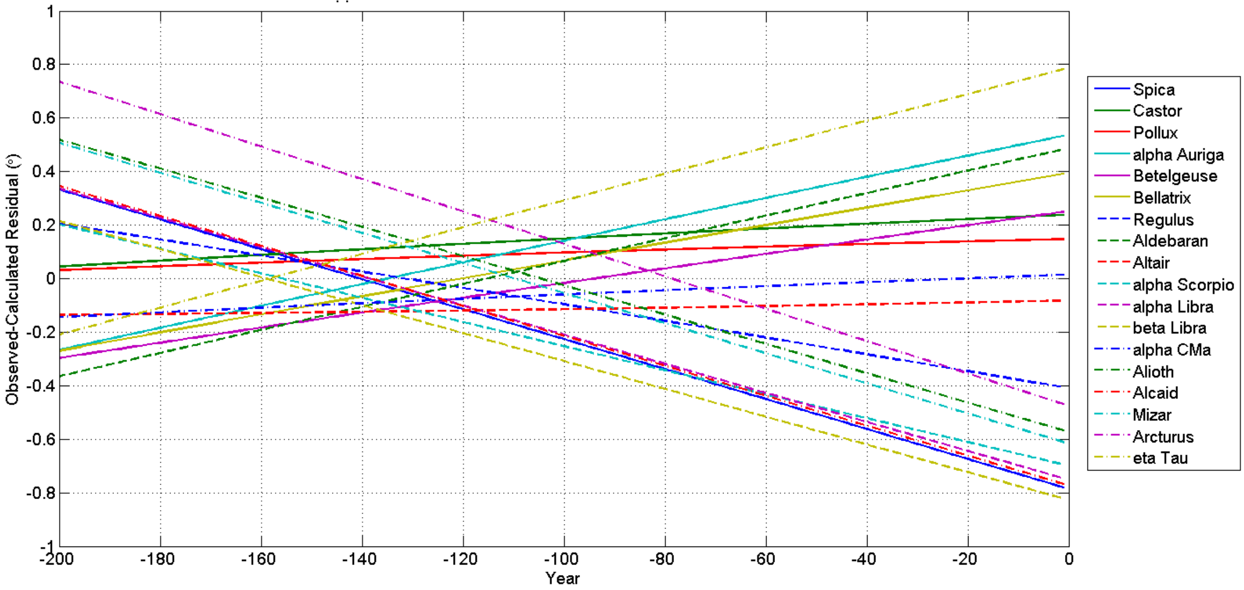

Figure 3: Residuals for Hipparchus as a function of year.

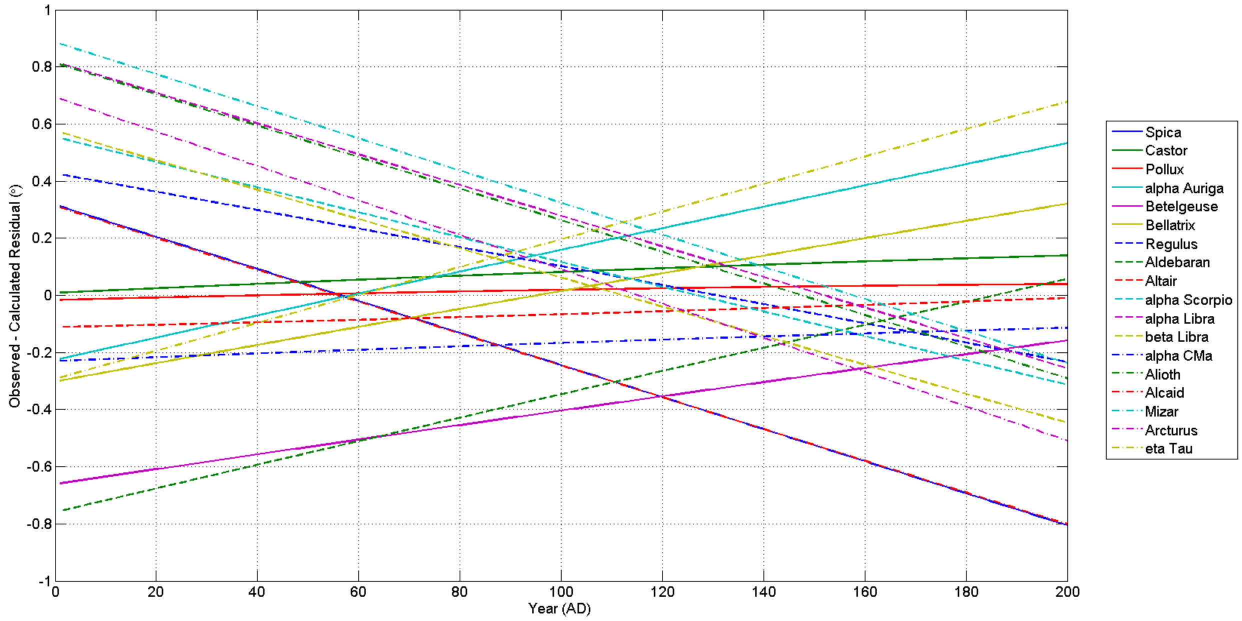

Figure 4: Residuals for Ptolemy(?) as a function of year.

is "… school of Timocharis." In this paper, we consider that the observers are as listed, but that others may have been involved. A special case is Ptolemy(?) as discussed in Section 4.

Historical information for the listed observers is as follows. Timocharis (Sarton, 1959: 53) is known from citations to his work by Ptolemy. He worked at Alexandria during the 290s and 280s BC, possibly in association with the Museum of Alexandria. The *Almagest* declinations mention Timocharis and Aristyllos together. Aristyllos may have been a student of Timocharis. Until recently, Timocharis and Aristyllos were thought to have worked during the same time period. Since the early 1980s, it has been clear that Aristyllos' declinations date approximately 30–45 years later than Timocharis' declinations. See Sections 3 and 5 for discussion.

Hipparchus (Sarton, 1959: 284−285) has specific references from Ptolemy in the time period 161–127 BC. He worked on the island of Rhodes. His lifespan can be estimated as c. 190 BC to c. 120 BC.

Ptolemy's lifespan has been estimated (Pedersen, 1974; Toomer, 1998) to be c. AD 100 to c. AD 175. The *Almagest* was written around AD 150 or perhaps a little later.

3 APPROACHES TO THE DATA

The earliest declinations were often not regarded as accurate. This idea goes back to Ptolemy himself. He notes that Hipparchus

> … had found very few observations of fixed stars before his own time, in fact practically none besides those recorded by Aristyllos and Timocharis, and even these were neither free

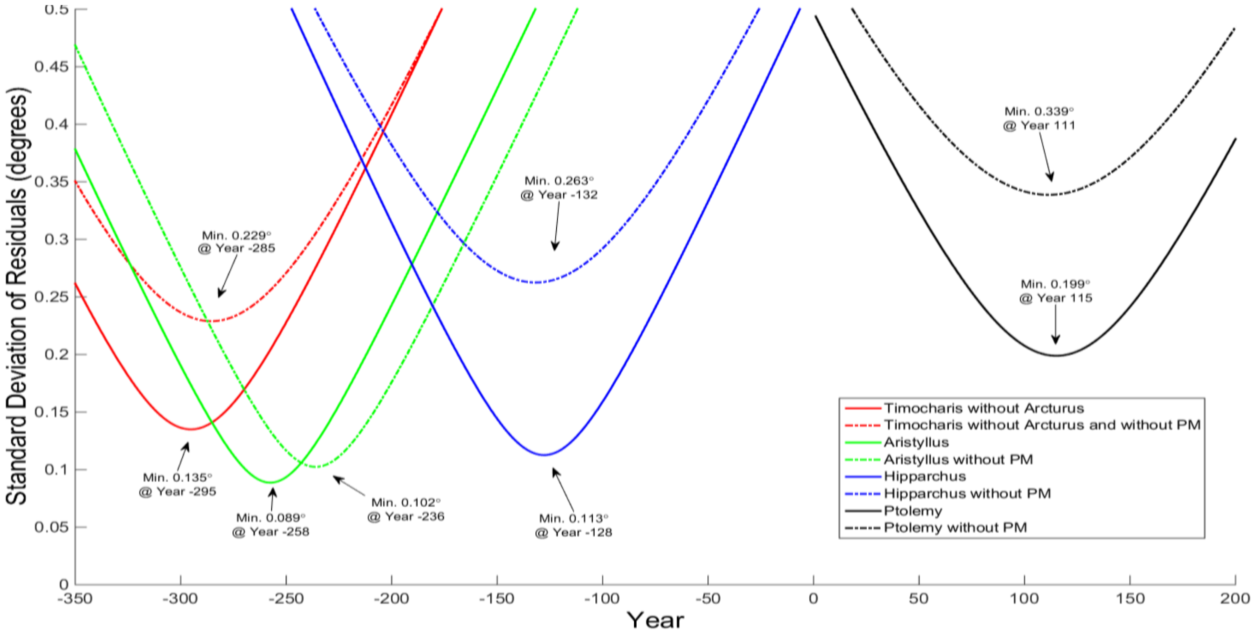

Figure 5: Standard deviation of residuals for Maeyama's method with and without proper motion.

> from uncertainty nor carefully worked out … Toomer, 1998: 321).

Ptolemy also writes (Toomer, 1998: 329) that "… the observations of the school of Timocharis are not trustworthy, having been made very crudely." This viewpoint has been repeated in modern times. Examples are the attribution of uncertainty or lack of accuracy to Aristyllos' observations (Taran, 1970) and to both Aristyllos' and Timocharis' observations by Neugebauer (1975: 287). Note that there was no known basis for this judgment since Pannekoek's (1955) paper (see just below) and we find (see Section 6) that these observations are accurate.

Pannekoek (1955) examined the accuracy by assuming the epoch and calculating the *C* position using simple precession formulae. Pannekoek took the relevant epochs to be 289 BC for Timocharis and Aristyllos, 129 BC for Hipparchus, and AD 137 for Ptolemy(?). He found mean errors of 0.15° for Timocharis (this value comes from dropping the observation for Arcturus; see Section 5); 0.12° for Hipparchus; and 0.22° for Ptolemy(?). Timocharis and Aristyllos were analyzed together using a single epoch, which we now know is incorrect. In addition, proper motion was not included. Pannekoek (1955) was apparently the first modern astronomer to show that the ancient declinations were accurate.

Maeyama (1984) used data from the Boss (1910) catalogue to calculate *C*. Then, he determined the RMS error in the residuals, *O* – *C* values, as a function of epoch. Finding the minimum in RMS error presumably fixes the epoch and thus the RMS error.

Maeyama (ibid.) analyses Timocharis and Aristyllos separately and finds epochs of 290 BC and 260 BC, respectively. See the additional discussion below and the statistical discussion in Section 5.

Proper motion is not mentioned anywhere in Maeyama's paper, yet it must have been included. His results are close to ours and a separate analysis with proper motion *not* included shows substantial differences as shown in Figure 5.

Results for RMS error and epoch from Maeyama (1984) are: Timocharis: 0.13°, 290 BC; Aristyllos: 0.087°, 260 BC; Hipparchus: 0.124°, 130 BC; and Ptolemy(?): 0.18°, AD 120. Note that Arcturus was dropped from the analysis for Timocharis by Maeyama and that the results for Ptolemy(?) are the raw or initial value. For example, Maeyama adjusts Ptolemy's(?) epoch to AD 130 by dropping three stars from the analysis. Maeyama notes that his historical sources indicate observational activity in the years AD 137/138. Also see the discussion in Maeyama, his Section 5.4.

We have repeated the analysis using Maeyama's method and the results are shown in Figure 5. Our accuracies (RMS) and epochs are: Timocharis: 0.135°, 295 BC; Aristyllos: 0.089°, 258 BC; Hipparchus: 0.113°, 128 BC; and Ptolemy(?): 0.199°, AD 115. We have also dropped Timocharis' observation of Arcturus from the analysis. See Section 5 for the statistical justification. Again, see below for additional discussion of the results for Ptolemy(?). Our analysis gives results close to those found by Maeyama (1984).

Dennis Rawlins produced a manuscript titled "Aristyllos' Date with Vindication and New Light on Ptolemy and the Roots of his Precession:

Table 2: Summary results.

	Accuracy (°)			Epoch			
Observer	Maeyama[1]	Rawlins[2]	$O - C = 0$[3]	Maeyama[1]	Rawlins[2]	$O - C = 0$[3]	Medians from Section 5
Timocharis[4]	0.135	0.135	0.135	295 BC	296 ± 11.0 BC	298 ± 13.2 BC	298 BC
Aristyllos	0.089	0.089	0.089	258 BC	256 ± 11.1 BC	259 ± 11.8 BC	253 BC
Hipparcus	0.113	0.113	0.113	128 BC	128 ± 7.3 BC	128 ± 8.4 BC	----
Ptolemy(?)[5]	0.199	0.199	0.199	AD 115	AD 115 ± 12.9	AD 117 ± 17.4	----
Ptolemy(E)[6]	0.023	---	---	AD 57	----	----	AD 56
Ptolemy(L)[6]	0.095	---	---	AD 128	----	----	AD 130

Notes:
1 Results from our updated calculation using Maeyama's method; see Section 3.
2 Results from our updated calculation using Rawlins's method: see Section 3.
3 Results from our approach as described in Section 3.
4 Timocharis's declination for Arcturus dropped from the analysis for all cases; see Sections 3 and 5.
5 All results for Ptolemy(?) are based on the entire sample.
6 The division of Ptolemy(?) into an early (E) and a late (L) subset is described in Section 4.

Studies of Hellenistic Star Declinations" in the early 1980s. It was circulated, but not published. Maeyama (1984) noted its existence. Rawlins has kindly supplied a copy of this manuscript, which we denote as Rawlins (1982a).

In addition to epoch and accuracy, Rawlins was also interested in checking for possible errors in the latitudes of the observers. He wrote an equation based on the analytical expression for precession. This equation with minor changes in notation is:

$$O - C = x + t(psin\varepsilon)cos\alpha = x + (E - E_o)Pcos\alpha \quad (1)$$

Here $O - C$ = the observed minus computed declination (Section 2); x = the error in the observer's latitude; $t = E - E_o$ = the difference in epoch from the assumed value; p = the annual precession; $P = p \sin \varepsilon$; ε = the obliquity of the ecliptic; and, α = the right ascension. The relevant quantity is P, and we have used 0.3338 arcmin/year, the same value that is used by Rawlins.

A least squares solution for a bivariate linear regression curve applied to the ensemble of stars for each observer yields the epoch and the error in the observer's latitude. With the epoch determined, the accuracy immediately follows. Rawlins explicitly includes refraction. Rawlins' results are: Timocharis: 0.15°, 296 BC; Aristyllos: 0.10°, 257 BC; Hipparchus: 0.12°, 132 BC; Ptolemy(?): Not available; see below. We note that Rawlins (1982b) quoted the separation between the epochs for Timocharis and Aristyllos in a discussion of Hellenistic astronomy.

Our repeat analysis using Rawlins' method yields: Timocharis: 296 BC ± 11.0; Aristyllos: 256 BC ± 11.1; Hipparchus: 128 BC ± 7.3; Ptolemy(?): AD 115 ± 12.9. The epochs found by both analyses are quite similar. Rawlins did not present results for the entire sample of 18 stars possible for Ptolemy. Our accuracies are given in Table 2 and come from the RMS residual curves in Figure 5. They are the same as the accuracies for Maeyama's method because the epochs are nearly the same and thus, close to the minima in the Figure 5 curves. Sirius was not used in the average for Ptolemy(?).

We return to the subject of the accuracies and epochs for Ptolemy's observations in Section 4. Rawlins (1994) has presented a later table of his results for epochs and observers' latitude. For Timocharis, Aristyllos, and Hipparchus, the epochs and latitudes are close to our values. For Ptolemy(?), Rawlins considers the observer to be Anonymous, and he gives an epoch of AD 131.

We can infer that the ancient observers knew their latitudes accurately. Our assumed latitude of 31.2° for Timocharis, Aristyllos, and Ptolemy(?) is appropriate for observations taken near Alexandria and the latitude of 36.2° assumed for Hipparchus is appropriate for observations taken from the island of Rhodes. The assumed latitudes enter the calculations directly through the refraction correction. For the epochs given in the previous paragraph, the accuracy of the observer's latitudes is: Timocharis, 0.012° (excluding Arcturus); Aristyllos, 0.003°; Hipparchus, 0.004°; and Ptolemy(?), 0.009°.

3.1 The Resulting Epochs

The trajectories for individual stars (e.g., Figures 1–4) cross the x-axis (or $O - C = 0$) and these crossings can be used to estimate the epoch and the spread in epochs. The crossing epochs are shown in Figure 6. The full $O - C$ program was not run outside the epoch ranges shown in Figures 1–4. Crossing times outside these epochs were determined from linear, quadratic, and quartic extrapolations. The dates from the different extrapolations are essentially the same except for Ptolemy(?)'s observation of Sirius. The linear extrapolation yields AD 389 while the quadratic and quartic extrapolations yield no zero crossing and a closest approach near AD 650. This is marked by the circle on Figure 6.

Our results for averages obtained using weighting by absolute value of the slope are as follows: Timocharis: 297.5 ± 13.2 BC; Aristyllos:

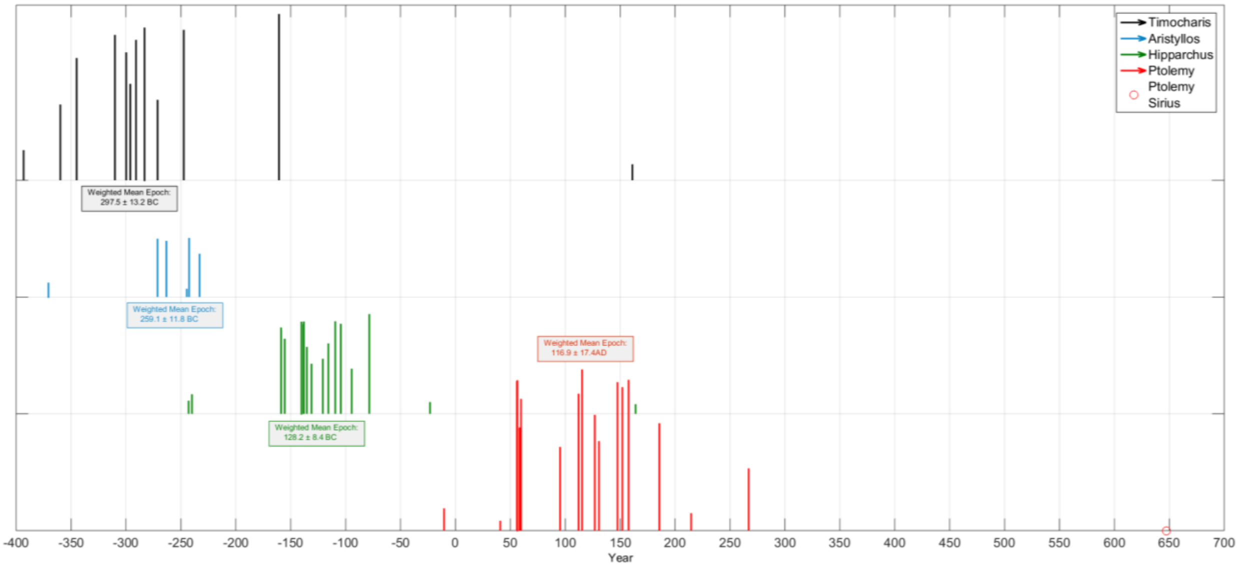

Figure 6: Dates of ($O - C$) = 0 for the different observers. The lengths of the bars are proportional to the absolute value of the slope.

259.1 ± 11.8 BC; Hipparchus: 128.2 ± 8.4 BC; and Ptolemy(?): AD 116.9 ± 17.4.

Inspection of Figure 6 shows that these dates are plausible. Figure 6 also shows grouping that will be discussed in Section 5.

4 PTOLEMY(?)

If we accept the probable dates for Ptolemy's life time of c. AD 100 to c. AD 175 and the likely date for the Almagest of c. AD 150 or perhaps a little later, we have a problem. The formal solutions for epoch in the range AD 115 to 120 are much too early for the observer to have been Ptolemy himself. Maeyama and Rawlins approach the situation by dropping observations from the analysis. Maeyama ultimately settles on AD 130. Although Maeyama (1984: 281) states that "… the names of different astronomers will only serve as a means to divide the available observations into different groups …" from the discussion later in the paper he (apparently) still considers that Ptolemy is the observer. Rawlins discusses values of AD 141 (based on 12 of 18 stars; dropping Ptolemy's six 'precession' stars, noted in Table 1) and AD 153 (based on 11 of 18 stars; in addition, dropping Betelgeuse) and considers the observer to be unknown, but not Ptolemy. Also recall that Rawlins (1994) gives an epoch of AD 131 for "Anonymous".

The computed measurement accuracies found for Ptolemy(?) in Section 3 are distinctly inferior to those found for Hipparchus; in addition, the spread in $O - C = 0$ dates found in Section 3 is much larger for Ptolemy(?) than for Hipparchus. Inspection of the $O - C$ vs. epoch plots for Hipparchus (Figure 3) and for Ptolemy(?) (Figure 4) shows a much tighter grouping for Hipparchus than for Ptolemy(?). Specifically, the clustering near 130 BC is clear for Hipparchus, but no single clustering is plausible for Ptolemy(?). There are, however, two plausible clustering around AD 65 and AD 125, as clearly shown in Figure 6. Inspection of this figure suggested splitting the sample into zero crossings before and after AD 100 and that three observations were 'unhelpful'. These are Betelgeuse, Aldebaran and Sirius. They have zero crossings later than AD 200 and small slopes.

The results for two distinct samples are instructive. The formal solutions (Maeyama's approach) date the earlier sample to AD 57 (Figure 7) and the later sample to AD 128 (Figure 8). In addition, the computed accuracy for each is much improved. The summary results are shown in Figure 9. The summary includes the three observations that we deem unhelpful.

The statistical issues involved in considering the Ptolemy(?) observations as two distinct groups are described in Section 5. A cluster analysis is presented and we show that it is reasonable to divide these observations into two groups.

Our analysis leads to the conclusion that these *Almagest* declinations are reasonably attributed to two periods, an early (E) one around AD 57 (possibly associated with the Museum of Alexandria) and a late (L) one around AD 128. The fact that this latter date falls within the dates for Ptolemy being an active observer, AD 124–141, is unlikely to be a coincidence. Hence, the later observations could have been taken by Ptolemy himself.

We are not the first authors to suggest problems or worse for the declinations attributed to Ptolemy. Unfortunately, we find ourselves in the long-running dust-up concerning the legitimacy of Ptolemy's observations. This situation

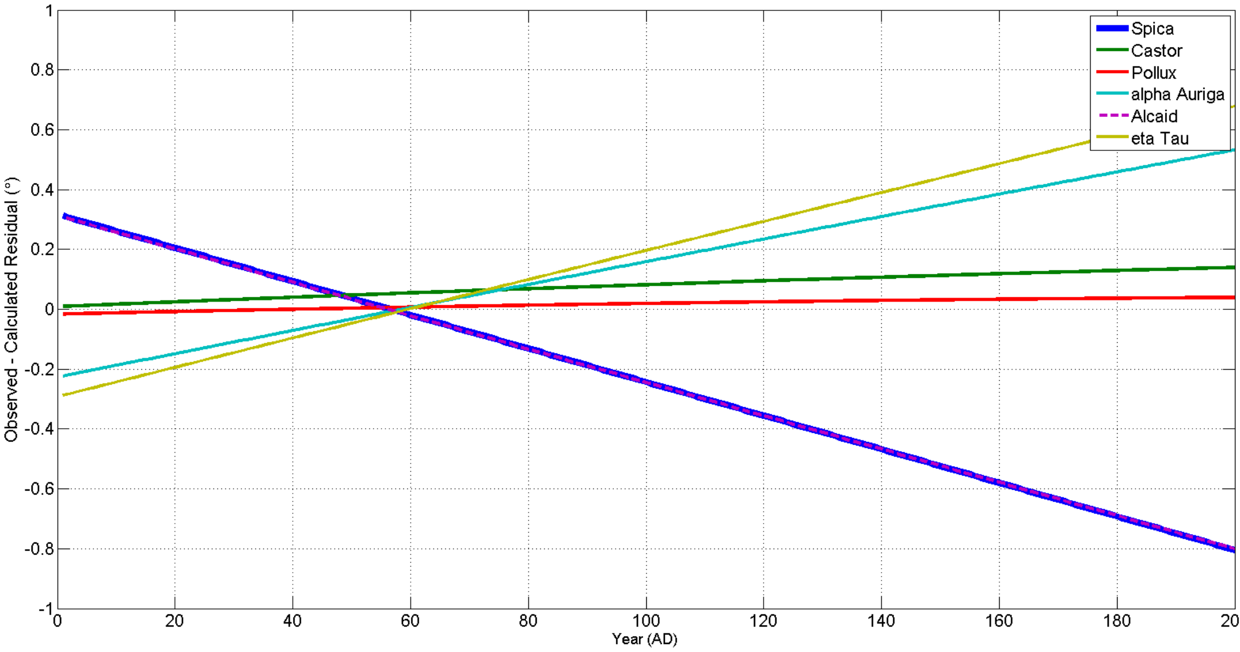

Figure 7: Residuals for Ptolemy's early group (E) as a function of year.

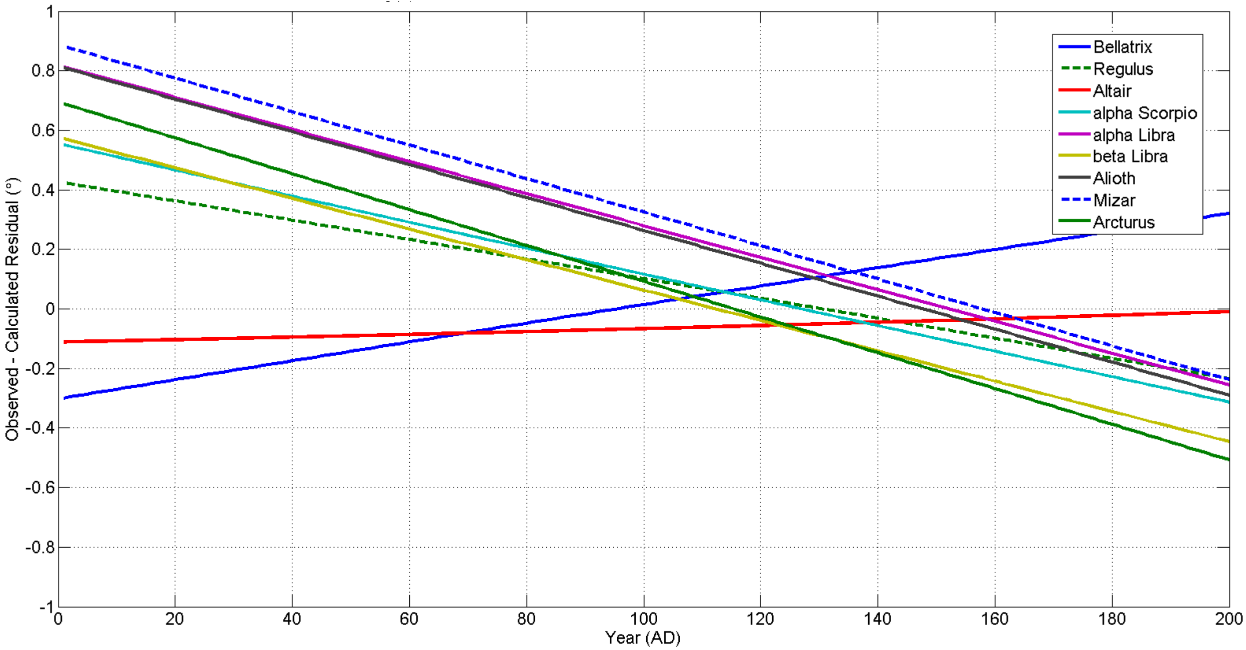

Figure 8: Residuals for Ptolemy's late group (L) as a function of year.

goes back at least to Delambre (1817; 1819) and was more recently reignited by R.R. Newton (1977). In our context, just after the declinations are given in the *Almagest* (Book VII, Chapter 3), Ptolemy selects just six stars (of the 18 available) to determine a value for precession. He finds 1° per 100 years, whereas the correct value is 1° per 72 years. Ptolemy offers additional evidence for his value elsewhere in the *Almagest* (e.g., Toomer, 1998: 338) and it is reasonable to believe that the six stars were selected because they yielded the desired result (see Duke, 2006 for additional discussion and references).

The situation has been nicely summarized by Evans (1998: 262):

> Few developments in science have so exercised the historians of science as Ptolemy's measurement of the precession rate. At stake is Ptolemy's reputation as an astronomer; at issue are his honesty and reliability as an observer.

Evans (1998) follows this quote with a short history of the issue of Ptolemy's reputation, primarily in the context of his Star Catalogue containing more than one thousand stars. We return to this subject after examining the statistical issues raised in our investigation. In the meantime, we note that Ptolemy does not literally claim to have observed all of these declinations by himself. The declinations are listed thusly: "As found by us …" (Toomer, 1998) or as "… we find." (Talia-

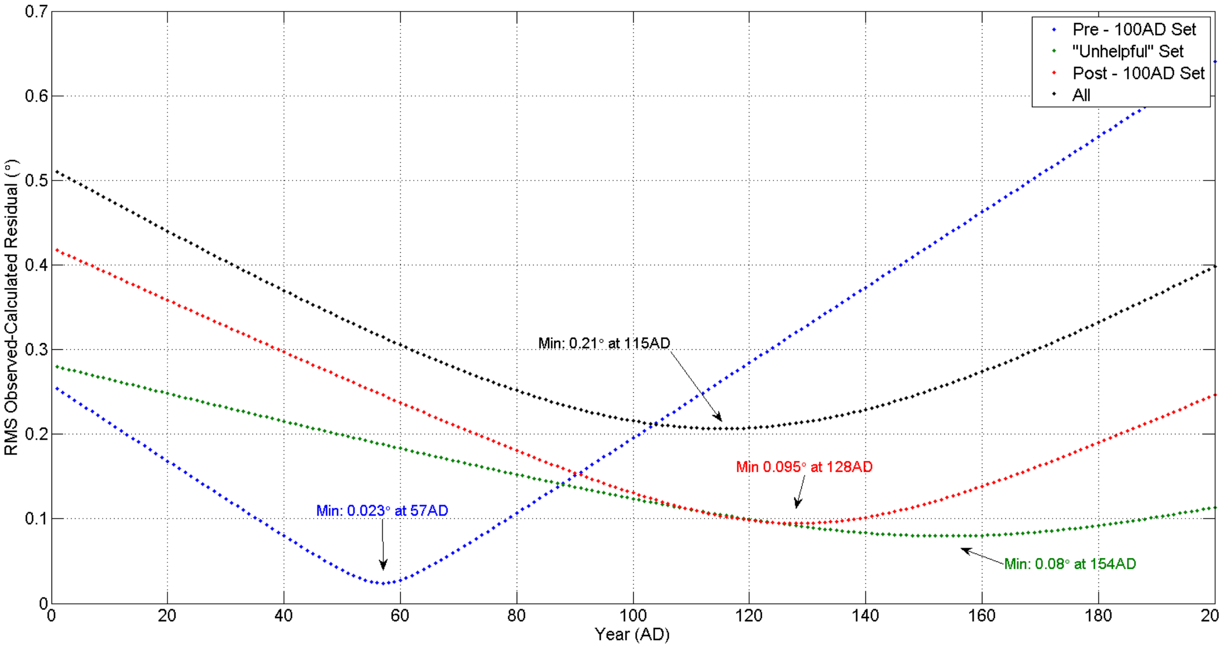

Figure 9: Standard deviation of residuals for the Ptolemy observations as a function of year showing the early (E) and late (L) groups.

ferro, 1952). But, we know from our consultation with Dr Garcia that the 'royal we' may be involved here. Finally, our groupings have no simple connection to Ptolemy's selected six stars. Of these, three fall in our early or AD 57 group, two fall into the later or AD 128 group, and one falls into our unhelpful group.

5 STATISTICAL ISSUES

In this Section, we examine several issues that are statistical in nature. First, our general approach is not to discard observations unless the decision is based on solid evidence. Timocharis' observation of Arcturus as reported is highly likely to be in error and has been discarded. The value is the same in all versions of the *Almagest* available to us. In addition, previous investigations (Maeyama, 1984; Pannekoek, 1955; Rawlins, 1982a) have also noted this. Our values show this point to be in error by 5.5σ for Timocharis' 11 observations or 7.7σ if the 17 observations of Timocharis and Aristyllos are taken as a group.

Are the dates for the observations by Timocharis and Aristyllos distinctly different? Until the early 1980s, as noted above, they were taken to be the same. Currently, the dates are considered to be different (Maeyama, 1984; Rawlins, 1982a; 1982b; 1994). Does a statistical analysis support this view?

Because the sample sizes were very small and the estimates of dates quite variable, we decided to investigate the crossing times using the Wilcoxon rank-sum test (rather than the more familiar two-sample *t* test) to determine whether the locations of the sample dates for Arystyllos and Timocharis differed significantly. The dates were rank ordered from lowest (earliest) to highest (latest) and the mean ranks were tested for differences. Two observations by Timocharis have been discarded, Arcturus, as noted above, and Altair. The later had an estimated crossing date of AD 518 and the slope was very small. If Timocharis and Aristyllos observed at approximately the same date, the mean ranking of the dates should be approximately the same in the two groups. The belief that Aristyllos observed later than Timocharis implies that Aristyllos' observations should have systematically higher rankings. For Aristyllos, five of the six observations had rankings that were ≥10 (only the value for Castor was small, rank = 2), while eight of the ten rankings for Timocharis were <10. The Wilcoxon statistic for the test was 69.00 with a Z approximation (Lehmann, 1975) yielding a probability of 0.0288. In other words, if the dates for the two observers were the same, a statistic this high or higher would be obtained by chance less than 3% of the time, suggesting that Aristyllos is quite likely to have observed at a later date than Timocharis. The best estimates for those dates from the approach taken here are probably the sample medians because the individual dates are highly variable. For Timocharis, the median date for the ten observations was 298 BC. For Aristyllos, the median date for six observations was 253 BC. These dates show a 45-year difference and the individual dates are remarkably close to the values obtained in Section 3 and given in Table 2.

Examining the difference between the two possible groups for Ptolemy(?) presents a rather

Table 3: Agglomeration schedule for Ptolemy's 14 stars.

Stage	Cluster Combined		Coefficients	Stage Cluster First Appears		Next Stage
	Cluster 1	Cluster 2		Cluster 1	Cluster 2	
1	1	15	.215	0	0	4
2	4	18	1.506	0	0	4
3	12	17	8.934	0	0	8
4	1	4	9.111	1	2	9
5	7	10	10.176	0	0	8
6	11	14	20.621	0	0	7
7	11	16	62.687	6	0	11
8	7	12	222.174	5	3	10
9	1	3	305.629	4	0	13
10	6	7	647.512	0	8	12
11	8	11	1079.407	0	7	12
12	6	8	1973.603	10	11	13
13	1	6	6675.667	9	12	0

different problem. Because the initial division date was chosen arbitrarily, it is inappropriate to do this kind of division and then test whether the division produces two meaningful samples based on the same criterion. We can use a hierarchical cluster analysis to examine the possibility of natural groupings.

Cluster analysis is a descriptive statistical procedure that is used to examine similarities among objects based on a distance parameter (Seber, 1984). Each star initially is in a cluster by itself. The two stars with the closest estimated dates or distances are combined first, creating a cluster with two objects in it. From the two initial clusters, now forming a single cluster, a heterogeneity coefficient is generated for the data set. Note that the initial value of the heterogeneity coefficient is zero when each star is in its own cluster. Then, the inter-cluster distances are recomputed and the two closest clusters are combined. This may be two other objects or the newly-formed cluster being combined with one other star. At each step, a new heterogeneity coefficient is calculated and the intercluster distances are calculated as well. This process continues until only a single cluster remains. Examining the heterogeneity coefficients, which increase as stars or clusters are added at each step, helps to determine natural groupings. The results can be presented in an *agglomeration schedule* to see the progression of the heterogeneity coefficients and to suggest natural orderings. Before proceeding to our results, we note that four stars were dropped from the final analysis. The crossing dates for these stars were determined using linear extrapolation (see Section 3) and seemed clearly problematic from the viewpoint of this analysis: Castor (21 BC), Altair (AD 226), Betelgeuse (AD 261), and Sirius (AD 389). These stars were included in an initial analysis, but their heterogeneity coefficients were so extreme compared to the rest of the observations that dropping them was appropriate. An analysis with similar dates for the first three stars and c. AD 650 for Sirius would have the same result.

Table 3 is an agglomeration schedule for the 14 stars. The heterogeneity coefficient increases from 0 to 0.215 after the first step and reaches a value of 6675.667 on the final step. The progression is reasonably smooth with only the last step showing an extremely large jump. This suggests that the remaining 14 stars might be properly divided into two clusters.

These results can be shown with a graphical representation called a *dendrogram*, Figure 10. The vertical lines show the stars combined into a cluster and the horizontal distance is a distance measure proportional to the heterogeneity coefficient. The dendrogram shows the same result as Table 3. The heterogeneities increase in a smooth manner, but the last step shows an increase from 8 to 25. This clearly suggests that two groupings of stars could be considered appropriate rather than a single grouping. In turn, this suggests that the observations reported by Ptolemy(?) fall into two natural groupings and that they were taken at different times. The medians of these two groups, excluding observations with dates below 0 and above AD 200, are AD 56 and AD 130 respectively.

These dates are compatible with our results from Section 4. We note that the groups determined here and in Section 4 are slightly different. An observation that appears to be associated with a group as selected in Section 4 may have a very small slope and fall outside the criteria for the cluster group analysis. Also, the two groups determined by the cluster analysis are not simply related to Ptolemy's selected six stars. Four of them fall into the early group and two into the late group.

6 CONCLUSIONS

Table 2 summarizes our results for our various approaches to the data. Specific values for epoch, accuracy and mean error are remarkably consistent for Timocharis, Aristyllos and Hipparchus. These observers clearly knew what they were doing. Historical information for them is not extensive, but nevertheless is consistent with

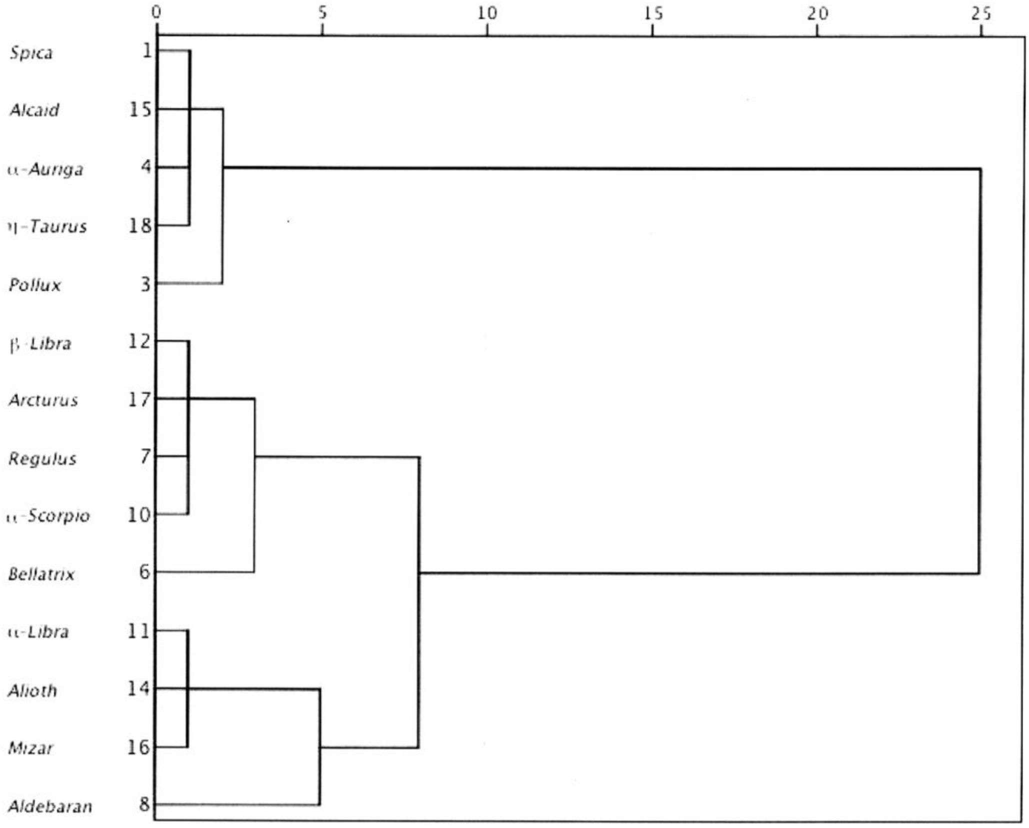

Figure 10: Graphic illustration of the cluster analysis for Ptolemy via a dendrogram using centroid linkage. See the text for discussion.

the derived epochs.

The straightforward results for Ptolemy(?) are another matter. The derived epoch for the entire ensemble of observations in the range AD 115–120 is incompatible with the historical evidence indicating a lifespan for Ptolemy of approximately AD 100–175. He would have been much too young. In addition, the accuracy of almost 0.2° is much worse than the earlier observers. Did the observational techniques deteriorate, or is there another explanation? Inspection of the residuals vs. epoch plot for Ptolemy(?) (Figure 4) and comparison with the plot for Hipparchus (Figure 3) shows that the trajectories for Ptolemy(?) are not clustered around the derived epoch (as they are for Hipparchus), but show possible clusterings at other epochs, specifically near AD 65 and AD 125. This point is also clearly shown in Figure 6. In addition, the spread in dates for Ptolemy(?) (Section 3) is much larger than for the other observers.

We have approached this question in two ways. In Section 4, we analyzed groups based on inspection of the trajectory diagram for Ptolemy(?), Figure 4, plus the zero crossing dates, Figure 6, and find support for observations taken circa AD 57 and AD 128. We have also divided the Ptolemy(?) observations into two samples (Section 5) based on a cluster analysis. The clusterings are qualitatively sound and the median dates for the two groups are AD 56 and AD 130. Thus, the observations were almost surely taken by observers ≈ 70 years apart. The early observer cannot be Ptolemy himself, but he certainly could have been the later observer. Support for this view comes from the determination by Neugebauer (1975) based on Ptolemy's dated observations in the *Almagest* that his active observing period spanned AD 124–141. The observations for Ptolemy(?) translated as "… we find …" (Taliaferro 1952) or "As found by us…" (Toomer 1998) does not necessarily imply a group effort. Again, we have consulted Dr Lorenzo Garcia on Greek usage. Use of the 'royal we' is customary throughout Classical and Hellenistic Greek. Thus, we have no idea from the text itself if others were involved in obtaining the observations circa AD 128. The 'we' concept may be appropriate for the observations circa AD 57. A likely source for the observations is the Museum of Alexandria.

Thus, we find that all of the declinations given in the *Almagest* are remarkably accurate with an RMS error ≈ 0.1° or about 6 arc min. Subsequent history shows how impressive they are. The accuracy would not be significantly improved until Tycho Brahe's work in the 16th century AD and his accuracy of approximately 1 to 1½ arc min (North, 2008: 327; Rawlins, 1993; Wesley, 1978).

The accuracy of the *Almagest* declinations implies considerable depth of astronomical understanding and sophistication in instrument-making technology, at least including metallurgy and the ability to make accurate fiducial markings. Support for such a technology in ancient Greece certainly exists (Evans, 1998: 80–83). Cicero (106–43BC) has described a mechanical device due to Archimedes (c. 287–212 BC), an orrery that showed the motions of the Sun, Moon, and five planets. Archimedes also wrote a book, now lost, on astronomical mechanisms. Cicero also noted an apparently similar orrery made by Posidonius (c. 135–51 BC). A splendid example is the Antikythera Mechanism (Charette, 2006; Evans et al., 2010; Freeth et al., 2006; 2008; Marchant, 2006; 2010; Wright 2007), a complex, multi-geared astronomical calculator recovered in 1901 in fragments from a Roman shipwreck. The device was probably made in the time period 150 BC to 100 BC. This extraordinary device surely was the result of a mature technological tradition. The instruments used to take the *Almagest* declinations are almost certainly part of this ancient Greek tradition.

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