A LUNAR ECLIPSE VOLVELLE IN PETRUS APIANUS' *ASTRONOMICUM CAESAREUM*

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Abstract: The workings and theory of an eclipse volvelle in Petrus Apianus' *Astronomicum Caesareum* is investigated. This paper also tries to explain how the volvelle was implemented from the theory and what values were given to the parameters that were used for the calculations. Results from model computations are presented.

Keywords: volvelle, lunar eclipse, Petrus Apianus

1 INTRODUCTION

Petrus Apianus (Figure 1; also Peter Apian), whose surname is a Latinized version of his original family name Bienewitz, was born the son of a shoemaker in Leisnig (Germany) in about 1501 and died in Ingolstadt (Germany) in 1552 (Galle, 2014). He began his studies at the University of Leipzig in 1516 but moved to the University of Vienna in 1519 where he studied mathematics, astronomy and astrology. In 1527 he was appointed a Professor of Mathematics at the University of Ingolstadt and soon achieved fame as an astronomer and astrologer. He published several works on comets and instruments for the calculation and astronomical observation, and he observed Halley's Comet. But his masterpiece was *Astronomicum Caesareum* (Apianus, 1540a), which was dedicated to the Roman Emperor Charles V (1500–1558). It is based in the Ptolemaic model of the Universe and the fundamental parameters are the same as in the Alfonsine Tables (1518) although it seems from preliminary computer calculations that I have made that Petrus Apianus in some cases made some small modifications. *Astronomicum Caesareum* contains a large set of ingeniously-constructed volvelles for computing the true locations of the Sun, Moon and planets, as well as an extensive set for different kinds of eclipse calculations. His work earned him an appointment as Imperial Mathematician.

Petrus Apianus (1540b) also published a manual in German for *Astronomicum Caesareum*. A review of *Astronomicum Caesareum* has been given by Gingerich (1971). At that time, about 120 copies of the original work were still extant. In 1967, a facsimile of *Astronomicum Caesareum* was published (Apianus, 1967).

There is a very complete compilation on different aspects of Apianus' life and work edited by Karl Röttel (1995) in connection with an exhibition on Apianus that was held in Leisnig in 1996 and Landrats-amt Neumarkt in 1997.

In this paper we study one of the volvelles (Figure 2) that Apianus (1967: 73) used for lunar eclipse calculations. As with all the other volvelles in Petrus Apianus' work, one cannot help but be impressed by the amount of mental and manual effort and skill that was used in creating this volvelle. Here, we try to investigate how this volvelle was constructed.

2 OVERVIEW OF THE VOLVELLE

At the top of the volvelle there is a panel for setting up the volvelle, given the anomalies of the

Figure 1: Petrus Apianus (courtesy: Instituto e Museo della Sciencia, Florence).

Moon and the Sun. There are then four separate panels grouped clockwise around the centre, the first for determining the size of the eclipse, N. PVNCTA ECLIPTICA, the second one for determining half the time of the totality, MORA MEDIA, the third one for determining the time of partiality, TEMPVS CASUS, and finally the last one for determining the angular movement of the Moon during the eclipse, MINVTA GRA. MOTVS LUNE. Each of these panels ex-
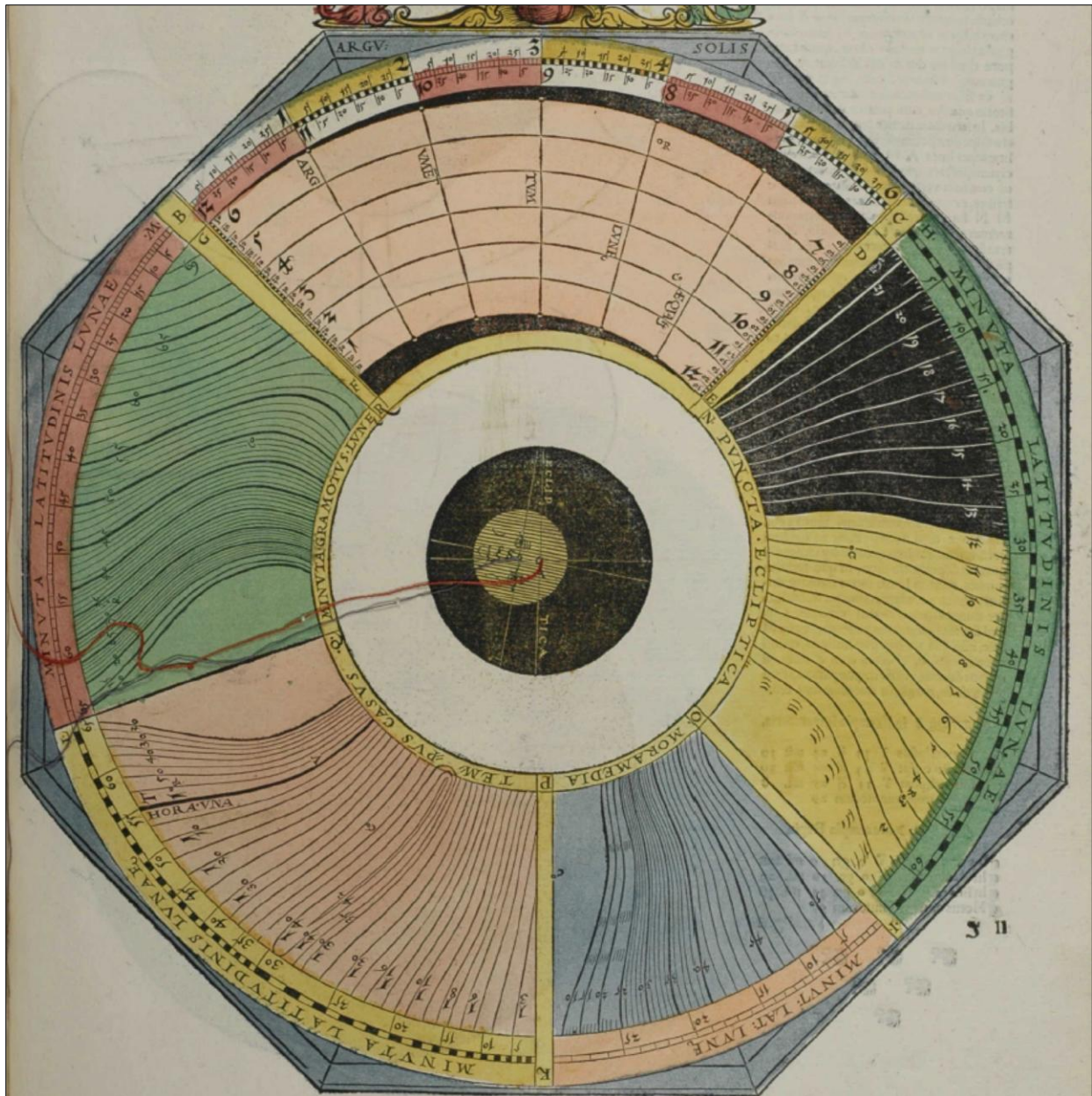
Figure 2: The eclipse volvelle (Photograph: Lars Gislén).

hibits an intricate set of curves. In the case of the size of the eclipse, each curve represents a certain eclipse size, for the *mora media* and *tempus casus* the curves show the half duration in hours and minutes of the totality and partiality respectively, for the *minuta motus* panel, the movement of the Moon in arc minutes during the eclipse.

The lunar anomaly is set in the top panel on the left and right hand scales, graduated from sign 1 to 6 and 7 to 12 respectively. The solar anomaly is set by the top scale along the edge of the volvelle. There are two different threads, red and blue, attached to different centres A and B. Centre A is the centre of the entire volvelle while B is slightly displaced to the left. The two threads presumably had small beads that could slide along the threads with some friction. You

first use the blue thread and stretch it along the left scale, setting the bead at the given anomaly of the Moon. For lunar anomalies with signs 7 to 12 the right hand scale is used. The red thread is then stretched and set against the solar anomaly on the upper scale. The blue thread is rotated until its bead crosses the red thread and the bead on the red thread is fixed at that point. Finally, the red thread is rotated to the respective panels and set against the given lunar latitude and a value is read off from the curve under the bead, possibly interpolating be-tween two curves.

The volvelle raises some questions. It is evident that the lunar and solar anomalies are not independent. So for instance a setting with lunar anomaly zero signs, solar anomaly zero signs gives the same result as a setting of lunar

anomaly 11 signs 10°, solar anomaly 6 signs. For this reason, I have treated the volvelle as having solar anomaly zero and refer the inclusion of the solar anomaly to the discussion at the end of the paper.

3 NOTATION

The lunar latitude is denoted by β. All angular measures are made in minutes of arc. The Moon's apparent radius is r and the radius of the shadow is R. As in the *Almagest* (Toomer, 1984: 254), it is assumed that $R = 2.6\,r$. The radius is a function of the Moon's distance from the Earth, which in turn is a function of the lunar anomaly, γ_M. Figure 3 comes from the Alfonsine Tables (1518: 234) and shows the apparent radii of the Sun, the Moon and the shadow as a function of their respective anomalies. Note that the Alfonsine Tables use sexagesimal notation for the anomaly, for instance 1:30 in the first column is 60 + 30 = 90. The last column in the table shows the correction to the radius of the shadow as a function of the solar anomaly γ_S. The shadow becomes slightly smaller as the Sun gets closer to the Earth, the largest shadow correction being −56″, see the Appendix at the end of this paper.

The very small influence on the theory from the inclination of the Moon's orbit is neglected.

4 THE DIFFERENT PANELS OF THE LUNAR VOLVELLE

4.1 Theory

4.1.1 *Puncta Ecliptica*, p, the Size of the Eclipse

This is expressed as the fraction of the lunar diameter that is obscured, in units such that a total eclipse has size 12 or larger. The mathematical expression is:

$$p = 12(R + r - \beta) / 2r = 21.6 - 6\beta/r \qquad (1)$$

It is easy to see that if $\beta = R + r$, the size of the eclipse is zero, while if $\beta = R - r$, the size is 12, the lower limit for a total eclipse. The maximum possible eclipse will be for $\beta = 0$ when the size is 21.6.

4.1.2 *Mora Media*, the Half Duration of the Totality, t_m

From Figure 4 the distance AB is seen by the Pythagorean theorem to be

$$m = \sqrt{((R - r)^2 - \beta^2)} \qquad (2)$$

In time units (minutes) $t_m = 60\,m / (\nu_M - \nu_S)$, where ν_M is the angular speed of the Moon per hour and ν_S the corresponding angular speed of the Sun. These speeds are a function of the respective anomalies. For the Sun this dependence is quite small and can be taken as constant as in the *Almagest* (Toomer, 1984: 306) where $\nu_M - \nu_S = \nu_M / (1+1/12)$.

Linee numeri cómunes		Semidiameter ☉		Semidiameter ☽		Semidiameter vmbre		Claria tio vmbre	
˙g	g	˙g	g	m	˙	m	˙	m	˙
0 0	6 0	15 40		14 30		37 42		0 0	
0 6	5 54	15 41		14 31		37 45		0 0	
0 12	5 48	15 41		14 32		37 48		0 0	
0 18	5 42	15 41		14 35		37 54		0 1	
0 24	5 36	15 43		14 37		38 1		0 2	
0 30	5 30	15 45		14 41		38 11		0 4	
0 36	5 24	15 48		14 45		38 22		0 6	
0 42	5 18	15 49		14 49		38 36		0 6	
0 48	5 11	15 51		14 57		38 52		0 8	
0 54	5 6	15 54		15 4		39 11		0 10	
1 0	5 0	15 58		15 12		39 31		0 13	
1 6	4 54	16 2		15 20		39 52		0 16	
1 12	4 48	16 0		15 29		40 16		0 18	
1 18	4 42	16 8		15 39		40 40		0 21	
1 24	4 36	16 11		15 48		41 5		0 23	
1 30	4 30	16 15		15 59		41 35		0 26	
1 36	4 14	16 20		16 12		42 7		0 30	
1 42	4 18	16 23		16 21		42 30		0 32	
1 48	4 11	16 26		16 34		43 3		0 34	
1 54	4 6	16 32		16 44		43 30		0 39	
2 0	4 0	16 35		16 56		44 2		0 41	
2 6	3 54	16 39		17 7		44 31		0 44	
2 12	3 48	16 41		17 17		44 57		0 46	
2 18	3 42	16 45		17 27		45 21		0 49	
2 24	3 36	16 46		17 36		45 46		0 53	
2 30	3 30	16 50		17 44		46 7		0 53	
2 36	3 24	16 50		17 51		46 25		0 53	
2 42	3 18	16 51		17 56		46 38		0 53	
2 48	3 12	16 53		18 0		46 49		0 55	
2 54	3 6	16 54		18 3		46 55		0 56	
3 0	3 0	16 55		18 4		46 57		0 56	

Figure 3: A table from the Alfonsine Tables.

4.1.3 *Tempus Casus*, the Half Duration of the Partiality t_c

This is the difference between the half the total time for the eclipse, AC, and half the time of the totality, AB. Again using the figure above we get

$$c = \sqrt{((R + r)^2 - \beta^2)} - \sqrt{((R - r)^2 - \beta^2)} \text{ if } \beta < R - r \quad (3a)$$
$$c = \sqrt{((R + r)^2 - \beta^2)} \text{ if } \beta < R - r \text{ (no total eclipse)} \quad (3b)$$

In time units (minutes) $t_c = 60\,c / (\nu_M - \nu_S)$.

Figure 5 shows a sketch of the *tempus casus* function. The function is here drawn with $r = 1$, $R = 2.6$. The slope of the curve gets infinite for $\beta = R - r$ with $c = 2\sqrt{(Rr)}$. Petrus Apianus' volvelle panel effectively shows a contour plot of the 'mountain ridge' in the figure as the Moon's anomaly varies.
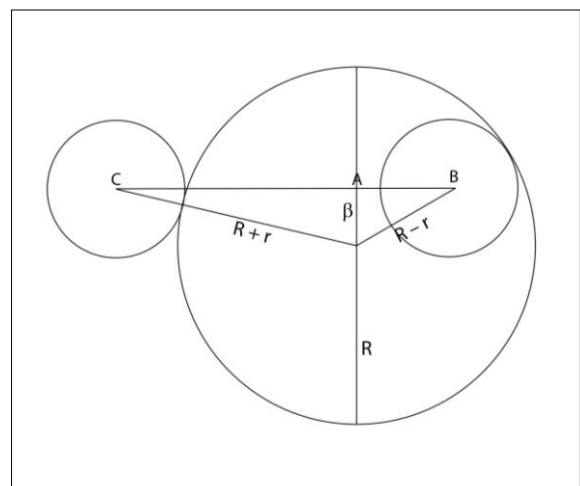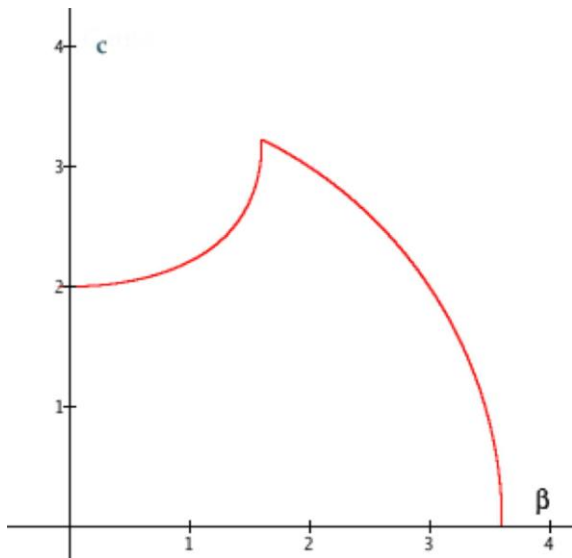

Figure 4: Eclipse geometry.

Figure 5: The *tempus casus* function.

4.1.4 *Minuta Motus*, the Moon's Movement During (Half of) the Eclipse

The Moon's angular movement, a, relative to the shadow disk is

$$a = \sqrt{((R + r)^2 - \beta^2)} \tag{4}$$

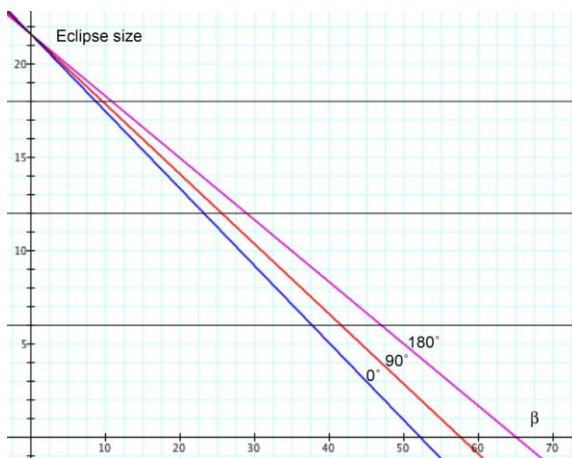The corresponding time is calculated as above by dividing this by the relative speed of the Moon
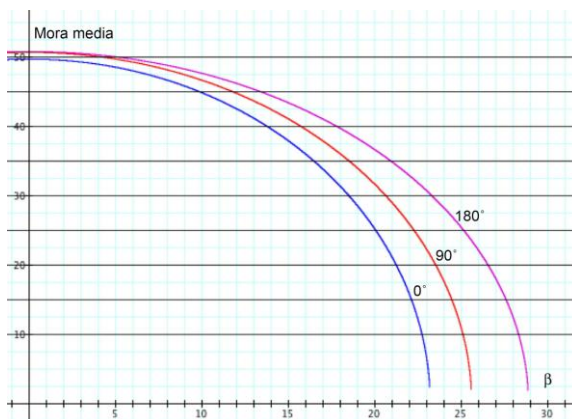

Figure 6: The eclipse size graph.

Figure 7: The *mora media* graph.

and the Sun. Multiplying this by the Moon's angular speed will give the Moon's movement,

$$\delta = a \, v_M / (v_M - v_S) \tag{5}$$

4.2 The Implementation of the Theory

The mathematical expressions above are somewhat complicated, especially for the *tempus casus* case. In order to draw the panel curves we want the variation of the lunar latitude as we move along a curve with constant value of for instance the duration of the totality. This means that we would have to invert relations (1)–(4), something that in the case of the *tempus casus* function is mathematically quite difficult. A much simpler—and I believe for Petrus Apianus more natural—procedure would be to graph the respective functions, select a value for the parameter of interest and read off corresponding β value manually. It is rather easy to plot curves for different R and r as the Moon's anomaly varies. It would also be enough to plot curve points in the volvelle panels for a few values of this anomaly and then connect these points by hand. In the graphs below I have only computed results for lunar anomalies 0°, 90° and 180°, in some cases where the panel curves are less linear I have also used intermediate lunar anomalies of 45° and 135°.

4.2.1 *Puncta Ecliptica*, the Size of the Eclipse

The three lines in Figure 6 were constructed by taking the values for r from the Alfonsine Tables for anomaly 0° (14′ 30″), 90° (15′ 59″) and 180° (18′ 4″) and inserting them in expression (1) and plotting the resulting three straight lines.

If we follow the line of, for instance, eclipse size 12 to the magenta curve (anomaly 180°) we get $\beta = 29′$. The red curve (anomaly 90°) gives $\beta = 26′$, and the blue curve (anomaly 0°) gives $\beta = 23′$. This corresponds excellently with the curve on the volvelle.

4.2.2 *Mora Media*, the Half Duration of the Totality

Again I have plotted three curves (Figure 7) using the tabular values of R and r and the formula (2) for anomaly 0°, 90° and 180°. A problem here is that I also need values of the lunar angular speed that depends on the anomaly. There is a table for these values in the Alfonsine Tables (1518: 190–191). However, using these values does not give a perfect fit, especially for the *mora media* panel.

I am not sure which version of the Alfonsine Tables was used by Petrus Apianus; there quite a few that are possible, with slightly different tabular values. Instead I have made a weighted least square fit of the curve values in the panels

and the theoretical values. The fit has to be weighted because the β scale in the panels is in many cases non-linear, for the *tempus casus* panel very much so. Table 1 shows the best fit values for $v_M - v_S$ in the *mora media* and *tempus casus* panels.

The values do not deviate very much from these you get from the Alfonsine Tables that I have consulted, except for lunar anomaly 180° (see Figure 7).

4.2.3 *Tempus Casus*, the Half Duration of the Partiality

In Figure 8, it should be mentioned that where the curves are more or less horizontal, the point where the curves cross a horizontal line is not very well defined and the value of β is not very precise. This is the case when the lunar latitude is small.

4.2.4 *Minuta Motus*, the Moon's Movement During Half of the Eclipse

Here (Figure 9) the ratio $v_M / (v_M - v_S)$ is essentially constant—its variation with the lunar anomaly is very small. I used values of v_M from the Alfonsine Tables and $v_S = 2.38$, the tabular value for solar anomaly 0°. Thus, the only important dependence comes from the variation of R and r.

5 RESULTS

Figure 10 shows the volvelle with some calculated points (red) using the procedures above. For the size panel I have calculated points for eclipse sizes 6, 12, and 10. In the *mora media* panel points are for times 45 and 25 minutes and in the *tempus casus* panel for times 1 hour and 1 hour 20 minutes. The white points mark the 'crest' of the *tempus casus* curve. In the *minuta motus* panel, points are calculated for 20, 45, 55, 60 and 65 minutes.

6 TWO EXAMPLES

In *Astronomicum Caesareum* there are two examples of eclipse calculations. The first is related to the year of birth of Emperor Charles V and was a partial lunar eclipse on AD 5 November 1500. Petrus Apianus gives the solar anomaly as 4 signs 23° 29′, the lunar anomaly as 9 signs 22° 59′, and the Moons latitude as 29′ [south]. The anomaly entry is marked by a small letter C that can be seen in the top panel of Figure 2. Moving to the *puncta ecliptica* panel we find the letter C marked corresponding to the Moon's latitude 29′ and the eclipse size can be read off as 10. In the *mora media* panel the corresponding point shows that there was no totality. In the *tempus casus* panel, point C gives

Table 1. Parameter values of $v_M - v_S$ for best fit.

Lunar Anomaly	0°	90°	180°
Mora media	28.05	30.3	34.11
Tempus casus	27.38	30.32	34.43
Alfonsine Tables	27.92	30.32	33.41

the half duration of the partiality as 1 hour 35 minutes and finally the *minuta motum* panel shows that the Moon moved 49′ during this time. I checked the result with a modern ephemeris program: eclipse size 10.6, half duration 1:36, and Moon's movement 56′.

The second example in *Astronomicum Caesareum* is the partial lunar eclipse on AD 15 October 1502, preceding the birth of King Ferdinand I, one of the brothers of Charles V. The solar anomaly was 4 signs 2° 12′, the lunar anomaly 6 signs 15° 33′, and the Moon's latitude 55′ north. The entry point is marked in the top panel by the letter R. This gives the eclipse size as 3, no totality, half duration of partiality 56 minutes, and Moon's movement 34′. The modern values are respectively 3.4, 56, and 37.

7 DISCUSSION

The above procedures explain and give results that agree very well with the different curve sets in the volvelle panels. So far, however, the influence of the solar anomaly has been neglected This will influence two things: the apparent angu-
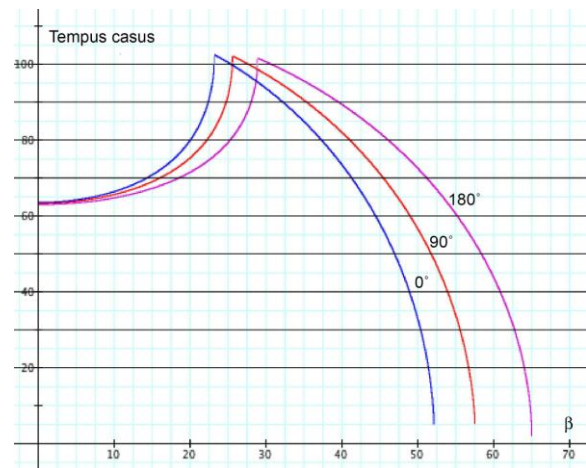

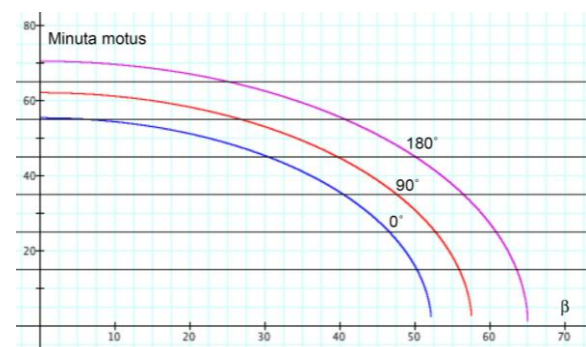Figure 8: The *tempus casus* graph.

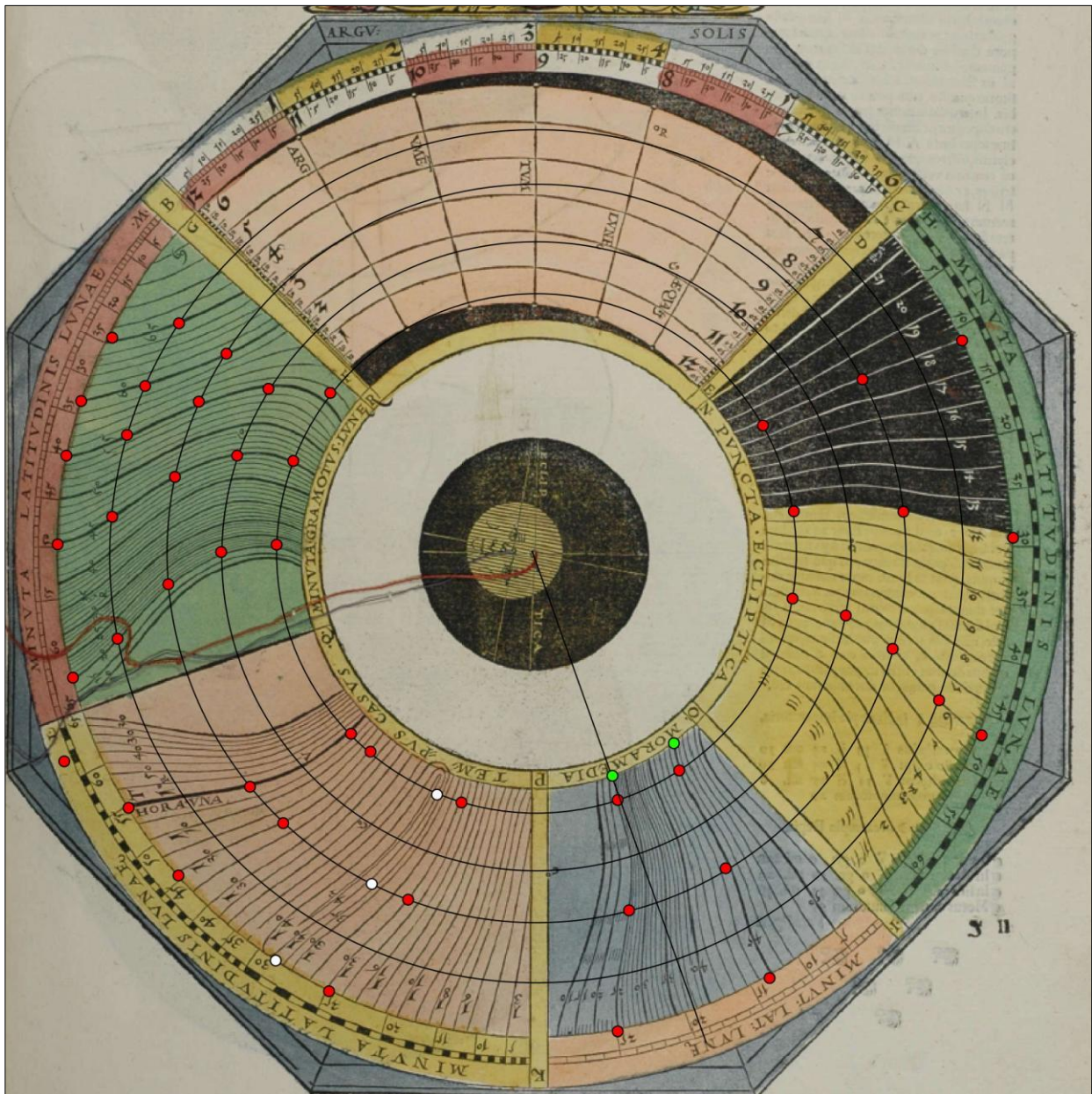Figure 9: The *minuta motus* graph.

Figure 10: Simulation results.

lar speed of the Sun and also the size of the shadow. The variation of the solar speed makes a very small impact on the results and I believe that it was purposely neglected by Petrus Apianus. The change in the radius of the shadow is also rather small but if neglected one would ask why the volvelle required a scale for setting the solar anomaly. However, the volvelle curves can be extremely well simulated for most of their extent, with the influence of the solar anomaly entirely neglected. Only the strip nearest to the centre of the volvelle is still undetermined. I will now consider in more detail the *mora media* panel.

As the solar anomaly increases from 0° to 180°, the shadow radius shrinks, slowly to begin with, more rapidly after 90° and has finally decreased by 56″ ≈ 1′ at 180°. As the elongation

speed is of order 30′ per hour, the time of the totality for $\beta = 0$ will be shortened by about $60 \cdot 1/30 = 2$ minutes. For larger values of β this time correction will be larger but even at $\beta = 20'$ it is only about 3 minutes. We also notice that, as the lunar and solar anomalies are set in the volvelle, they are not independent. If we examine the volvelle it is evident that the panel curves in general show a rather abrupt small change of direction towards smaller values of β in the strip closest to the centre. I believe that the solar anomaly correction was only implemented by Petrus Apianus in this strip. I have simulated this by points corresponding to lunar anomaly 0° and solar anomaly 180° at the inner border of the *mora media* panel where for instance the point on the curve representing a 45 minute duration has been calculated for 47 min-

utes, lunar anomaly 0° and solar anomaly 180°, and then corrected by 2 minutes, 47 – 2 = 45. The simulated point is marked by a green dot. A similar procedure gives another green dot on the 25 minutes curve. Both these points agree very nicely with the panel curves.

In the *tempus casus* panel, the curves are the result of the difference between two square roots and we expect the time correction to be smaller. A similar, but slightly more involved, computation as for the *tempus casus* panel, indeed shows that the time correction in this case is very small, in general much less that one minute. This is also evident from an inspection of the panel curves, there is no or very small change in the direction of these curves in the strip closest to the inner border of the panel. The only panel posing a problem is the *minuta motus* panel where the curves in the strip deviate in the wrong direction—the curves on the volvelle indicate that the Moon moves a larger angular distance as the shadow radius shrinks, which must be wrong. I cannot explain this, although a possible explanation may be that there was an error in the layout of the curves in this part of the panel.

8 ACKNOWLEDGEMENTS

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10 APPENDIX: THE SHADOW CORRECTION

We refer to Figure 11 showing the Sun, the Earth and the shadow plane where the Moon is located. D is the Sun-Earth distance, d the Earth-Moon distance, and x the distance from the shadow plane to the shadow apex. R is the radius of the Sun, r the radius of the Earth, and S the radius of the shadow. The apex angle is small, of the order of 0.5 and we can use the approximation that the sine of this angle is equal to the tangent of this angle and also equal to the angle itself, expressed in radians. We also see that the shadow becomes smaller when the solar distance decreases.

From equal triangles we have

$$(D + d + x) / R = (d + x) / r = x / S$$

The first equality gives $x = D \cdot r / (R - r) - d$

Inserting this in the last equality we get

$$S = r - (R - r) \, d / D$$

The apparent angular size of the shadow (in radians) as seen from the Earth is

$$\alpha \approx S / d = r / d - (R - r) / D$$

At the maximum distance of the Sun where the Sun-Earth distance is D_0 we have

$$\alpha_0 = S / d = r / d - (R - r) / D_0$$

The change in angular size is $\Delta\alpha = \alpha_0 - \alpha = (R - r) (1/ D_0 - 1/D)$

In the Ptolemaic model $D = D_M \cdot \sqrt{(1 + e^2 + 2e \cos \gamma)} = D_M (1 + e) \sqrt{(1 - 4e \sin^2 (\gamma/2) / (1 + e)^2)}$.

where D_M is the mean solar distance and e the eccentricity of the Sun, and γ the anomaly of the Sun.

At maximum distance where $\gamma = 0$ we have $D_0 = D_M (1 + e)$. This gives

$$\Delta\alpha = (R - r) (1 - 1/\sqrt{(1 - 4e \sin^2 (\gamma/2) / (1 + e)^2)}) / (D_M (1 + e)).$$

The eccentricity is a small quantity and we can Taylor expand the second term in the bracket skipping higher orders of e:
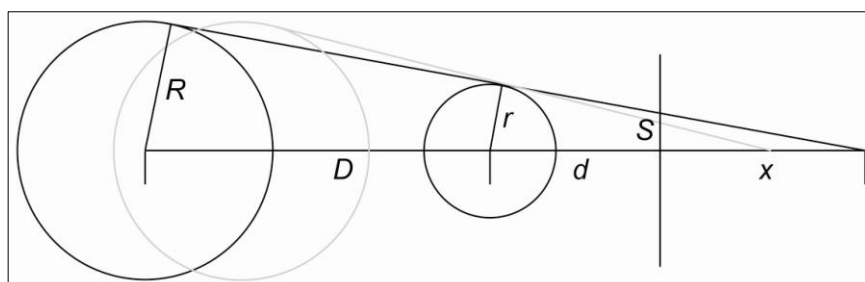

Figure 11: The eclipse shadow.

$1/\sqrt{(1 - 4e \sin^2 (\gamma/2) / (1 + e)^2)} \approx 1 + 2e \sin^2 (\gamma/2) / (1 + e)^2$

Thus we finally get

$\Delta\alpha = -2e (R - r) \sin^2 (\gamma/2) /(D_M (1 + e)^3).$

We now insert Ptolemy's values $R = 5.5$, $r = 1$, $D_M = 1210$, $e = 2.5 / 60 = 0.0417$ (Toomer, 1984: 158, 257) and convert to arc seconds by multiplying with the factor $3600 \cdot 180/\pi$

$\Delta\alpha = -56.6'' \sin^2 (\gamma/2).$

The value used in the Alfonsine Tables in Figure 2 is a rounded down value of 56″.

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