APIANUS' LATITUDE VOLVELLES – HOW WERE THEY MADE?

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Abstract: This paper studies the working and construction of the volvelles in Petrus Apianus' *Astronomicum Caesareum* that describe the latitudes of the planets. It is found that they can be constructed using a graphical method.

Keywords: volvelle, planetary latitudes, Petrus Apianus.

1 INTRODUCTION

Petrus Apianus' *Astronomicum Caesareum* contains a large set of complicated and ingenious volvelles that can be used to compute the longitudes of the planets, the Sun and the Moon, as well as the latitudes of the planets and the Moon. Besides there are volvelles for finding different astrological quantities and for determining the date of religious seasons like the Easter and Passover. In an earlier paper (Gislén, 2016) I studied the working and construction one of Apianus' lunar eclipse volvelles and also provided some biographical data on him.

In this paper I will study the volvelles used for computing the planetary latitudes in *Astronomicum Caesareum*. I will study one superior planet, Mars, and the two inferior planets, Venus and Mercury. Apianus used the theory of planetary latitudes as implemented in the *Almagest* (Toomer, 1984). The Ptolemaic theory of the planetary latitudes is very complicated, and below I give a much condensed description of it; further details can be found in Neugebauer (1975), Pedersen (1974: 355), and Swerdlow (2005).

2 DESCRIPTION OF THE VOLVELLES

In order to use the latitude volvelles you need two input parameters: the longitude λ of the epicycle centre, counted from the top of the deferent (see below), and the anomaly angle γ. The Mars volvelle (Figure 1) has a rim with two sets of graduations, the inner rim being graduated counter-clockwise 0˚ to 180˚ from the top left to the top right, then going back clockwise from 180˚ to the left top 360˚. This is the entry of the longitude of the centre. Radially you set the anomaly angle starting at the periphery of the central disk at 0˚ and reaching the rim at 180˚, then returning back to the central disk at 360˚. This scale is displayed in the wedge-formed area at the top of the volvelle. There is a thread going from the centre of the volvelle with a small bead that can slide along the thread.

The working is as follows. First you use the anomaly scale to set the position of the bead on the thread. Then the tread with the bead is set against the longitude of the centre on the rim and then the latitude is read off from the line found below the bead, if necessary interpolating between two adjacent lines. The red area of the volvelle signifies northern, positive (*septentrionalis*) latitudes while the green area signifies southern, negative (*meridionalis*) latitudes. For the Venus and Jupiter volvelles the colours are reversed.

The Saturn and Jupiter volvelles are very similar but the rim graduation is displaced taking into account that the ascending node of Saturn is assumed to have an ecliptic longitude of 50˚ and that of Jupiter of –20˚. These values are the same as those used in the *Almagest* and the Toledan Tables.

The Venus volvelle (Figure 2) has an outer longitude rim graduated counter-clockwise from 0˚ to 360˚ and is to be used for anomalies from 0˚to 180˚. The inner rim is graduated clockwise from 0˚ to 360˚ from the bottom of the volvelle and is to be used for anomalies from 180˚ to 360˚.

The Mercury volvelle (Figure 3) is divided into two sections, a left part and a right part. The left rim is graduated counter-clockwise from 0˚ to 360˚ with the zodiacal signs written with Latin numbers and is to be used for anomalies from 0˚to 180˚. The right rim is graduated clockwise from 0˚ to 360˚ with Arabic numbers and is to be used for anomalies from 180˚to 360˚.

3 THEORY

The planetary latitude in the Ptolemaic scheme used by Apianus is calculated using two input variables, the longitude of centre counted from the top of the deferent circle and the anomaly. Ptolemy then uses these variables as entries in a set of tables with two columns for each planet (Toomer, 1984: 632). Identical tables can be found for instance in Al-Battani (Nallino, 1903(II): 140) and several versions of the Alfonsine Tables. The Handy Tables (Halma, 1822–1825), the Toledan Tables (Pedersen, 2002:1309) and Al-Khwarizmi (Suter, 1914:139) use a different scheme.

3.1 Superior Planets

The superior planets have a deferent circle that is inclined by a fixed angle relative to the ecliptic plane (see Figure 4). The nodes are located at the crossings between the deferent circle plane and the ecliptic plane. The epicycle in turn is deviated from the deferent plane by an angle relative to a line in the deferent plane from the deferent centre to the epicycle centre. This deviation is maximum when the epicycle centre is at the top/bottom of the inclined deferent and zero at the nodes. The Ptolemaic procedure to compute the latitude for a superior planet is to use Table 1 with the anomaly, γ, as an argument. For Mars, the first column, C_1, is used for longitude of centre arguments less than 90˚ and larger than 270˚, the second one C_2, for longitude of centre arguments between 90˚ and 270˚.

The longitude of centre argument, λ, is as stated above, the longitude of the epicycle centre, measured from the top of the deferent circle. Mathematically the latitude is then computed from

$$\beta = C_{1,2}(\gamma)\sin(\lambda + 90^\circ) \quad (1)$$

In the *Almagest* the last sine function is represented by a separate column.

The original unit of the tables is degrees: minutes, and I have converted this to decimal units in an extra third column for each planet in Table 1.

3.2 Inferior Planets

The inferior planets have a more complicated mechanism to account for the latitude. As for the superior planets the deferent circle is inclin-

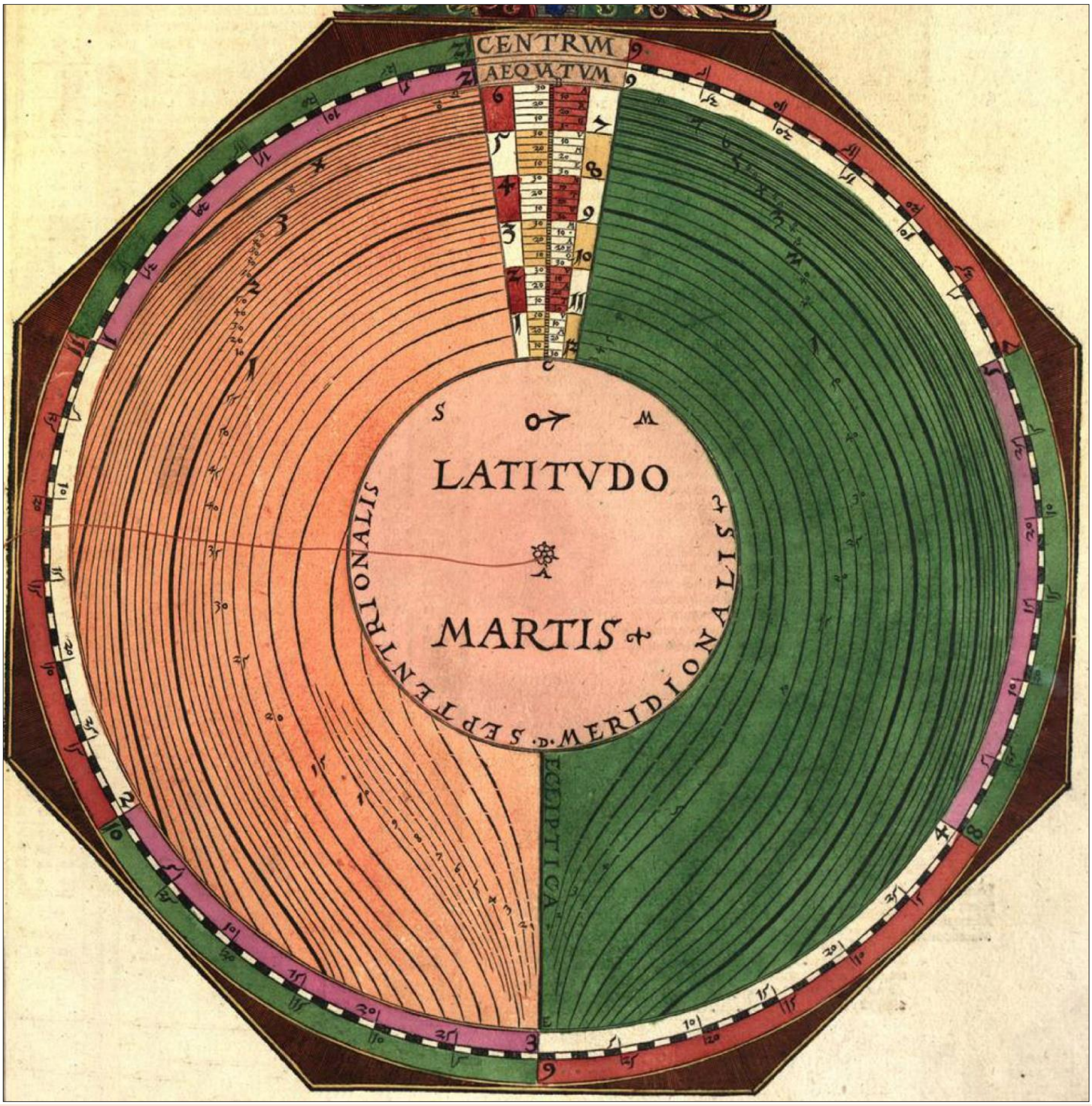

Figure 1: The Mars volvelle.

ed by an angle relative to the ecliptic plane, but this inclination is variable, being zero when the planet is at the nodes and maximum/minimum at right angles to the nodes. Secondly, the epicycle is, as for the superior planets, deviated relative to the deferent plane, but the deviation is zero at the top/bottom of the deferent and maximum/minimum at the nodes. Thirdly, the epicycle has a rocking motion perpendicular to the epicycle deviation. The rocking angle (slant, obliquity) is zero at the nodes and maximum/minimum at the top/bottom of the deferent. This complicated motion is then approximated by Ptolemy as a sum of three separate latitudes:

$$\beta = C_0 \sin^2(\lambda + 90^\circ) + C_1(\gamma) \sin(\lambda) + C_2(\gamma)\, f \sin(\lambda + 90^\circ) \quad (2)$$

The first term describes the inclination of the deferent plane, the second term the deviation of the epicycle, and the third term the rocking motion. C_0 is a fixed angle being 0°10' ≈ 0.167° for Venus and –0°45' = –0.75° for Mercury. $C_1(\gamma)$ and $C_2(\gamma)$ are to be taken from Table 1 with the anomaly as an argument. For Mercury the second and third terms in (2) are taken with the opposite sign.

The factor f is 1 for Venus but for Mercury it is 0.9 if $\lambda < 180^\circ$ and 1.1 if $\lambda > 180^\circ$.

The combined effect of the three terms in (2) is "... to give the epicycle a heaving, pitching, and rolling motion like that of a ship in a heavy sea." (Pedersen, 1974: 370).

In the C_1 table I have changed the sign of the values of γ for values larger than 90° and less than 270° in the decimal column. This makes

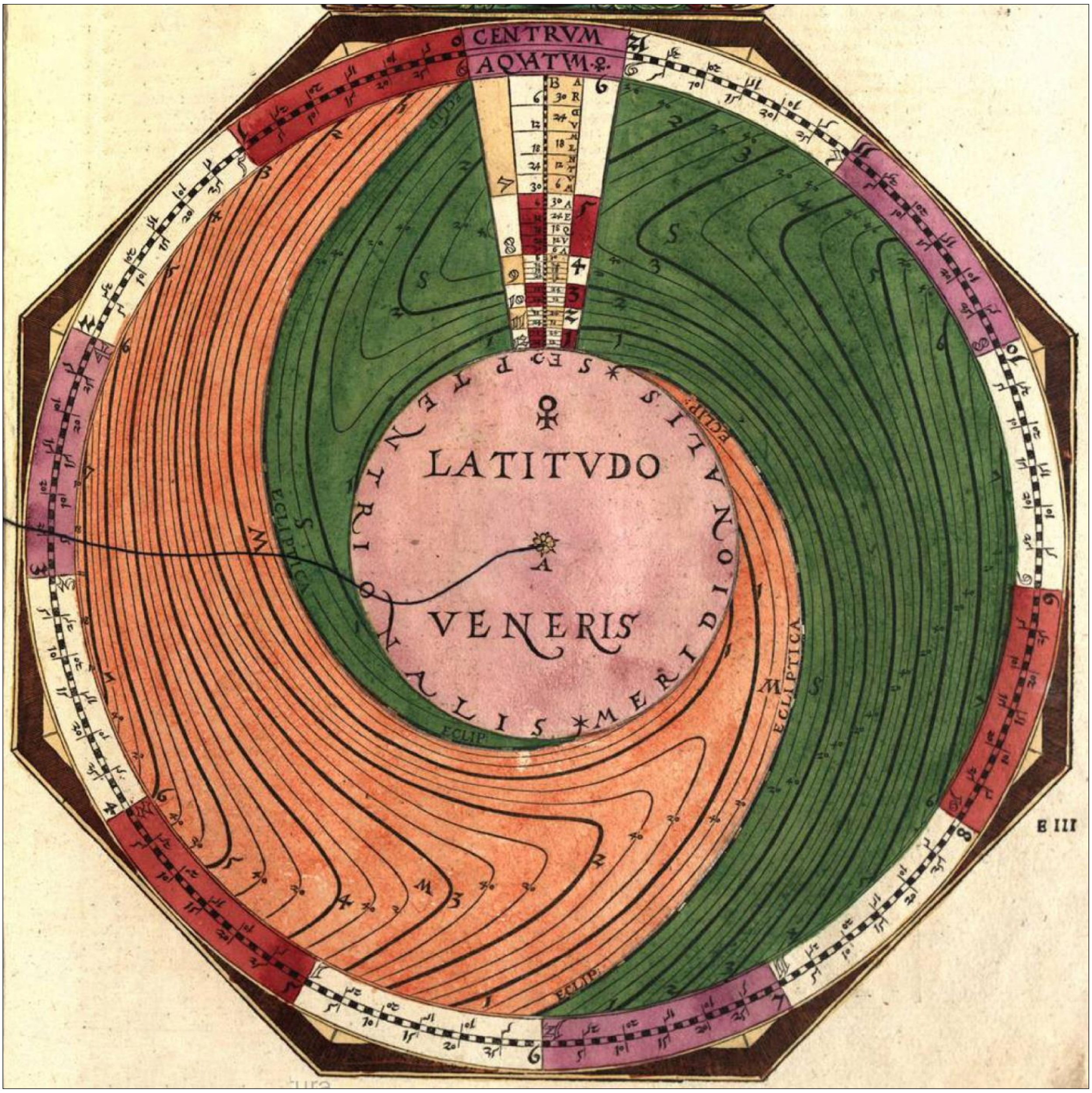

Figure 2: The Venus volvelle.

some of the subsequent calculations easier. In the *Almagest* this is taken care of by a special rule.

On my website http://home.thep.lu.se/~larsg/Site/Welcome.html there is a Java application (LatitudeViewer1.jar) that can be freely downloaded. It illustrates in a qualitative way the complicated motion of the deferent and epicycle for the superior and inferior planets in the Ptolemaic model. You will need to have the Java Runtime Environment (JRE) installed on the computer in order to run the file. The JRE can be freely downloaded from https://www.java.com/en/download. On a Macintosh you may need to change your security settings in System Preferences/Security & Privacy/Open Anyway button to be allowed to run the program.

4 DISCUSSION AND CONCLUDING REMARKS

It is interesting to speculate how Apianus constructed the quite intricate set of lines showing the latitudes. One way would be to invert the mathematical relations above, something that is analytically impossible, but could be done numerically. Another and more likely way for Apianus would be to try to graph the relations and then use the graphs to extract the necessary data. I used Microsoft Excel to make tables for the three planets Mars, Venus, and Mercury for a selected set of values of γ and λ and then graphed these tables. Figures 5, 6, and 7 show the results.

For Mars and Venus, you use the anomaly $\gamma' = 360 - \gamma$ if $\gamma > 180^\circ$.

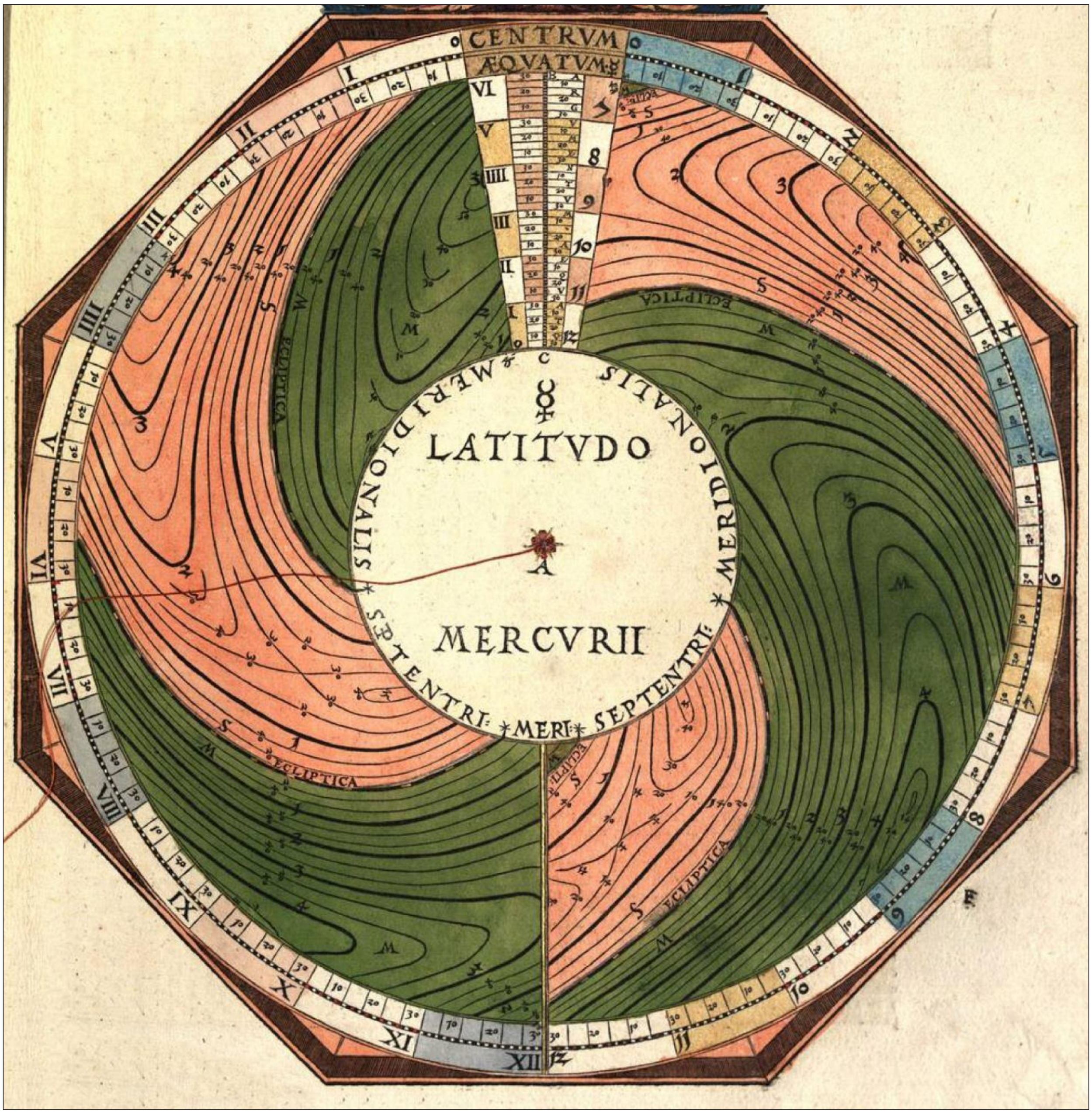

Figure 3: The Mercury volvelle.

It is quite tedious to do the computations leading to these graphs but mathematically it is rather simple and could be done by even an inexperienced person given a set of simple instructions. In order to construct his volvelles with some accuracy, Apianus would certainly have to draw the graphs in a larger scale than can be represented in this paper. But I have found it quite possible to use the graphs to find specific points (λ, γ) in the volvelle plane and then to connect the points by lines and reproduce

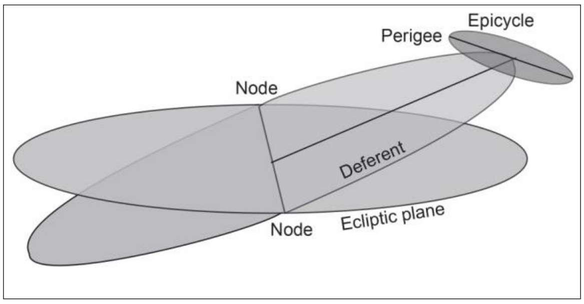

Figure 4: Deferent inclination and epicycle deviation.

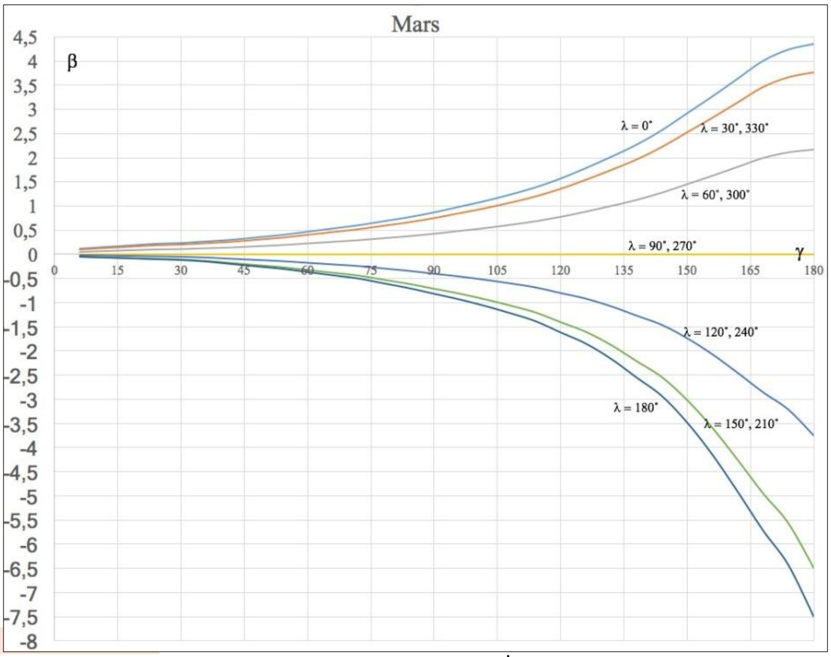

Figure 5: (γ, λ) graph for Mars.

Table 1: Fundamental tables

Anomaly		Mars						Venus						Mercury					
□		C1			C2			C1			C2			C1			C2		
0	360	0	5	0.08	0	2	0.03	1	3	1.05	0	0	0	1	46	1.77	0	0	0.00
6	354	0	7	0.12	0	3	0.05	1	2	1.03	0	8	0.13	1	45	1.75	0	11	0.18
12	348	0	9	0.15	0	4	0.07	1	1	1.02	0	16	0.27	1	44	1.73	0	22	0.37
18	342	0	11	0.18	0	5	0.08	1	0	1.00	0	24	0.40	1	43	1.72	0	33	0.55
24	336	0	13	0.22	0	6	0.10	0	59	0.98	0	33	0.55	1	40	1.67	0	44	0.73
30	330	0	14	0.23	0	7	0.12	0	57	0.95	0	41	0.68	1	36	1.60	0	55	0.92
36	324	0	16	0.27	0	9	0.15	0	55	0.92	0	49	0.82	1	30	1.50	1	6	1.10
42	318	0	18	0.30	0	12	0.20	0	51	0.85	0	57	0.95	1	24	1.40	1	17	1.28
48	312	0	21	0.35	0	15	0.25	0	46	0.77	1	5	1.08	1	16	1.27	1	27	1.45
54	306	0	24	0.40	0	18	0.30	0	41	0.68	1	13	1.22	1	8	1.13	1	35	1.58
60	300	0	28	0.47	0	22	0.37	0	36	0.60	1	20	1.33	0	59	0.98	1	44	1.73
66	294	0	32	0.53	0	26	0.43	0	29	0.48	1	28	1.47	0	49	0.82	1	51	1.85
72	288	0	36	0.60	0	30	0.50	0	23	0.38	1	35	1.58	0	38	0.63	2	0	2.00
78	282	0	41	0.68	0	36	0.60	0	16	0.27	1	43	1.72	0	26	0.43	2	7	2.12
84	276	0	46	0.77	0	42	0.70	0	8	0.13	1	50	1.83	0	16	0.27	2	14	2.23
90	270	0	52	0.87	0	49	0.82	0	0	0.00	1	57	1.95	0	0	0.00	2	20	2.33
96	264	0	59	0.98	0	56	0.93	0	10	-0.17	2	3	2.05	0	15	-0.25	2	27	2.45
102	258	1	6	1.10	1	4	1.07	0	20	-0.33	2	9	2.15	0	31	-0.52	2	28	2.47
108	252	1	14	1.23	1	13	1.22	0	32	-0.53	2	15	2.25	0	48	-0.80	2	29	2.48
114	246	1	23	1.38	1	23	1.38	0	45	-0.75	2	20	2.33	1	6	-1.10	2	30	2.50
120	240	1	34	1.57	1	37	1.62	0	59	-0.98	2	25	2.42	1	25	-1.42	2	29	2.48
126	234	1	47	1.78	1	51	1.85	1	13	-1.22	2	28	2.47	1	45	-1.75	2	26	2.43
132	228	2	1	2.02	2	10	2.17	1	38	-1.63	2	30	2.50	2	6	-2.10	2	20	2.33
138	222	2	16	2.27	2	33	2.55	1	57	-1.95	2	30	2.50	2	26	-2.43	2	11	2.18
144	216	2	34	2.57	2	56	2.93	2	23	-2.38	2	28	2.47	2	47	-2.78	2	0	2.00
150	210	2	55	2.92	3	29	3.48	3	13	-3.22	2	22	2.37	3	7	-3.12	1	45	1.75
156	204	3	16	3.27	4	9	4.15	3	43	-3.72	2	12	2.20	3	26	-3.43	1	29	1.48
162	198	3	38	3.63	4	55	4.92	4	26	-4.43	1	55	1.92	3	42	-3.70	1	10	1.17
168	192	4	0	4.00	5	43	5.72	5	24	-5.40	1	27	1.45	3	54	-3.90	0	48	0.80
174	186	4	14	4.23	6	26	6.43	6	24	-6.40	0	48	0.80	4	2	-4.03	0	28	0.47
180	180	4	21	4.35	7	30	7.50	7	12	-7.20	0	0	0.00	4	5	-4.08	0	0	0.00

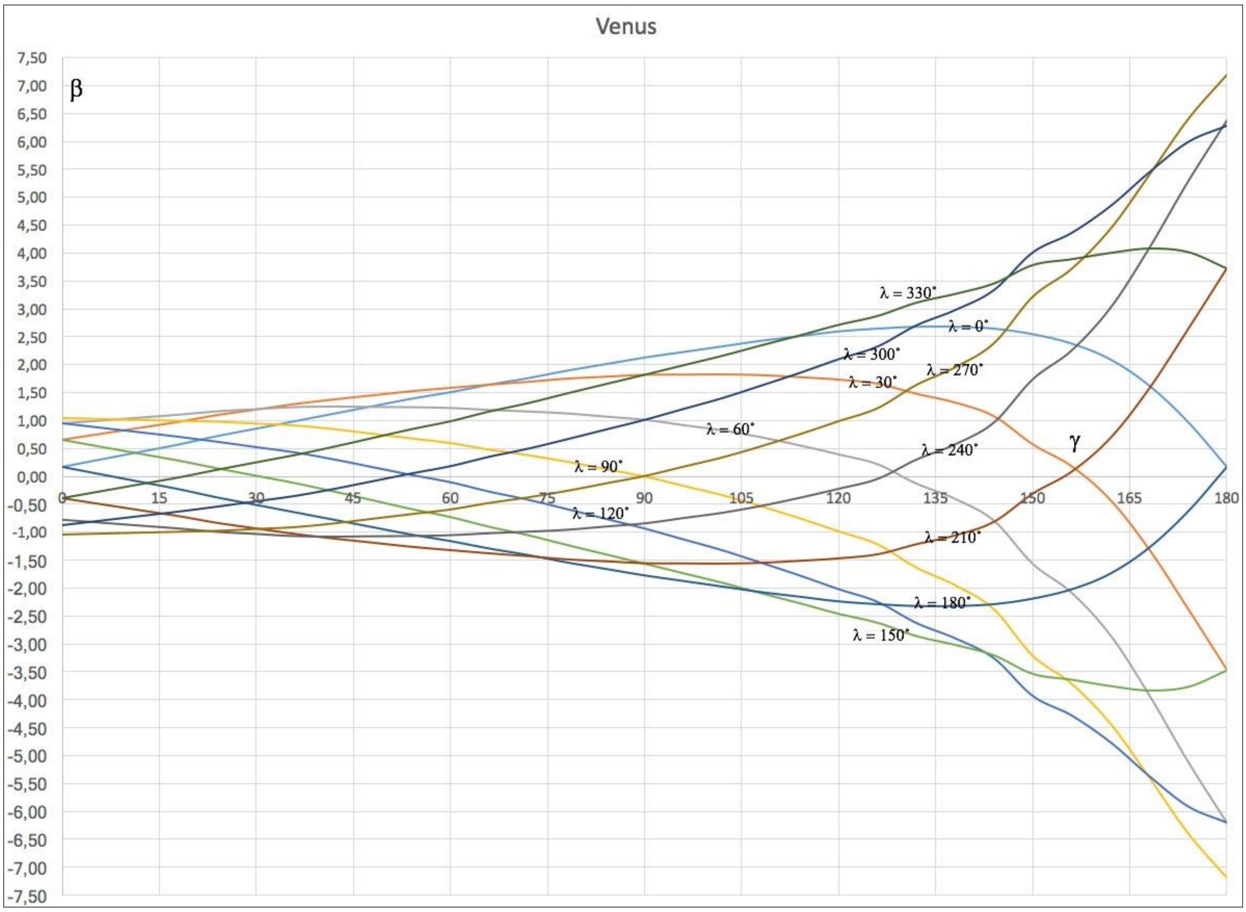

Figure 6: (γ, λ) graph for Venus.

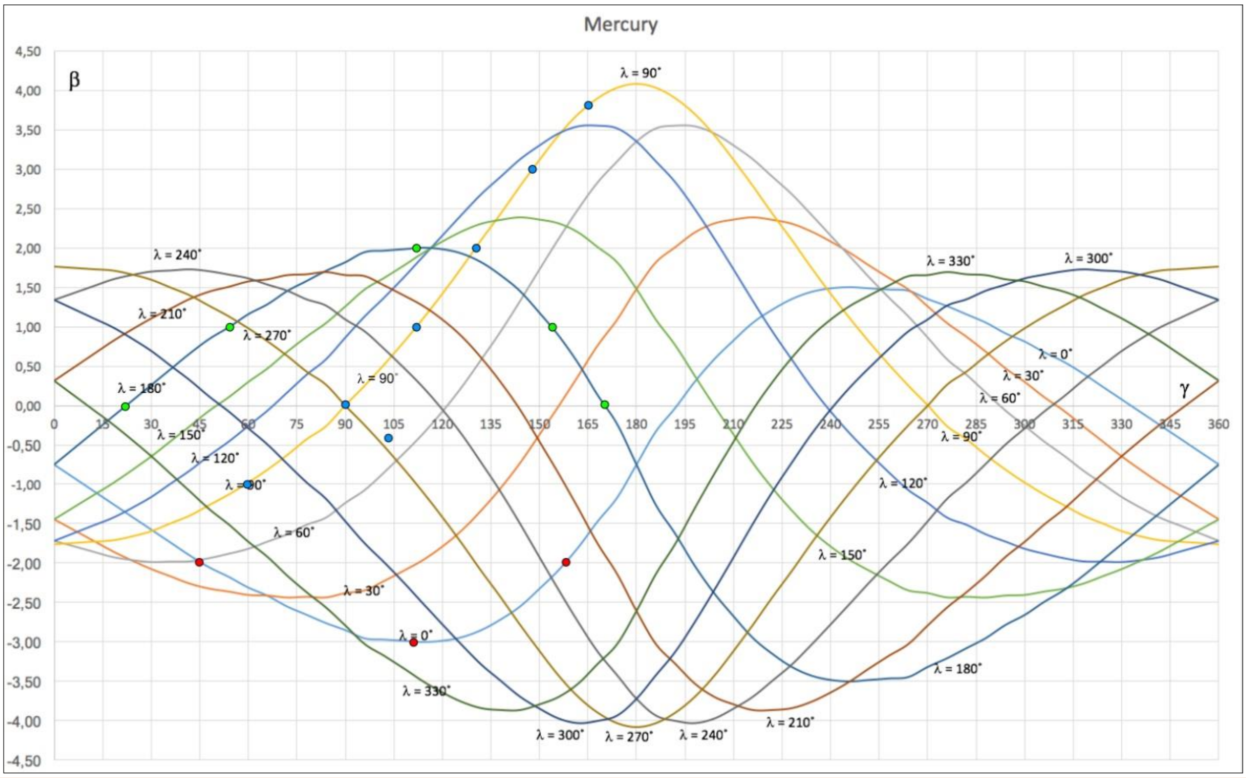

Figure 7: (γ, λ) graph for Mercury.

Apianus' results quite well. It is even possible to interpolate between the curves and in that way find points for intermediate values of γ and λ. As an illustration of how the construction may have been done, we can use the Mercury graph, Mercury being the most complicated case.

Following the light blue curve ($\lambda = 0^\circ$) in Figure 7, it crosses $\beta = -2^\circ$ for $\gamma = 45^\circ$ (1 sign 15°). This point is marked by a red dot in Figure 8 and in the graph. The curve then just touches $\beta = -3^\circ$ for $\gamma = 110^\circ$, then crosses $\beta = -2^\circ$ again for $\gamma = 158^\circ$.

The yellow curve ($\lambda = 90^\circ$) crosses $\beta = -1^\circ$ for $\gamma = 60^\circ$, passes $\beta = 0^\circ$ for $\gamma = 90^\circ$, then $\beta = 1^\circ$ for $\gamma = 90^\circ$, $\beta = 2^\circ$ for $\gamma = 130^\circ$, $\beta = 3^\circ$ for $\gamma = 147^\circ$, and finally $\beta = 4^\circ$ for $\gamma = 172^\circ$. These points are marked in blue.

The dark blue curve ($\lambda = 180^\circ$) crosses $\beta = 0^\circ$ for $\gamma = 22^\circ$, then $\beta = 1^\circ$ for $\gamma = 55^\circ$, almost touches $\beta = 2^\circ$ for $\gamma = 115^\circ$, again crosses $\beta = 1^\circ$ for $\gamma = 153^\circ$ and $\beta = 0^\circ$ for $\gamma = 170^\circ$. These points are marked in green.

Given some patience it is no doubt possible to use this procedure to construct the latitude curves of the volvelles. If one does a detailed check it is found that Apianus has some errors in his latitude curves in the volvelles of the inferior planets where the latitude curves change direction, like for Mercury around $\lambda = 60^\circ$, $\gamma = 15^\circ$. But this is an exception; in general the points generated agree very well with the volvelles. The craftsmanship and elegance of these volvelles once again confirms the impression that Petrus Apianus had one of the most interesting and creative minds of the Middle Ages.

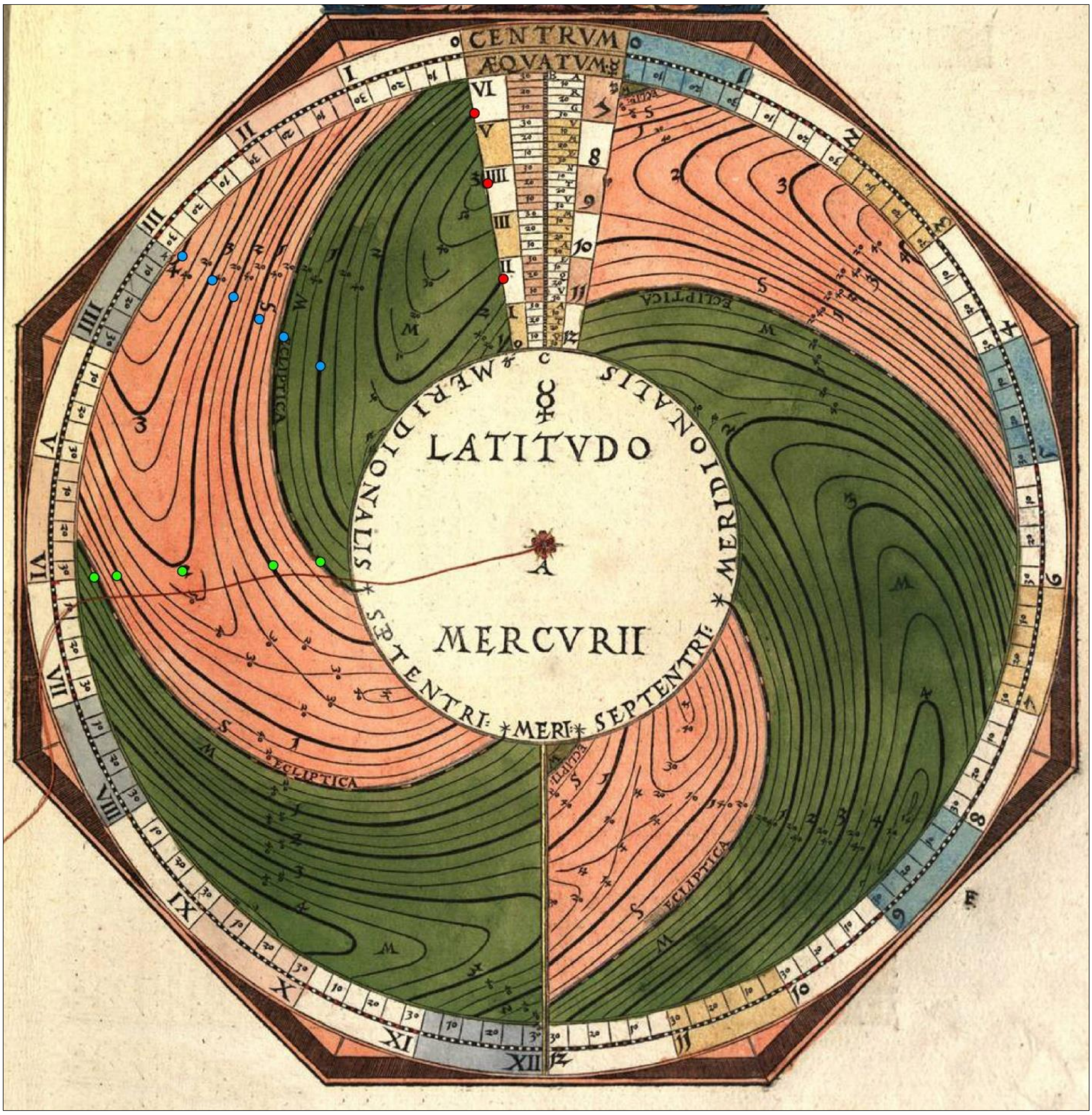

Figure 8: Mercury verification.

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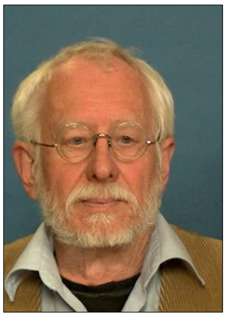

for a Ph.D. in the Faculté des Sciences, at Orsay, France. He has been doing research in elementary particle physics, complex system with applications of physics in biology and with atmospheric optics. During the last fifteen years he has developed several computer programs and Excel sheets implementing calendars and medieval astronomical models from Europe, India and South-East Asia (see http://home.thep.lu.se/~larsg/).