# **THE CALENDARS OF SOUTHEAST ASIA. 5: ECLIPSE CALCULATIONS, AND THE LONGITUDES OF THE SUN, MOON AND PLANETS IN BURMESE AND THAI ASTRONOMY**

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**Abstract:** Many of the calendrical records in Southeast Asia contain information on the longitudes of the Sun, the Moon and the planets—something that is valuable for the dating of these records. Both the Burmese and the Thai use calculation schemes for the longitudes of the Sun, the Moon, and the planets that are almost identical to the original *Sūryasiddhānta* schemes. After the change to the Thandeikta calendar the Burmese changed some of the parameters involved, in general following those of the modern *Sūryasiddhānta*.

**Keywords:** History of astronomy, calendars, eclipse calculations, longitude of the Sun, longitude of the Moon, longitude of the planets, Burmese astronomy, Thai astronomy

# **LONGITUDES OF THE SUN AND THE MOON AND THE LATITUDE OF THE MOON**

The mean longitude of the Sun in the Thai and original Burmese calendars $1$  is calculated using the excess of solar days in units of  $1/800<sup>th</sup>$  of a day at the beginning of the solar year, adding the number of elapsed day of the year multiplied by 800 and dividing by 24350. The quotient gives the zodiacal sign (rasi) involved.<sup>2</sup> The remainder is divided by 811, the quotient is the degrees in the sign (*angsa*), the remainder is further divided by 811, the quotient is the arc minutes (*lipda*). The reason for the numbers 24350 and 811 is the following: a zodiacal sign contains 30 degrees that is  $1/12^{th}$  of 360°. A year contains 292207 parts in units of  $1/800<sup>th</sup>$  of a day. Dividing this by 12 we get 24350, i.e. one zodiacal sign corresponds to 24350 parts of  $1/800<sup>th</sup>$  of a day. Then one degree corresponds to  $24350/30 = 811$ . The calculation procedure is only approximate but accurate enough and typical of the integer kind of calculations with 'magical' numbers that is used in Burma and Thailand. There are 3 arc minutes extra subtracted from the mean longitude of the Sun, the reason is given below.





The true longitude of the Sun, *Ltrue*, can mathematically be written as a function of the mean longitude, *Lmean*, by

$$
L_{true} = L_{mean} - \arcsin[e \times \sin(L_{mean} - w)], \qquad (1)
$$

where *e*, the eccentricity of the Sun, is 14/360, and *w* is the longitude position of the apogee of the Sun, taken as 80°. These are both *Sūryasiddhānta* values (Billard, 1971). The second term of this expression, the equation, is interpolated using a table given in arc minutes, the *chaya* (see Table 1*)*, with entries for every 15 degrees of angle in the first quadrant, 0–90°. Due to the symmetry of the sine function only values in the first quadrant are needed in the table though the equation is negative in the third and fourth quadrants.

The Thai and Burmese computations for the lunar longitude may appear confusing because the Southeast Asian calendarists effortlessly switch between using solar day and lunar day units (and sometimes get confused themselves). For example, in the Burmese and in La Loubère's (1691) description the *avoman*, *a*, is computed from the elapsed lunar days or *tithis*, *t*, by the relation

$$
a = [(t \times 11) + 650] \mod 703, \tag{2}
$$

while, in Wisandarunkorn (1997) and Faraut (1910), it is computed using the elapsed *ahargaņa*, *h*, by

$$
a = [(h \times 11) + 650] \mod 692. \tag{3}
$$

The *avoman* is the excess over whole *tithis* for elapsed solar days, or *ahargaņa*. A *tithi* is 692/703 of a solar day and thus 11/703 shorter than a solar day in units of a solar day. On the

other hand, a solar day is 703/692 *tithi* and thus 11/602 *tithi* longer in units of a lunar day. The difference between the two expressions above is that the first one gives the *avoman* in units of 1/703 of a solar day while the second expression gives the *avoman* in units of 1/692 of a lunar day.

In a similar way *tithis* can be converted to *ahargaņa* and *vice versa* with the two relations:

$$
t = h + [(h \times 11) + 650]/692,
$$
  
and 
$$
h = t - [(t \times 11) + 650]/703.
$$
 (4)

The *tithis* measure the elongation of the Moon from the Sun, each *tithi* corresponding to 360/30 = 12° of elongation. The *avoman* is a measure of the fraction of the *tithi*. The scheme for converting the *avoman* into a fractional lunar elongation of the day is to take the *avoman*, divide by 25, add the *avoman* to the quotient and divide by 60. This will give the fractional lunar elongation in degrees. The procedure is equivalent of multiplying the *avoman* by 1.04 and dividing by 60. In the first relation above, equation (2), the maximum value of the *avoman* is 703, corresponding to a solar day. If we multiply this by 1.04 and divide by 60 we get 12.185°, which is almost exactly the daily movement of the Moon, 360/29.530583 = 12.191°. Using the second expression for the *avoman*, a value of 692 corresponds to one *tithi*, and  $692 \times 1.04/60 =$ 11.995°, which again is almost exactly the correct value of 12° for the movement of the Moon in one *tithi*. Thus, by using the number 25 the fractional lunar longitude of the day is 'magically' produced. If the number of whole elapsed *tithis* multiplied by the daily *tithi* motion of 12° is added to this fractional longitude, the result is the mean elongation of the Moon from the Sun. Alternatively, if the *ahargaņa* multiplied by the Moon's motion in a solar day can be added, the result will be the same.

Adding the mean solar longitude to the elongation longitude will give the mean lunar longitude. Then 40 minutes of arc are routinely subtracted from the mean longitude of the Moon as with the 3 minutes of arc were subtracted from the mean longitudes of the Sun. These corrections have no clearly obvious purpose, but an explanation can be found. The mean daily motion of the Sun is 59′ and that of the Moon is 790'. Thus, 3' of the Sun corresponds to  $24 \times 3$  $/59 = 1.22$  hours of time and 40 of the Moon corresponds to  $24 \times 40/790$ , also 1.22 hours of time. Converted to geographical longitude where 15° corresponds to one hour, 1.22 hours equate to 18.3° in longitude, which is roughly the longitude difference between Ujjain, the prime meridian of India and Western Burma, and indicates that the subtracted minutes of arc is a correction introduced when the Indian calculation methods were transferred to Burma (something already

that the French astronomer Cassini noticed). One notes, however, that the operation survived in Faraut's (1910) version of the scheme, whose home ground was Cambodia, whereas the adjustment properly applies only to Burma. If it had indeed been required, the adjustment would have needed to be twice the applied amount to apply to Cambodia. It is clear, then, that the function of the adjustment was lost sight of, but it nonetheless continued to be integral to the reckoning.

The longitude of the lunar apogee is not fixed but is determined by the *uccabala*, and the current value has to be calculated using the following formula:

*uccabala* = (*ahargana* + 2611)mod 3232. (6)

The *uccabala* is converted to the angular position of the apogee, where 3232 days correspond to 360°. In the original *Sūryasiddhānta* it is assumed that the lunar apogee (*mandocca*) makes 488219 revolutions in 1577917800 days, i.e. the period is 1577917800/488219 = 3231.9878 days.

The true longitude of the Moon is calculated in the same way as for the Sun but now with the *Sūryasiddhānta* lunar eccentricity 31/360 (Billard, 1971) and using the current value of the lunar apogee. The equation is again given in terms of a table or *chaya* (see Table 1).

The Burmese Thandeikta scheme for the Sun and the Moon is a little different from that of the Makaranta. The mean longitude in minutes of arc of the Sun is calculated from the expression

 $L_{mean}$  = (1000 ×  $k_0$  + 800000 × *sutin* – 6 × *sutin*)/13528, (7)

where  $k_0$  is the New Year *kyammat* and *sutin* is the number of elapsed days of the solar year. The last two terms express the mean daily solar movement (800,000–6)/13528 = 59.136162 minutes of arc.

The mean longitude of the Moon is calculated, as before from the *avoman* but now with the expression

 $L_{mean}$  = *tithis*  $\times$  12  $\times$  60 + (*avoman* + 7/173 $\times$ *avoman*) – 52′, (8)

which gives the longitude in arc minutes. The factor  $7/173 \approx 1/24.714$  is very nearly the same as 1/25 that is used in the Makaranta scheme. The extra correction of 52' is partly a geographical longitude correction from Ujjain and partly a secular correction. The lunar apogee is calculated using the approximation that the daily motion of the apogee is  $3 \times 1800/808$  = 6.68317′, which is effectively the same as the Makarata and Arakanese value.

The Thandekta equations are calculated ac-





cording to the modern *Sūryasiddhānta* where the eccentricities vary in size,

 $e = e_0 - 20/(360 \times 60) \sin(L_{mean} - w)$ , (9)

and  $e_0$  is 14/360 for the Sun and 32/360 for the Moon. The solar apogee is fixed at longitude 77° 18′. This gives the following *chayas* for the Sun and the Moon (see Table 2). This table is also divided into smaller steps of 3.75°.

The latitude of the Moon is computed by assuming an inclination of the lunar orbit of 4.5° and assuming that plane geometry can be used. If the distance from the node is ∆, this would give the lunar latitude *β* as

 $\beta = \Delta \times \tan 4.5^{\circ} \approx \Delta \times 4.5 \times \pi/180 \approx \Delta \times 4.5/60$ . (10)

It is also used when the tangent of a small angle is approximately equal to the angle expressed in radians and the value of  $\pi$  has been set to 3.

### **2 DAY LENGTH AND** *LAGNA*

Day length and *lagna*, the ascendant or rising

Table 3: The day length and oblique ascension of the Sun for a geographical latitude of 15° 45′.



sign, are closely related. The *lagna* is used in Southeast Asian astrological records to give the time of day. Day length (Thai: *thinpraman*) can be computed using the concept of ascensional difference, that is the excess of daytime over the day length at the equinoxes. At the equinoxes the Sun for any location on the Earth moves during the daytime in the sky in a 180° section of a great circle, and the day length is 12 hours. At other times of the year the Sun will move in a parallel circle and the day length will be either longer or shorter. The ascensional difference *A*, can be computed by the formulae

 $\sin \delta = \sin \epsilon \times \sin \lambda$  (11)

and  $\sin A = \tan \varphi \times \tan \delta$ , (12)

where *λ* is the ecliptic longitude of the Sun, *δ* the declination of the Sun, *ε* is the obliquity of the ecliptic (in Indian tradition assumed to be 24°), and *φ* is the geographical latitude of the location on Earth. The day length (in degrees) is then calculated by 180° + 2*A*. Note that *A* is negative when the declination of the Sun is negative.

The Burmese and Thai astronomers used the time units *nadi*/*nayi*/*nati* and *vinadi*/*vinati*/ *bizana*. There are 60 *nadi* in a day and night and each *nadi* is equal to 60 *vinadi*, i.e. there are 3600 *vinadi* in a day and night. This makes the conversion from the angular measure degrees to *vinadi* very simple, as it is achieved by a multiplication of the degrees by 10.

The *lagna* is a concept inherited from India and is the rising sign of the ecliptic at a given time. In Western astrology it is the ascendant. In order to calculate the lagna another quantity is needed, the oblique ascension, *Ω*. The scheme to calculate the oblique ascension is the following:

Given the longitude of the Sun we can calculate its right ascension, *α*, by

$$
\tan \alpha = \tan \lambda \times \cos \varepsilon. \tag{13}
$$

The oblique ascension of the Sun is then the difference between the right ascension and the ascensional difference

$$
\Omega = \alpha - A. \tag{14}
$$

Table 3 shows the day length and oblique ascension of the Sun for a geographical latitude of 15° 45′ where the argument in the left column is the longitude of the Sun. At sunrise the *lagna* of the Sun is of course the longitude of the Sun. As the Sun rises the oblique ascension will increase by the time converted to degrees, and the longitude of the *lagna* can be obtained by inverse interpolation from the table for oblique ascension.

Figure 1 shows the Thai way of displaying these tables. The circle is divided into twelve sectors, one for each zodiacal sign, with Aries at the top and the other signs following counter-



Figure 1: Thai day length and oblique ascension table (after Wisandarunkorn, 1997: 165).

clockwise around the circle. The outer band of numbers presents the day lengths in *vinadi*, the inner band shows the differences in oblique ascensions. Intermediate day lengths are found by interpolation. The oblique ascension is found, as an example will show, by starting at the top of the figure, adding the numbers in the inner band up to the given sign, and then interpolating the final addition.

Worked example: Find the daylength when the longitude of the Sun is 30° and the geographical latitude is 15° 45′. Using formula (11) we get *δ* = 11.73°. Formula (12) gives *A* = 3.36° and day length 186.8° or 1868 *vinadi*. This is also the result from the figure.

Worked example: Find the *lagna* using Figure 2 when the Sun's longitude is  $70^{\circ} = 2 \times 30^{\circ}$ + 10° and the time is 1 *nadi* = 60 *vinadi* (24 minutes) after sunrise. We first add differences  $244 + 272 = 516$  for the two signs up to 60°. The next difference is 312 and interpolating with the remaining 10 $^{\circ}$  we get an additional 312/3 = 104 and a total of  $516 + 104 = 620$ . Adding the time after sunrise, 60, we get 680. We now start subtracting the numbers in the figure as far as we can go: 680 – 244 – 272 = 164. The next difference is 312 and corresponds to a longitude difference of 30° thus interpolating we get (30 × 164)/312 = 15° 46´. The *lagna* is 60 + 15° 46′ = 75° 46′.

Figure 2 illustrates the concepts of ascensional difference, day length and oblique ascension. The figure shows the celestial sphere as seen from a place on Earth. The sky seems to rotate along an oblique axis that makes an angle of the geographical latitude with the horizontal plane. The origin of the equatorial system is  $γ$ , the vernal equinox. The Sun, S, is supposed to be just rising. In the equatorial coordinate system it has a right ascension, the angle  $\gamma$ CP and a declination, the angle PCS. The point P has the same right ascension as the Sun at S. The angle



Figure 2: Ascensional difference, day length and oblique ascension (diagram: Lars Gislén).

PCA is half the day length and the angle  $\Omega$ CP is the ascensional difference. Subtracting the ascensional difference ΩCP from the right ascension of the Sun, γCP gives the angle γCΩ, the oblique ascension of the Sun.

There is sometimes a simplified way of calculating the *lagna* by a standardised table (Table 4) for the rising times of the different zodiacal

Sign	Rising Time(nadis)	<b>Rising Time (Minutes)</b>	
Aries	5	120	
Taurus	4	96	
Gemini	3	72	
Cancer	5	120	
Leo	6	144	
Virgo		168	
Libra		168	
Scorpio	6	144	
Sagittarius	5	120	
Capricorn	3	72	
Aquarius	4	96	
<b>Pisces</b>	5	120	

Table 4: Rising times.

Mahayuga	Days	Sūrvasiddhānta	Thai
1577917800	<b>Rotations</b>	Period	Period
Mercury	17937000	87.969995	8797/100
Venus	7022388	224.69818	224.7
<b>Mars</b>	2296824	686.999875	687
Jupiter	364220	4332.32058	$12997/3 = 4332.33$
Saturn	146564	10766.0667	10766
Rahu	232226	6794.7508	6795
Ketu			679

Table 5: Planetary periods.



Figure 3: A Thai amulet showing Rahu eating the Sun (Gislén Collection).

sign given for instance in Wisandarunkorn (1997: 12). It is not true for any location but has the advantage that it can be used for any location.

Example: Find, using the standardised table, the *lagna* when the Sun's longitude is 70° (i.e. 10° into Gemini, thus  $2 \times 30^\circ + 10^\circ$  and the time is 5 *nadi* (120 minutes) after sunrise. We first add the rising times for Aries and Taurus, 5 + 4 = 9. The next rising time is 3 *nadi*, of which we require one third and by interpolating get an additional 1 *nadi* and a total of 10 *nadi*. Adding the time after sunrise, 5 *nadi* as given, we get 15 *nadi*. We now start subtracting the numbers in the table as far as we can go:  $15 - 5 - 4 - 3 = 3$ *nadi*. The next rising time is 5 *nadis* and corresponds to a longitude difference of 30° thus interpolating we get  $(30 \times 4)/5 = 24^\circ$ . The *lagna* is 90°+ 24° = 114°, Cancer 24°.

### **3 PLANETARY LONGITUDES**

For the planets the quite complicated computational scheme in *Sūryasiddhānta* is used (Billard, 1971: 76). In Faraut (1910: 214-221) the description of the scheme for Mars uses seven pages of text. The mean longitudes of the plan-

Table 6: Planetary parameters.

Planet	е		W
<b>Mercury</b>	28/360	132/360	$220^\circ$
Venus	14/360	260/360	$80^\circ$
<b>Mars</b>	70/360	234/360	$110^\circ$
Jupiter	32/360	72/360	$160^\circ$
Saturn	60/360	40/360	$240^\circ$

ets are computed using somewhat simplified values for the *Sūryasiddhānta* periods, see Table 5. The Thandeikta scheme uses 20383/3 = 6794.333 for the period for Rahu, the Moon's ascending node.

Rahu was considered to be a demon that devoured the Sun during eclipses (Figure 3). It is equivalent to the Western notion of the Dragon's Head. It has a retrograde constant motion. Ketu, while borrowed from Hindu astrology, is different from its original version. Hindu astronomy considers Rahu and Ketu to be the ascending and descending [lunar nodes,](http://www.gpedia.com/en/gpedia/Lunar_node) respectively, but Southeast Asian astrology considers Ketu to be a theoretical planet orbiting the Earth with a speed ten times that of Rahu, moving in the same retrograde direction and with only astrological significance.

The true longitudes of the five planets are computed with the complicated scheme below (Billard, 1971; Faraut, 1910). The modern mathematical formulation is given with comments. The input parameters are: the mean longitudes of the planet, *L*, and Sun, *S*. Each planet also has three fixed parameters: the eccentricity, *e*, the radius, *ρ,* of the excentre, and the longitude of the apogee, *w*. These parameters are shown in Table 6.

The true longitude of an outer planet is then computed using the scheme below:

- 1)  $\eta = S L$ , the elongation
- 2)  $c_1 = \arcsin(\rho \times \sin(\eta)/\sqrt{[(1 + \rho \times \cos(\eta))^2 + (\rho \times \sqrt{2})^2]}$ sin<sub>η</sub>)<sup>2</sup>], first correction
- 3)  $w_1 = w c_1/2$ , first corrected apogee
- 4)  $\alpha_1 = L w_1$ , corrected anomaly
- 5)  $c_2$  = arcsin( $e \times \sin \alpha_1$ ), second correction
- 6)  $w_2 = w_1 + c_2/2$ , second corrected apogee
- 7)  $\alpha_2 = L w_2$ , second corrected anomaly
- 8)  $c_3$  = arcsin( $e \times \sin \alpha_2$ ), third correction
- 9)  $L_1 = L c_3$ , corrected mean longitude
- 10)  $\eta_1 = S L_1$ , corrected elongation
- 11)  $c_4 = \arcsin(\rho \times \sin(\eta_1/\sqrt{[(1 + \rho \times \cos(\eta_1))^2 + (\rho \times \cos(\eta_1))^2$  $\times$  sin $\eta_1$ <sup>2</sup>], fourth correction
- 12)  $L_{true} = L_1 + c_4$ , the true longitude.

For the inner planets the roles of the Sun and the planet are interchanged.

The mathematical functions, in (5) and (8), and in (2) and (11) are given by Faraut (1910) in the form of *chayas* for each planet, albeit with many printing errors and lacunae. The first func-



Figure 5: Thandeikta *chaya* for Mars with transcription (after Anonymous, 1953).

1534

641

2302

1611

656

2257

1687

669

2208

1760

679

2154

1456

623

2342

tion is symmetric in the intervals [0, 90°] and [90°, 180°] such that its value for an angle in the first interval is the same as for the complementary angle in the second interval and it is only necessary to table it for the first interval, the *montol* table. The second function does not have this symmetry and is given as two separate tables, then *mangkar* and *korakat*, one for each angular interval. Figure 4 shows these tables for Mars.

1129

524

2418

 $\leftrightarrow$ 

 $\leftarrow$ 

1213

553

2413

1295

579

2398

1376

602

2374

The symmetric function is located in the top middle with values for 15°, 30°, 45°, 60°, 75°, and 90° in the direction of the arrows. The table then continues in descending order for angles 90° to 180°. The asymmetric function is located to the top left  $(0^\circ-90^\circ)$  and right  $(90^\circ-180^\circ)$  as shown by the arrows. The two bottom values,

1982 and 1983, should have been the same, showing that Faraut (1910) was ignorant of the astronomical background of these tables.

1832

686

2096

1902

691

2033

1969

692

1969

The Thandeikta scheme for calculating the true longitudes is the same as the one above but the eccentricities and radii of the excenters vary as function of the anomaly just as in the modern *Sūryasiddhānta* (Burgess, 2000) and the planetary *chayas* become different. Figure 5 shows the Thandeikta *chaya* for Mars corresponding to Figure 4 with transcription, taken from a Burmese manuscript (Anonymous, 1953). Each vertical of the twenty-four column represents 3.75°.

### **4 ECLIPSE CALCULATIONS**

Eclipse calculations require the true longitudes of the Sun and the Moon. Also, the position of the lunar node has to be calculated in order to calculate the ecliptic latitude of the Moon, which is essential to know when determining the size and duration of the eclipse. These quantities are calculated for two sequential days that cover the eclipse. By interpolation, the time is found when the Sun and the Moon coincide in longitude for a possible solar eclipse or when the longitude of the Sun and the Moon differ by 180° for a possible lunar eclipse. If the Moon is sufficiently close, less than 12° to either the ascending or the descending node there can be an eclipse. A solar eclipse requires corrections for parallax. Due to the finite and different distances of the Moon and the Sun from the Earth there will be an apparent change in the relative positions of the Sun and the Moon for an observer on the Earth (topocentric) relative to the positions observed from the centre of the Earth (geocentric) that has to be taken into account and corrected for. Once these corrections are calculated and applied, the circumstance of the eclipse, sizes of the solar, lunar and shadow disks, eclipse duration, and the times for the start and end of the eclipse can be computed (Eade and Gislén, 1998; Gislén, 2015).

# **4.1 Parallax in Longitude**

Parallax corrections are only necessary for the calculation of a solar eclipse. A lunar eclipse looks the same for all observers on the Earth where the Moon is above the horizon and the eclipse events are simultaneous for all observers and can because of this fact be used to determine geographical longitude differences between locations. It was an important tool for the French Jesuits who visited Siam in CE 1685 when they determined the longitude of Lopburi using the lunar eclipse of 11 December 1685 (Gislén, 2004; Gislén et al., 2018).

# 4.1.1 Thailand and Early Burma

Figure 6 shows the situation at the time of a geocentric conjunction between the Sun and the Moon. It is assumed that the Moon and the Sun move in the equatorial plane of the Earth and that we see the Earth from the North Pole. The Earth, like the Moon and the Sun, rotates anti-

clockwise around C in the figure. At the geocentric conjunction, the Moon, M and the Sun, S, lie on a straight line from the centre of the Earth, C. However, for the observer at O, the sight lines to the Moon and the Sun will not coincide: there is an angle between the lines OM and OS. This is the parallax that will displace the Moon and the Sun relative to each other. In general, the parallax *π*, of an object is given by *π*  $= \pi_0 \times \sin H$ , where  $\pi_0$  is the horizontal parallax, the parallax when the hour angle *H* is 90° and the celestial object is at the horizon. The horizontal parallax in the *Sūryasiddhānta* astronomical system is  $\pi_M = 53'$  for the Moon and  $\pi_S = 4'$ for the Sun, actually defined as the angle they move in 4 *nadi*. After some time, the Earth has rotated the angle *∆H* and the observer, now at O', will see that the Moon and the Sun have moved a little counter-clockwise and that the new positions of the Moon, M' and the Sun, S' will coincide as seen from O' and that there is a topocentric conjunction. The angle *α* is given by

*α* = *H* + *∆H* – *v<sub>M</sub>* × *∆H*/21600′ +  $\pi$ <sup>*M*</sup> × sin(*H* + *∆H*), (15)  $\alpha = H + \Delta H - v_S \times \Delta H/21600' + \pi_S \times \sin(H + \Delta H),$ (16)

using the new positions M' and S' for the Moon and the Sun respectively, and assuming that the radius of the Earth is small compared to the distances to the Moon and the Sun. Here *v<sup>M</sup>* and *v<sup>S</sup>* are the angular daily motions of the Moon and the Sun expressed in minutes of arc. Setting these two expressions equal we get

$$
\Delta H = 21600' \times (\pi_M - \pi_S)/(v_M - v_S) \times \sin(H + \Delta H). \tag{17}
$$

If now the mean values for the daily motions are inserted,  $v_M$  = 790',  $v_S$  = 59', we get

$$
\Delta H = 24^{\circ} \sin(H + \Delta H). \tag{18}
$$

This relation can be found also in al-Khwārizmī (Neugebauer, 1962). It is a transcendental equation for *∆H*, the observer's correction to the geocentric conjunction time, and has to be solved by iteration. First *∆H* is set to zero in the right member of the equation, and a new value for *∆H* is computed, this value is inserted in the right member and so on. This iteration converges



Figure 6: Parallax in longitude (diagram: Lars Gislén).



#### Table 7: Comparison of longitudinal parallax.

converges very rapidly. Expressing *H* in the time unit *nadi* (1 *nadi* = 6°) and converting *∆H* to the movement in minutes of arc in longitude of the Moon by multiplying by the factor 790/360, these iterations generate the values shown in Table 7 (where the minutes of arc have been rounded to the nearest integer). The parallax correction is negative for times before noon and positive after noon.

This table compares the iteration result with two of the sources for Thai eclipse calculations. As can be seen, Wisandarunkorn (1997) agrees very well with the third and fourth iteration. Faraut's (1910) table lacks the first few values and is clearly defective at the end. Making his values start at three *nadi* and deleting some of his last values gives something similar to the second iteration.

#### 4.1.2 Late Burma

The recipe to calculate the longitudinal parallax from the hour angle of the eclipsed bodies is given in a Burmese manual (U Thar-Thana, 1937). Figure 7 shows an extract from the manual text. Here is an English translation of the text in Figure 7:

*Method to compute the parallax correction.*

Convert the *nadi* and *vinadi* to *vinadi*. Set down the *vinadi* in two places. Multiply the upper one by 7 to make the numerator. Add 600 to the lower one to make the divisor. Divide the numerator and the denominator, the quotient is the parallax correction in *nadi*. The remainder, multiplied by 60 is the *vinadi* .…

The parallax in longitude is called *lambanata* (လမ္ဗနတ). Burgess (2000) gives the Sanskrit term as *lambana* (लम्बन), meaning ‗hanging down'. In mathematical language the calculation scheme can be written as:

$$
N = 7 \times V/(600 + V),
$$
 (19)

where *V* is the hour angle (time from noon) expressed in the time unit *vinadi* and *N* is the parallax time correction in *nadi*. *V* is always taken to be positive, but the resulting *nadi* corr-

# လမ္အနတနၥရတွက်နည်း။

မျှေးတူ နာရီ,8နာရီကို 8နာရီပုံပြု၍ နှစ်ထပ်ထား၊အထက် ကို သတ္တ(၅)ခုမြောက်၊ တည်ကိန်းဖြစ်၍။ အောက်ကို နဘာ, သည,ဆ(၆၀၀)ထောက်၊စားကိန်းဖြစ်၍။တည်ကိန်းကိုတည်၊ စားကိန်းနှင့်စား၊ လ§ကား မချလမ္မနာရီ။သေသကို ခ,ဆ $(\mathbf{\epsilon}\,\mathbf{\mathrm{o}})$ မြှောက်။စားမြဲစား၊လ§ကား 8ိနာရီဖြစ် ၍။ ယင်းမချလမ္အနာရိ– စသည်တွင် မဈနတ,နာရီ–စသည့်ကိုနော။အောက်က မနေရာ သည့်အတိုင်းတက်။လမ္တနတနာရီ,8နာရီဖြစ်ဤ။

Figure 7: Extract from a Burmese eclipse manual (after U Thar-Thana, 1937: 22).

ection is added to the conjunction time if the conjunction occurs after noon otherwise it is subtracted. In order to compare it with the Thai parallax above we convert the *vinadi* into *nadi* and the time correction into minutes of arc and get Table 8 to compare with Table 7.

If we compare the Thai and Burmese variants of parallax with the result from modern astronomy the Burmese formula is remarkably good (Gislén, 2015). In reality the correction varies a little with the longitude of the Sun, but the Burmese formula gives good mean values.

#### **4.2 Parallax in Latitude**

#### 4.2.1 Thailand

A rather precise theoretical expression used in Indian and early Islamic astronomy for the parallax in latitude  $\pi$ <sup>β</sup> of an object in the ecliptic is the expression (Neugebauer, 1962):

$$
\pi_{\beta} = (\pi_M - \pi_S) \sin(\delta_N - \varphi), \tag{20}
$$

Table 8: Burmese parallax correction in longitude.



where  $\delta_N$  is the declination of the nonagesimal, the highest point of the ecliptic, and *φ* the geographical latitude. Using standard trigonometrical formulae, and the relation between declination and longitude, this can be written as

$$
\pi_{\beta} = (\pi_M - \pi_S)(\text{sinc}\cos\varphi \sin\lambda_N - \cos\delta_N \sin\varphi). \tag{21}
$$

Here the longitude of the nonagesimal is  $λ<sub>N</sub> = Λ$ – 90°, where *Λ* is the longitude of the ascendant, the *lagna*, and *ε* is the obliquity of the ecliptic, in Indian astronomy assumed to be 24°. The Thai scheme splits this expression into two terms. The first of these terms,  $(\pi_M - \pi_S)$ sin $\varepsilon \cos \varphi \sin \lambda_N$ , can be simplified using an approximation. For locations in Mainland Southeast Asia the geographical latitudes are such that the cosine term is close to 1 and varies little with the geographiical latitude. The value used for the factor  $(\pi_M$ *πS*)sin*ε*cos*φ* is set at 19′, giving *φ* the value of 17.6°, presumably a kind of mean geographical latitude for the region. The function for this parallax term is given as a very crude table {9′, 16′, 19′} for arguments 30°, 60° and 90°.

The second term  $\pi' = (\pi_M - \pi_S) \cos \delta_N \sin \varphi$ still depends on the declination of the nonagesimal. As  $\delta_N$  lies in the interval [0, 24°], the value



Figure 8: Geographical parallax (diagram: Lars Gislén).

of  $cos \delta_N$  will lie in the interval [0.91, 1], and the factor  $(\pi_M - \pi_S) \cos \delta_N$  will lie in the interval [44.6′, 49′]. Without providing any explanation, Wisandarunkorn (1997) gives a value of 13′ 44″ for this second parallax term. The anonymous manuscript (Anonymous MS.) uses the value 13′ 40″. Faraut's description (1910: 176) of how to compute this parallax, 8", is cryptic: "On retranche toujours 2, de 50 **=** 48, que l'on multiplie avec 2 **=** 96, que l'on divise par 12 **=** 8*.*'' However, there is a possible explanation, which is based on a crude approximation. Assume that the Moon and the Sun move in the equatorial plane and are located in the meridian of the observer. Figure 8 then describes the situation for the Moon. Assuming that the combined parallaxes of the Moon and the Sun are small relative to the geographical latitude, the geographical parallax is given by

$$
\pi = (\pi_M - \pi_S) \sin \varphi = 49' \sin \varphi. \tag{22}
$$

Wisandarunkorn (1997) gives his tables for day length and *lagna* for a geographical latitude of

about 16° N. Inserting this in the formula gives a parallax of 13′ 30″. Faraut (1910) gives a table of day lengths for a geographical latitude for 9° 40′ N. Inserting this value in the formula gives a parallax of 8′ 14″. Both of these numbers are close to the values actually used by these sources.

# 4.2.2 Burma

The Burmese handling of the parallax in latitude is more sophisticated. The time from noon in *nadi* is multiplied by 6 in order to convert it to degrees, the hour angle. The result is then added to the longitude of the Sun. The result is a rather good approximation of the longitude of the nonagesimal *λN*, at least for locations in Burma. A table is then used to calculate the declination *δ<sup>N</sup>* of the nonagesimal, using the correct relation sin $\delta_N$  = sinεsin $\lambda_N$  with  $\epsilon = 24^\circ$ , the Indian obliquity. The parallax in latitude is then calculated using the correct expression *π<sup>β</sup>*  $=(\pi_M - \pi_S)\sin(\delta_N - \varphi)$ , also with help of a table, where  $\pi_M - \pi_S = 49'$ . The agreement with a modern calculation is very good (Gislén, 2015).

# **5 ANALYSIS OF A THAI TRADITIONAL SOLAR ECLIPSE CALCULATION**

The solar eclipse of 18 August 1868 is one of the more interesting events in Thai astronomical history and also one of the most important eclipses in the history of solar physics (see Orchiston, 2020). The duration of the totality was exceptionally long and it was the first solar eclipse where spectroscopic observations were made, leading to the discovery of helium (Nath, 2013). It was observed at several locations on the Earth, from Aden in the west to Indonesia in the east, by Austrian (Aden), English (India), German (Aden and India), French (India and Siam), and Dutch (Indonesia) astronomers (see Launay, 2012; Mumpuni et al., 2017; Orchiston et al., 2017). One of the French contingents of astronomers observed the eclipse from Wah-koa in Siam (under the black spot in Figure 9) in the presence of King Rama IV (Mongkut) of Siam (Orchiston and Orchiston, 2017). King Rama IV (Saibejra, 2006) had personally made calculations for this eclipse using Western methods, not being satisfied with the traditional way of calculating eclipses. Figure 9 shows a modern calculation of the centrality path of the eclipse. The black dot shows the size of the area on the Earth where the eclipse was total, and the red circle the area in which it was partial at the time of the totality in Wah-koa. It was total at that location about twenty minutes before noon, local time.

For eclipses the traditional Thai calculations use a special set of parameters for the Sun and the Moon. In the ordinary day-to-day calculations in South-East Asia the astronomical parameters were based on the *Sūryasiddhānta* and



Figure 9: The solar eclipse 18 August 1868 (diagram: Lars Gislén).

midnight was used as the time reference. It is evident from the source manuscripts that the South East Asian astronomers used such dayto-day calculations to spot the occurrence of an eclipse and then switched to a more accurate set of parameters to perform the calculation of the eclipse circumstances. The parameters used for eclipse calculations are shown in Table 9 (Gislén and Eade, 2001). The precision given is 1/10000000 of a minute of arc.

The solar parameters except for the epoch longitude are exactly the values used in the *Aryabhatiya* (Billard, 1971: 77). Also, the use of 6:00 hours as the time reference points to the *Aryabhatiya*. However, the lunar parameters differ slightly from Aryabhāta's values. Billard (ibid.) describes a Hindu calendar that was introduced around CE 1000 and mentioned in the anonymous *Grahacarānibandhanasamgraha* (Haridatta, 1954). In Billiard's notation it is called k.(GCNib)B. The calendar scheme makes the following amendments to Aryabhāta's lunar parameters (Billard, 1971: 143):

Moon's longitude – (*S* – 444) × 9/85′, and Moon's apogee – (*S* – 444) × 65/134′, and Node – (*S* – 444) × 13/32′,

where *S* is the Śaka year. A simple check shows that this correction generates lunar parameters that precisely match those used by the Southeast Asian calendarists.<sup>3</sup>

Also, the epoch values correspond exactly to the values generated by the change above with the exception of that for the lunar apogee: 17651′ instead of 17641′; the Thai symbols for 4 and 5 look very similar and are easily confused. Finally, the epoch chosen for the eclipse calculations is exactly the day 1550000 current since

the epoch of the Kaliyuga, 18 February 3102 BCE, certainly not a coincidence.

By a lucky coincidence, we have a Thai calculation of the 18 August 1868 solar eclipse of Wah-koa and shown in Figure 10 that was drawn from an anonymous Chiang Mai manuscript (Anonymous MS). The calculation follows exactly the traditional calculation scheme for a solar eclipse with the 63 steps given by Wisandarunkorn (1997:  $190-204$ ). The calculation in the manuscript is shown as a series of numbers accompanied by a Thai technical label that in most cases has a Sanskrit or Pali origin. A detailed transcription of the numbers in the manuscript is given here in the Appendix, in Section 12.1. Below is merely a crude layout of the calculation scheme.

The Thai date of the eclipse is written at the top of the page, 3 1/– 10 1230 Chulasakharat. The first digit 3 stands for the weekday, Tuesday. The second and third digits indicate that the day is the first of the waxing Moon in month 10, which is the month Bhadrapada in the numbering of Central Thailand. The last number is the year in the Chulasakharat Era. The first line of the calculation shows the number of years elapsed since the epoch, 725. Line 3 shows the value 265098, the number of elapsed days from the epoch. The following lines 4-15 show the







Figure 10: Traditional Thai solar eclipse calculation.

calculated mean longitudes of the Sun, the Moon, the Moon's apogee, and the node respectively for this day and the day following, the eclipse occurring within this time. Lines 16-26 calculate the true longitudes, the true elongation and the true daily motions. After having checked if an eclipse is possible, line 28, the conjunction time and longitudes are interpolated in lines 29– 35. The eclipse is assumed possible if the Moon is within 12° of the lunar node, both for solar and lunar eclipses. The lengths of day and night are calculated in lines 36–41. In lines 42– 45 the longitude parallax corrections are calculated and in 46– 49 applied to the conjunction time and conjunction longitude. Lines 50–57 cal-

culate the first approximation to the lunar latitude using the Moon's distance from the node. After having calculated the *lagna* and the nonagesimal (lines 58–63), the parallax corrections to the latitude are calculated to give the apparent latitude, 3′ 19″, line 64. Lines 65–70 calculate the sizes of the solar and lunar disks and the size of the eclipse. Finally, the last lines compute the eclipse duration and the times of the start and end of the eclipse.

The calculation of the semiduration of the eclipse uses a scheme that is special for Thai eclipse calculations. It has not been possible to determine the origin of this scheme, but it is very crude and may be quite ancient. The lunar

latitude in minutes of arc is subtracted from 31′, the sum of the mean lunar and solar disk radii, and with this argument Table 10 is entered and the duration interpolated.

For the eclipse, the computed lunar latitude is 3′ 19″. Subtracting this from the sum of the mean solar and lunar radii, 31′, we get 31′ – 3′ 19″ = 27′ 41″. Interpolation in the table gives the duration as 5 *nadi* 41 *vinadi*, the actual value given in the manuscript.

There is a similar table to use for lunar eclipses where the lunar latitude is subtracted from 54′, which is the sum of the mean radii of the disks of the Moon and the mean shadow disk radius, where the disk radius of the shadow is taken to be 2.5 times that of the radius of Moon's disk. The table used for lunar eclipses is shown as Table 11. Faraut (1910) has a similar table, but for unknown reasons it has slightly different numbers.

According to the traditional calculation above for Bangkok, the eclipse is not total. The reason is that the parallax correction in latitude is too large, the real effective parallax correction in latitude being only of the order of 2′. The geocentric lunar latitude is about –2′, which gives a topocentric latitude of zero and a total eclipse. The reason for the error is that in order to compute the *lagna* correctly it is necessary to use the tropical longitude of the Sun, i.e. including the precession, which in this case is 21° 51′, and this is not done. The neglect of the precession in the calculation can be due either to its having fallen out of the instructions or that the instructions were out-dated. Recalculation with the precession included gives indeed a total eclipse.

### **6 A BURMESE LUNAR ECLIPSE CALCULATION**

Burmese solar eclipse calculations are much more complex (Gislén, 2015). They use the modern *Suryasiddhānta* parameters. The precision of the calculation is in seconds of arc, precession is included, and the duration of the eclipse is calculated using an iterative scheme that corrects for the changing ecliptic latitude of the Moon during the eclipse.

The layout for a lunar eclipse is shown in Figure 11. The first two numbers in the top left column give the year, first one in the Burmese era, 1296 (၁၉၆), then in the Kaliyuga era 5035 (၅၀၃၅). The Gregorian date of the eclipse is 26 July 1934. The following lines show the new year *kyammat,* 85 (၈၅), the new year weekday  $(1 \ (s)$  = Sunday), and the number of elapsed days of the year, 102 (၁၀၂). Then follow the calculated mean longitudes of the Sun, the Moon, the apogee and the node. The longitudes are also given here with a precision of seconds of arc.

Table 10: Solar eclipse duration.

Argument	Duration/Nadi	
3	$\mathcal{P}$	
6	3	
12		
20	5	
31		

Then the true longitudes are calculated, top centre column, lines 2 and 3, and the true daily motion and the true motion in elongation, right column line 2. The rest of the top right column is devoted to a computation of the tropical longitude of the Sun, day length and the noon shadow (see Section 7 below).

The bottom left column calculates the conjunction times and the conjunction longitudes. Knowing the angular distance between the Moon and the node, in this case the ascending node, the lunar latitude is calculated assuming an inclination of 4.5″ of the lunar orbit relative to the ecliptic. This value of the inclination of the lunar orbit is standard in many Indian astronomical texts.

Also standard in both Thailand and Burma is the calculation of the sizes of the lunar and shadow disks. These sizes are assumed to be proportional to the daily motion, *v,* of the stellar object, i.e. the diameter, *D*, is assumed to be

$$
D = D_{mean} \times v_{true}/v_{mean}.
$$
 (23)

Given the apparent radii, *r* and *R*, of the Moon and the shadow respectively and the lunar latitude, *β*, the first approximation to the (half) duration, *d*, is calculated using the Pythagorean theorem:

$$
d = \sqrt{(R - r)^2 - \beta^2}
$$
 (24)

for the total phase and

$$
d = \sqrt{(R + r)^2 - \beta^2}
$$
 (25)

for the partial phase. If  $R - r < \beta$ , the first relation will not give a real number and the eclipse is only partial. The geometry of the eclipse is shown in Figure 12.

The latitude of the Moon is not constant during the eclipse and using the computed dura-







Figure 11: Burmese lunar eclipse calculation.

tion as a first approximation, new values of the lunar latitude are computed for the beginning and end of the eclipse and new semi-durations are calculated and taken as final. This calculation is correct mathematically, is very similar to the calculations done in the modern *Sūryasidddhānta* (Burgess, 2000), and is much more sophisticated than the Thai calculation.

This lunar eclipse was partial. The beginning of the eclipse was not visible from Yangoon as the Moon had not yet risen, and the middle of the



Figure 12: Eclipse geometry (diagram: Lars Gislén).

eclipse occurred about 10 minutes after sunset. Comparing the calculated times with a modern ephemeris the agreement is very good, the error being only about five minutes.

### **7 SHADOW CALCULATIONS**

These calculations are specific for Burma where there are instructions in astronomical manuals of how to do such calculations (Gislén and Eade, 2014). However, there are a few examples also from the Thai region:

… in the third month, on the fourth day of the waning moon … when the shadow (of the gnomon) cast by the morning sun measured fifteen feet, in the rksa of Citra, in the year seven hundred thirty-one of the Era.

… the eighth day of the day of the waxing moon of the first month, Thursday, the auspicious day and time, in the afternoon when the shadow of the gnomon marked exactly six padas.

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It is difficult to evaluate these records as there are too many unknown facts. The height of the gnomon is not known, and being a Thai record it is not known if the solar longitude and day length are corrected for precession. It seems that the shadow calculations or observations had mostly astrological significance and were of little practical use. Some records even give ‗Moon shadows'. They appear in horoscopes, but also accompany eclipse calculations (Gislén, 2015).

Primitive shadow calculations appear already in Mesopotamia (Neugebauer, 1975(1): 544; Ôhashi, 2011). More sophisticated are the Indian calculation schemes using the formula

$$
d/(2t) = (s - s_0)/G + 1, \qquad (26)
$$

where  $s_0$  is the noon shadow length, G, the gnomon height, *s* the shadow length, *t* the time from sunrise or sunset, and *d* the day length (Abraham, 1981). The same formula can also be found in the *Romakasiddhānta* in Varahamihira's *Pañcasiddhantika* (Neugebauer, 1970; Sastry, 1993). Using a half day length *D* = *d*/2 and measuring time *T* from noon  $T = D - t$ , we can rewrite this expression as it is used in the Burmese calculations:

$$
s = s_0 + G \times T/(D - T).
$$
 (27)

It is easy to see that this expression makes sense. At noon,  $T = 0$ , and  $s = s_0$ , the noon shadow. At sunrise or sunset, the denominator is zero and the shadow becomes infinite as it should.

However, in an astronomically correct calculation the factor *G* is not constant but will depend on the longitude *λ* of the Sun and the geographical latitude *φ*, as well as on the gnomon height. There is also a dependence on the time *T* from noon. In the Burmese scheme the dependence on the geographical latitude is neglected since this variation is not excessive in the Mainland Southeast Asia area (Mauk, 1971; Thi, 1936). The noon shadow depends on the declination of the Sun (in turn being a function of the solar longitude), and on a geographical latitude *φ*, which cannot be neglected there. The half day length will depend on the longitude of the Sun and the geographical latitude. We are then left with the formula

$$
s = s_0(\lambda, \varphi) + G(\lambda, T) \times T/(D(\lambda, \varphi) - T). \tag{28}
$$

The calculation of the noon shadow, *s*<sub>0</sub>, and the half day length is done by having a table for a set of towns in Southeast Asia. Table 12 below shows a table for Yangoon with a transcription. What is tabulated is the difference between the noon shadow and the equinoctial noon shadow, the *bawa* (ဘဝါ).

The number to the far right, 126, is the equinoctial noon shadow for Yangoon, using a gnomon with height  $G = 7$  subdivided into 60 subunits. The equinoctial noon shadow is *G* tan  $\varphi$ , where  $\varphi$  is the geographical latitude. Thus, tan *φ* = 126/420, giving *φ* = 16° 42′, which is close to the modern geographical latitude 16° 52′.

The column on the left in the table stands for multiples 1, 2, and 3 of 30° of the solar latitude. The next column is used for calculating the day length, and shows the excess *vinadi* to be added or subtracted to the equinox half day length of 15 *nadi* or 900 *vinadi*. In this column, due to symmetry, multiple 2 is equivalent to 4, 8, and 10, and multiple 1 is equivalent to 5, 7, and 11. The numbers in the last two columns show the *bawa*. You start going up the first column then back down, continue up the last column and then back down remembering that there is a hidden row with zeros at the top for solar longitude 0° and 180°. The *bawa* is to be added or subtracted from the equinoctial shadow, depending on the declination of the Sun.

For the multiplier *G*(*λ*,*T*) there is a special double-entry table for longitude and *nadi* from noon, as shown in the upper Table 13. The column numbers are the *nadi* from noon.

Example. Calculate the shadow three *nadi* after noon in Yangoon when the Sun's longitude is  $60^\circ = 2 \times 30^\circ$ . The excess day length from Table 13 is 69 and as the Sun is north of the equator the excess is added to the equinox half day length of 900 *vinadi* to give 969 *vinadi*. The *bawa* is 155. As the Sun is moving north in declination the noon shadow becomes shorter and we should take the difference between the *bawa* and the equinoctial shadow to get the noon shadow:  $155-126 = 29$ . From top Table 13, with arguments 60° for the Sun (Taurus) and



#### Table 12: Shadow table for Yangoon.



Table 13: Multiplier tables (after Thi, 1936).

three *nadi*, we get the multiplier 363, where 3 *nadi* is equal to 180 *vinadi*. Using formula (25) we get the shadow of a 420 unit gnomon as

$$
s = 29 + 363 \times 180/(969 - 180) = 112. \tag{29}
$$

By inverting the relation it is possible to get an expression for the multiplier

$$
G(\lambda, T) = (s - s_0) \times (D(\lambda, \varphi) - T). \tag{30}
$$

The bottom Table 13 shows the result for *G*(*λ*,*T*) from a modern calculation for geographical latitude 22°, with a result that is rather similar to the Burmese table. However, it is not known how the Burmese constructed their table.

Using equation (28) it is possible to solve for the time after noon:

$$
T = D(s - s_0)/(G + s - s_0).
$$
 (31)

This can be used to find the time, given the shad-ow length—a procedure that is found in some manuscripts. However, the quantity *G* is itself a function of time. The problem is solved by iteration, first *G* is set to 7 (or 420) and a preliminary time, *T*, is computed. A new *G* is taken from the multiplier table using this time and a second approximation time can be calculated that can be used to find a new *G*, and so on.

In the Burmese calculations the scheme is also used to calculate Moon shadows by, instead of using *D* for half the length of the night, letting *T* be time from midnight and using the longitude of the Moon. This neglects the fact that the Moon does not move on the ecliptic. Such Moon shadows are obviously purely artificial, and show that the system had become a kind of 'number magic'.

# **8 PRECESSION**

For the shadow and day length, and also for *lagna* correct calculations, it is necessary to use the tropical longitude of the Sun, i.e. to take precession into account. However, it seems that Thai calculations ignore precession, something that seems very probable also for the earlier Burmese Makaranta calculations. The Burmese Thandeikta scheme uses the Indian model for precession that assumes that the correction for

precession is a zig-zag function with an amplitude of 27° and a period of 7200 years. The zigzag function starts at zero 88 years before the epoch of the Kaliyuga epoch and decreases linearly and becomes –27° after 1800 years. It then increases linearly to +27° for 3600 years, then decreases linearly to reach zero 1800 years later. In order to compute the correction for precession the following scheme is used.

The year in the Burmese era is converted to the Kaliyuga era by adding 3739. The epoch constant of 88 years is added. The result is divided by 1800 and the remainder is saved. The quotient tells which part of the zig-zag function that is actual. For all reasonable historic eras this part is where this function is positive and rising. The remainder is multiplied by 9 and divided by 10. The reason for these two numbers is that the zig-zag function increases linearly by 27° in 1800 years. 27° is 27 × 60 = 1620′. 1800 times 9/10 is precisely equal to this number. The quotient will then be the number of minutes of arc of precession. The remainder is multiplied by 6, and gives the seconds of arc of precession. The quotient is divided by 60. The quotient will be the degrees of the correction for precession, the remainder is the minutes of arc. The extreme of the zig-zag function is 1800. Multiplied by 9 and divided by 10 and then by 60 gives 27°, the amplitude of the correction for precession. The rate of precession is 27/18 = 1.5° per century, not far from the correct modern value of about 1.4° per century.

Example: Compute the correction for precession for the year 1297 in the Burmese era.

 $1297 + 3739 + 88 = 5124.$ 5124/1800 = 2, remainder 1524 1524  $\times$  9/10 = 1471, remainder 6, 6  $\times$  6 = 36  $1471/60 = 22$ , remainder 51 The correction for precession is 22° 51′ 36″.

# **9 CONCLUDING REMARKS**

The astronomical calculations show a great influence from Indian astronomy, in particular from the original *Sūryasiddhānta*. The later Burmese Thandeikta calculations are adaptations of the

modern *Sūryasiddhānta* and show more sophistication in, for instance, the eclipse calculations. The Thai eclipse calculations use methods taken from the Indian *Aryabhāta* canon.

### **10 NOTES**

- 1. This is the fifth paper in a series reviewing the traditional calendars of Southeast Asia. The first paper (Gislén and Eade, 2019a) introduced the series; Paper #2 (Gislén and Eade, 2019b) was about Burma, Thailand, Laos and Cambodia, with emphasis on the first two nations; Paper #3 (Lân, 2019) was about Vietnam; and Paper #4 (Gislén and Eade, 2019c) about Malaysia and Indonesia.
- 2. For specialist terms used in this paper see the Glossary in Section 12.3.
- 3. For example, in the Aryabhata canon the lunar apogee makes 232226 rounds during a period of 4320000 years, or 1577917500 days (Billard, 1971: 78). This gives a mean motion of 232226/1577917500  $\times$  360  $\times$  60 = 6.6831950 minutes of arc per day. The correction per year is –65/134′. The daily correction will then be  $-65/134 \times 4320000/$ 1577917500 = –0.0013280′. Thus, the corrected mean motion is 6.6831950 – 0.0013280 = 6.6818670, the value used in Table 9.

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# **12 APPENDICES**

**12.1 The Solar Eclipse of 18 August 1868: A Detailed Recomputation**



We refer to the manuscript page shown in Figure





10. The solar and lunar equations used are given above in Table 14.

CS 1230, day 1, Tuesday, month 10 (Bhadrapada). The numbers to the left in the calculation below refer to steps in Wisandarunkon (1997: 190-200). Bold numbers are those found in the anonymous manuscript, and they all agree with numbers computed by following the steps in Wisandarunkorn. The numbers on the right give the line number in the manuscript.





Sign is  $<$ 6 thus parallax north



parallax

- $3/3 = 1$ , second quadrant  $-56 3:20:11 = 2:9:49$ Sign =  $2$ , parallax 16<sup>'</sup>, difference  $19' - 16 = 3'$  $3 \cdot 9:49 / 30 + 16 = 16:59 \approx 17$  $51 \quad 16:59 + 0:0 = 16:59$ <sup>52</sup> 13:40 Geographical latitude  $16:59 - 13:40 = 3:19$  $53 \quad 57 \cdot 31 / 59 = 29.56$ 54a  $801 \cdot 31 / 790 = 31:25$ 54b  $858 \cdot 31 / 790 = 33:40$  $56$   $(29:56 + 33:40)$   $/2 = 31:48$  $57 \quad 31:48 - 3:19 = 28:29$  $58$   $29:56 - 28:29 = 1:27$  $59 \quad 31 - 3:19 = 27:41$  $5 + (27:41-20) / (31 - 20) = 5 + 7:41 / 11 = 5:41$ 
	-
- 60  $5:41 / 2 = 2:50$
- $61.62$   $14.9 2.50 = 11.19$ ,  $14.9 + 2.50 = 16.59$ 
	- 63  $16:59 15:58 = 1:1$

In order to compute the *lagna* correctly it would be necessary to use the tropical longitude for the Sun and the Moon, i.e. including the precession which in this case is 21° 51′ = 1311′. The tropical longitude of the Sun at the conjunction will then be  $1311 + 7395 = 8706$ . Repeating the calculation using this value gives a total eclipse. It is not clear why precession has been neglected here.

### **12.2 Comparison of Thai Traditional Eclipse Timings**

In order to compare the quality of traditional eclipse calculations we include Tables 15 and 16, which list the calculated timings of the end of totality of a number of solar and lunar eclipses found in the records. All of these eclipses were visible (weather permitting) in Southeast Asia.

The Thai month names are abbreviated by their first three letters, and the references mentioned in column 2 in both tables are as follows:

- 1 Astrologers Notebook, 1808.
- 2 Astrologers Notebook, 1808.
- 3 Astrologers Notebook, 1891.
- 4 Astrologers Notebook, MS #159.
- 5 Astrologers Notebook, MS #159.
- 6 Wisandarunkorn, 1997.
- 7 Faraut, 1910.
- 8 Anonymous [ca. 1868].

The eclipses in bold font were visible in Southeast Asia. The quality as regards to the tim-

$3/3 = 1$ , second quadrant $\rightarrow 6 - 3:20:11 = 2:9:49$	Complementary angle	60, 61
Sign = 2, parallax 16', difference $19' - 16 = 3'$	Interpolation	
$3.9:49 / 30 + 16 = 16:59 \approx 17$	Latitude parallax correction	62
$16:59 + 0:0 = 16:59$	Second latitude north	63
13:40	Geographical latitude correction	
$16:59 - 13:40 = 3:19$	True latitude	64
$57 \cdot 31 / 59 = 29:56$	Solar disk diameter	65
$801 \cdot 31 / 790 = 31:25$	Elongation diameter	66
$858 \cdot 31 / 790 = 33:40$	Lunar disk diameter	67
$(29:56 + 33:40) / 2 = 31:48$	Sum of radii	68
$31:48 - 3:19 = 28:29$	Eclipse size	69
29:56 – 28:29 = <b>1:27</b>	Crescent $> 0$ , not a total eclipse	70
$31 - 3:19 = 27:41$	Duration argument	
$5 + (27:41-20) / (31 - 20) = 5 + 7:41 / 11 = 5:41$	Interpolation	
5:41	Eclipse duration	71
$5:41 / 2 = 2:50$	Semiduration	72
$14:9 - 2:50 = 11:19$ , $14:9 + 2:50 = 16:59$	Start/end of the eclipse	73, 74
$16:59 - 15:58 = 1:1$	Time from noon	75

Table 15: Comparison of solar eclipse timings.

Date		<b>CS</b>	C <sub>S</sub>	
(CE)	Ref	Date	Time	Time
16 May 1817	1, 5	1 Jye 1179	13:00	12:58
9 Nov 1817	1, 5	1 Kar 11794	06:30	05:50
14 Apr 1828	1, 3	1 Vai 1190	17:00	16:51
9 Nov 1836	1.5	30 Asv 1198	08:00	06:06
9 Oct 1847	1.3	30 Bha 1209	15:48	15:54
18 Aug 1868	2, 8	1 Bha 1230	11:40	11:49
6 Jun 1872	$\qquad \qquad -$	1 Ash2 1234	09:12	08:31
11 Nov 1901	6.7	1 Kar 1263	14:58	15:29

Table 16: Comparison of lunar eclipse timings.





timing is quite good: Figure 13 shows the correlation. The horizontal axis is the time after midnight for the Thai prediction, the vertical axis the time according to modern calculation. A perfect correlation would be that all the eclipse timings fell on a straight line.

The deviation from modern times has a standard deviation of 30 minutes.



### **12.3 Glossary**

*ahargaṇa* The number of elapsed days since the epoch.

*apogee* The location in a planet's orbit where it is farthest from the Earth.

*ascensional difference* The excess, positive or negative, over a half day length of six hours.

*avoman* Thai อวมาน, Burmese အဝမာန်. The excess of lunar days over solar days in units of 1/692 of a lunar day modulus 692. It increases by 11 units each solar day. It is used to determine when to add intercalary days in the calendar. Sometimes in Burmese astronomy the avoman is expressed in units of 1/703 of a solar day.

*chaya* Thai ฉายา, Burmese ဇယး. The original meaning of the word is 'shadow' but it is generally used as the name of a table often being a table for the correction of mean longitude to true longitude.

*eccentricity* A quantity that measures the deviation from circularity of a planetary orbit.

*ecliptic longitude* A coordinate used together with the *ecliptic latitude* and determining a position in the zodiac.

*gnomon* A vertical pole casting a shadow of the

Sun. It can be used for determining the time of the day. In Indian astronomy the gnomon often has a length of 8 units, in Burmese astronomy it is often 7 units long.

*kyammat* Burmese ကြမွတ်. A quantity that gives the excess of solar days over whole solar days.

*lagna* An Indian term for the ascending zodiacal sign.

*lipta* An Indian term for minutes of arc. Of Greek origin,  $\lambda \varepsilon \pi \tau \sigma \nu$ .

*Makaranta* Burmese boogpo. A Burmese cal-

endar similar to the Thai calendar but with a Metonic intercalation with 7 intercalary years in each 19-year period.

*mean longitude* The ecliptic longitude of a planet calculated assuming that the planet moves with constant angular velocity. This calculation is made by reference to its revolution period and to the *ahargaṇa*. Tables of correction, the *chaya* tables, are then used to convert this mean longitude to a *true longitude*.

*nadi* An Indian time measure with 60 *nadi* in a day and night. In Thai it is *nathi* and in Burmese *nayi*. It corresponds to 24 minutes of an hour.

*nonagesimal* The highest point of the ecliptic in the local sky.

*noon shadow* The length of the shadow of a vertical gnomon at noon at a particular location and at a specific date.

*oblique ascension* An astronomical quantity used in *lagna* calculations.

*parallax* A nearby object will be displaced as viewed from two different points. A simple way of experiencing parallax is to view the location of a nearby object relative to the distant horizon when viewed alternatively by the left and right eyes. As the Sun and the Moon are located at different distances from the Earth two observers on different places on the Earth will not see the Sun and the Moon in exactly the same places. In solar eclipses it is necessary to correct for parallax caused by the observer not being located at the centre of the Earth.

*Precession* Due to the gravitational influence of the Sun and the Moon on the equatorial bulge of the Earth, the rotation axis of the Earth will trace out a cone similar to that of a spinning top on a table. This will cause the vernal equinox of the ecliptic to move slowly backwards. The rate of change in tropical longitudes due to the precession of the equinoxes, about 1° in 72 years.

*rasi* An Indian term corresponding to the Western zodiacal sign.

*sutin* The number of days that have passed since the start of the given year.

*Thandeikt*a Burmese . The calendar used from about 1200 BE (1868 CE) in Burma.

*thinpraman* Thai ทินปรามาณ. The length of half a day, which depends on the time of the year.

*tithi* Originally a time unit being a lunar day or 1/30<sup>th</sup> of a synodic month, in Southeast Asia astronomy being 692/703 of a solar day. It can also refer to the lunar day number in a month and also the relative position of the Moon relative to the Sun, the possible 360° divided into 30 tithis, each one covering 12°.

*uccabala* A measure of the position of the Moon's apogee. It increases by one unit a day to a maximum of 3232.

vinadi An Indian time measure being 1/60<sup>th</sup> of a *nadi*.



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