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PETRUS APIANUS' VOLVELLE FOR FINDING THE EQUINOXES

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Abstract: In this paper the mathematics behind the equinox volvelle in Petrus Apianus' *Astronomicum Cæsareum* is investigated.

Keywords: Petrus Apianus, *Astronomicum Cæsareum*, equinox volvelle

1 INTRODUCTION

This is the fifth and final paper (see Gislén, 2016; 2017a; 2017b; 2018) in a series on the volvelles in Petrus Apianus' *Astronomicum Cæsareum* (1540),

> … a magnificent book with more than thirty volvelles or equatoria, a kind of paper computer, that, using the detailed instructions and examples in the text, allowed readers to calculate astronomical, chronological and also astrological phenomena. (Gislén, 2018: 135).

The *Astronomicum Cæsareum* has been described as "… undoubtedly one of the most extraordinary books in the world." (Gislén, 2018: 199). Gislén (2018: 135) also notes that such "… computing devices were quite popular during the fifteenth and sixteenth centuries."

Petrus Apianus (Figure 1) was born Peter Bienewitz on 16 April 1495 in Leisnig in Saxony, Germany, and after studying astronomy and mathematics he joined the University of Ingolstadt as a mathematician and printer. He remained in Ingolstadt until his death on 21 April 1552. For further biographical details of Apianus see Draxler and Lippitsch, (2012), Galle (2007) and Gingerich (1971).

This paper examines the equinox volvelle in Apianus' *Astronomicum Cæsareum*.

2 DESCRIPTION OF THE EQUINOX VOLVELLE

For this paper I have used the digital copy of the *Astronomicum Cæsareum* in the Astronomy Library at the Vienna University. This copy has a high resolution and the scanned pages are almost free of distortions. The equinox volvelle, (Figure 2) is the penultimate volvelle in the first part of the *Astronomicum Cæsareum*. It consists of a bottom plate, the mater, and a moveable top disk that can be rotated around a central axis. The mater is graduated by the fourteen days on which the equinox can occur in the selected time interval CE 1300–3000. Each day is sub-graduated by 24 hours from midnight to noon to midnight. Night and day portions are indicated by an outer circular band with white and black partitions. In the middle of the night part are the words MEDIA NOX or just NOX, and in the middle of the day part the words DIES or MERI(DIES) or just M. The letters R and C stand for sunset and sunrise respectively (see Figure 3). In the innermost circular band of the volvelle are eighteen fixed century indexes marked 1300 to 3000 for years after Christ. The index for the index 1500 is marked with M. When the specified year is a leap year, the February dates at the top of the mater should be 28 and 29 instead of 27 and 28.

Figure 1: Petrus Apianus

The moveable top disk has five equally spaced tabs marked B, C, D, E, and F. Each of the tabs represents twenty years of the century, grouped in five concentric circular bands, each band having a sequence of four years. In a circular band immediately below the tab there is a scale from 12 hours midnight over noon to 12 hours midnight. To the right of this scale are the letters OC (occident, west) and to the right OR (orient, east). This scale is used for geographical time difference corrections for locations west or east of Ingoldstadt, the meridian used for the *Astronomicum Cæsareum*. Finally, there is a

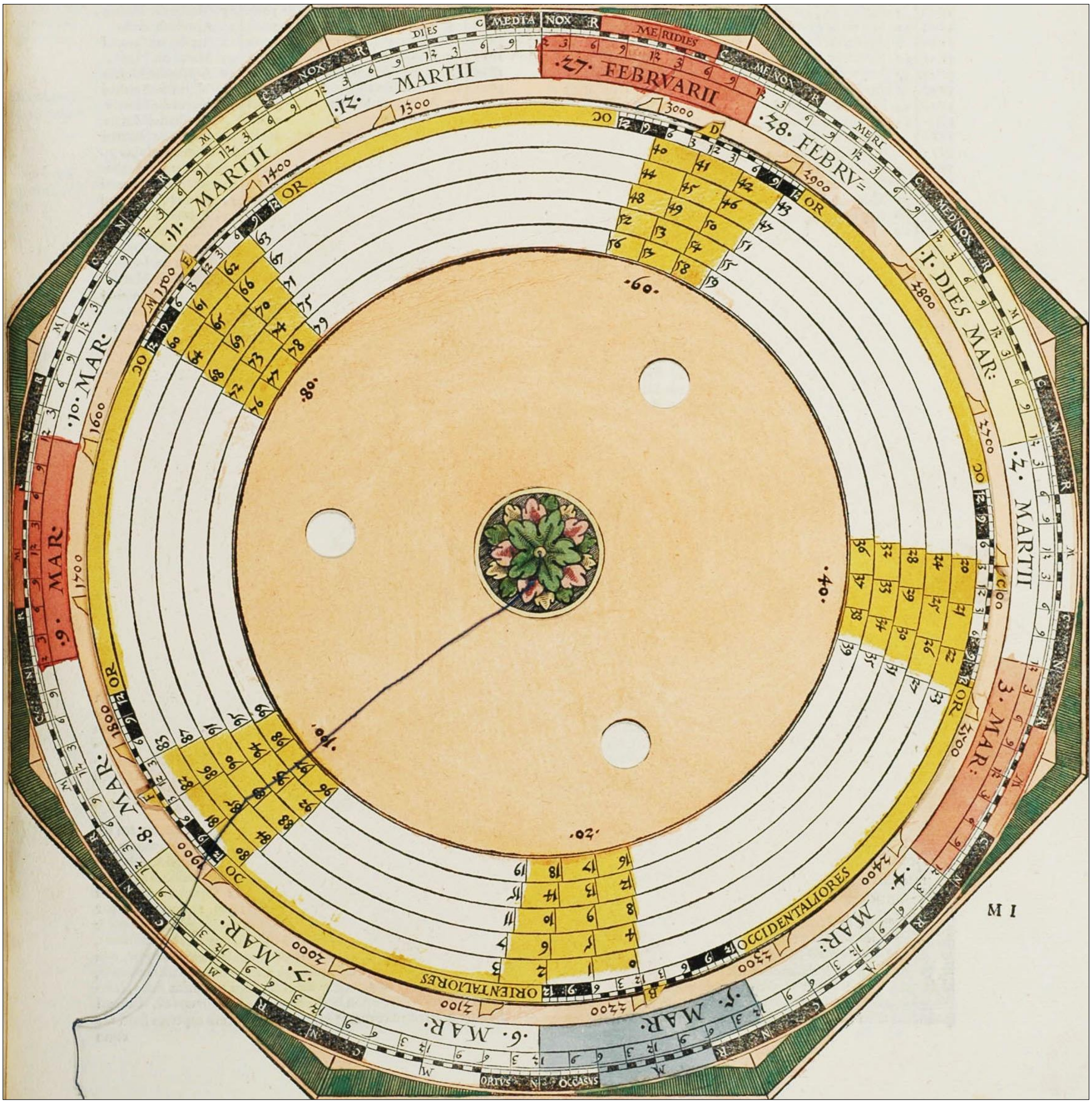

Figure 2: The equinox volvelle.

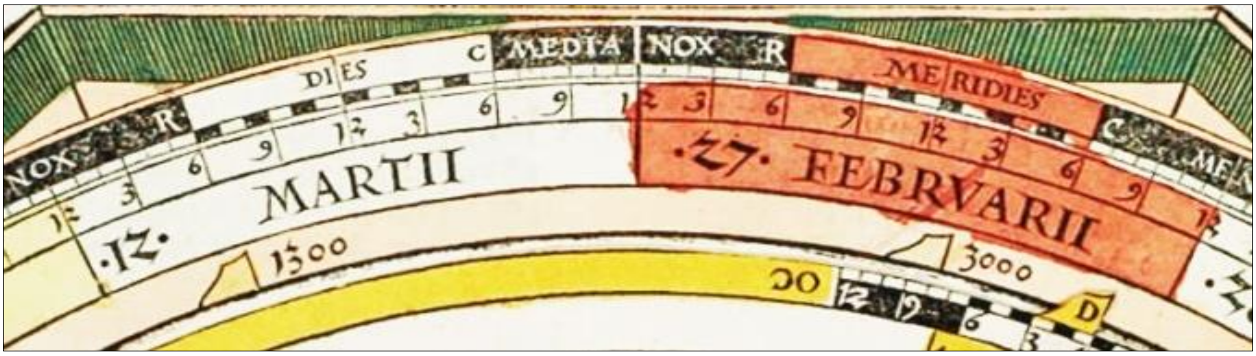

Figure 3: Detail of the mater.

thread from the centre of the volvelle to be used for reading off the time of the equinox.

The instrument uses the Julian calendar with a mean year length of 365.25 days.

3 HOW TO USE THE VOLVELLE

The working of the volvelle is very simple. The tab of the top disk with the 20-year group containing the specified year in the century is placed on top of the index of the chosen century. The thread from the centre is aligned with the line representing the specified year in the 20-year group and the time and day of the spring equinox are read out from the mater scale. If necessary, the time is corrected for east-west longitude time difference using the time scale below the top disk tab. To find the corresponding autumn equinox you add six months, three days, and 42 minutes. Due to the difficulty of reading the time scale the result has an effective precision of about 10 minutes. In the examples given by Apianus in the *Astronomicum Cæsareum*, he states the times with a precision of minutes, and he clearly got his results by some other means of computation.

4 THEORY

The volvelle in principle only uses the mean motion of the Sun but what is wanted is the true equinox when the true Sun has longitude 0˚. The volvelle is based on the assumption that the correction added to the solar mean longitude in order to get the true solar longitude, the equation of centre, is fixed during the time interval covered by the volvelle. The equation of centre is determined by the anomaly, the difference between the mean solar longitude and the solar apogee. In the Ptolemaic model used by Apianus the apogee is fixed relative to the fixed stars but moves with the precession of the equinox. In the Alfonsine and Apianus models the precession has two components, one steady motion that takes the equinox a full turn of 360° in 49,000 years, and a periodic oscillation, the trepidation, with an amplitude of 9° and a period of 7,000 years. The trepidation is a false hypothesis brought along by medieval astronomers who found that their measurements of the precession differed from Ptolemy's value of precession and then assumed that the value of precession varied (see Dreyer,1953; Neugebauer, 1962). The precession means that the solar apogee moves and that the solar equation of centre is not constant. However, it turns out that by coincidence, for the centuries CE 1300 to CE 1700, in the model of precession above, the equation of centre is maximum and almost constant (see Figure 4). I used the scheme of the Alfonsine Tables (Gislén, Poulle) to compute the equinoxes and added 1 hour 29 minutes as given in the *Astronomicum Cæsareum* as the longitude time difference between Toledo and Ingolstadt. The result agrees very well with the volvelle values for century years 1300 to 1700 but then diverges progressively from the volvelle values to make a difference of about 15 minutes for CE 3000. The mean solar tables in the *Astronomicum Cæsareum* are based on the Alfonsine tables but with a correction to the mean solar longitude at the epoch that corresponds to the geographical longitude time difference between Toledo and Ingoldstadt.

It is very tedious to compute the time of the true equinoxes by hand using tables and I speculate that Apianus computed the time for one century by tables and then used the extrapolation shown below for the others, assuming a constant equation of centre. It turns out that such an assumption agrees very well with the layout of the volvelle.

In the following I use the notation a, b, c, d … where 'a' is the zodiacal sign with Aries = 0, 'b' the degrees within the sign, and 'c', 'd' and so on representing each 1/60 of the preceding unit. For time I use the notation hour: minute. In the present investigation there is no need to retain the seconds, although I have used them in the internal calculations.

As there are fourteen days along the periphery of the volvelle, each day represents an angle of 360°/14 or 25.714° of the circumference. In the *Astronomicum Cæsareum* the Sun's mean daily motion is v_S = 0;59,08,19,37,19,13,56 = 0.985646398°. This gives a year length of 360 / v_S = 365.2425461 days. Now, 100 Julian calendar years contain 36525 days

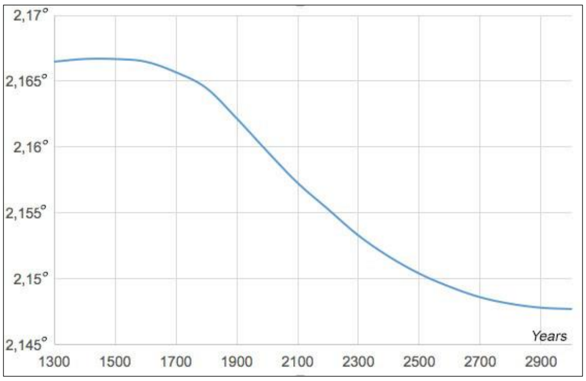

Figure 4: The equinox equation of centre.

while 100 solar years have 36524.25461 days. The difference is 0.74539 days and this corresponds to an angle of 25.714 · 0.74539 = 19.167° between the centuries. In Figure 5 I have added 18 computer-generated century lines, starting with the year 1300 using this fact. As can be seen the agreement is excellent.

In a 20-year Julian period there are 7305 calendar days, but 20 tropical years have 7304.850922 days. The difference is 0.149078 days or 3:35 hours. In the top disk of the volvelle, the years within a century are divided into 5 groups each with 20 years. Within each group of 20 years there are 5 leap year cycles, each with 4 years. On the top disk these five groups are placed with the index tabs equally spaced along the edge of the disk. The index tab B with years 0–19 is aligned with year 0. The index tab C of the years 20–39 is displaced

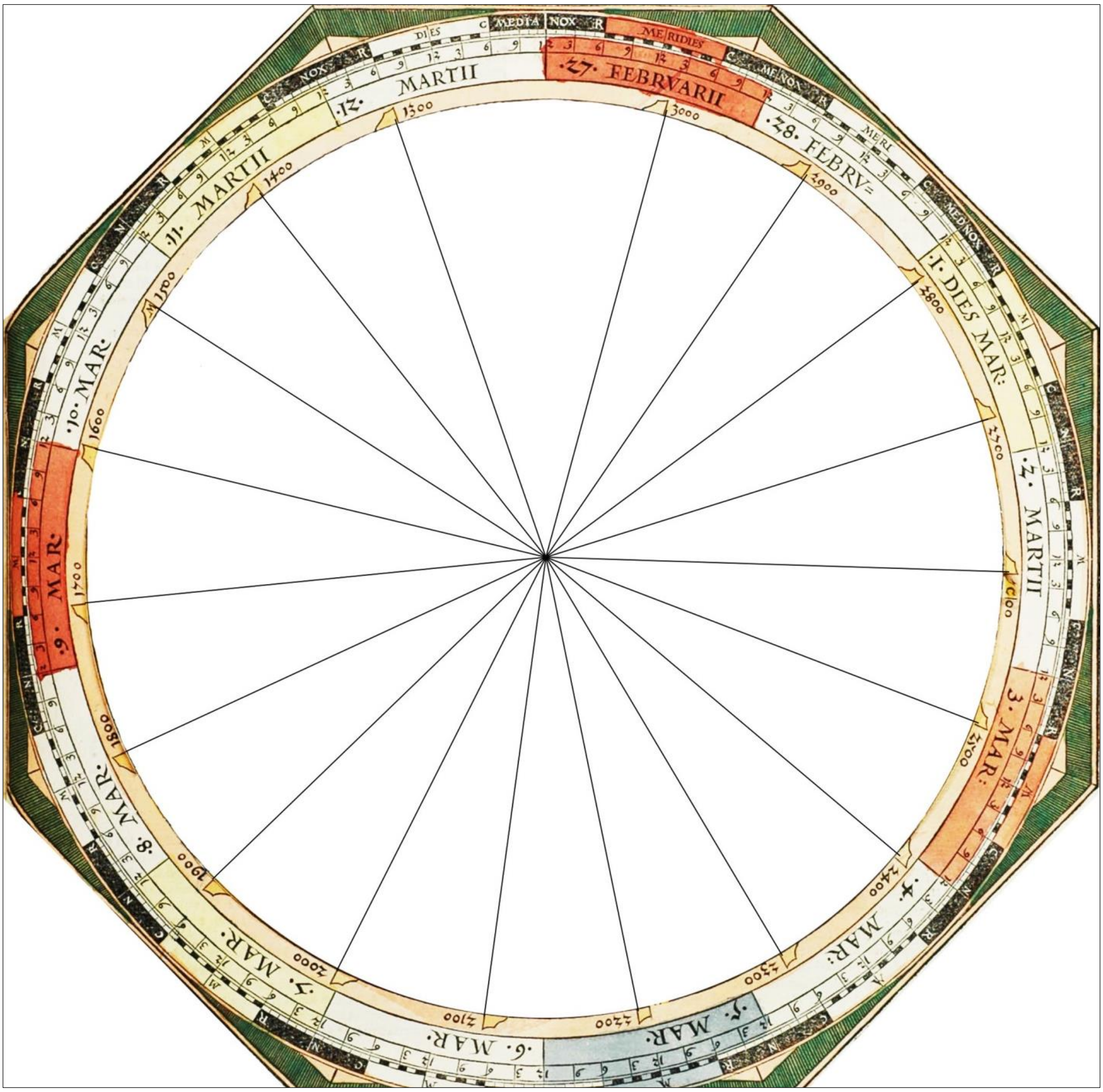

Figure 5: Predicted locations of the index tabs.

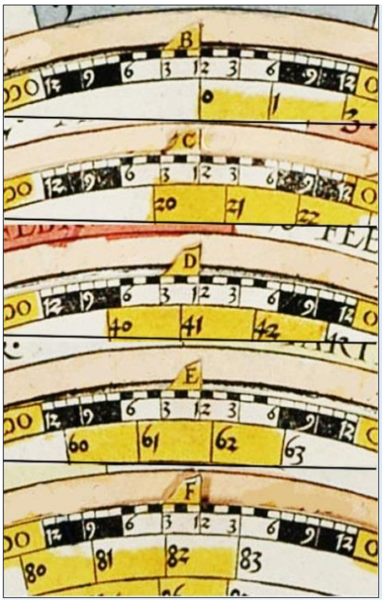

Figure 6: The tab displacements.

3:35 hours clockwise with respect to year 20 and similarly further forwards in turn by 2 · 3:35 = 7:09 hours for the tabs D for year 40, by 3 · 3:35 = 10:43 in E for year 60, and by 4· 3:35 = 14:18 in F for year 80. Figure 6 show this progressive displacement.

In a 4-year period there are 1461 calendar days and 1460.970184 solar days. The difference is 0.029816 days or 0:43 hours. In one ordinary year the difference between the civil days and the solar days is 0.2425461 = 5:49 hours. In each group of four years the equinox time is moved forward by this amount for each year. At the end of the fourth year it would have moved in total 4 · 5:49 = 23:17 hours but because of the leap day it is also moved back 24 hours and starts the next 4-year period 0:43 hours earlier than the previous 4-year group. This is illustrated in Figure 7 by the green com-

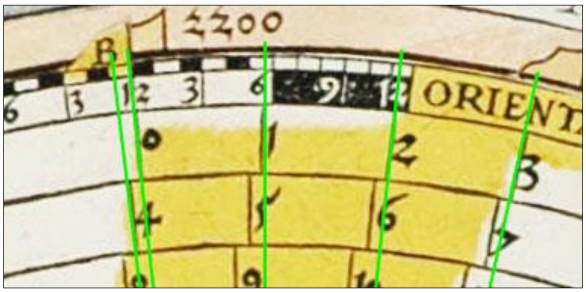

Figure 7: The year location.

puter-generated lines for the years 60, 61, 62, 63, and 64. The agreement is excellent.

5 CONCLUDING REMARKS

This volvelle is another example of the creativity and ingenuity of Apianus although perhaps its practical usage is limited because unfortunately, by comparison with a modern calculation, it turns out that the trepidation correction in the time interval CE 1300–1700 is large and causes the volvelle equinox times to be about ten hours early. The error then becomes progressively smaller and by CE 3000 it is about one hour.

A working model of the volvelle with instructtions for its construction can be downloaded from the following web site:

http://www.thep.lu.se/~larsg/EquinoxVolvelle.pdf

6 NOTES

1. Petrus Apianus was also known by his Anglicised name, Peter Apian.

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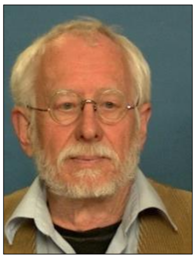

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