Journal of Astronomical History and Heritage, 22(3), 545–552 (2019).

THE RATIONALE FOR *ŚRĪRGUṆAMITRĀDIVĀKYAS* AS DESCRIBED IN THE *LAGHUPRAKĀŚIKĀ*

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Abstract: *A vākya* is a Sanskrit term used for a sentence in general and in an astronomical context it is used to represent a numerical value corresponding to an astronomical parameter. There are different classes of *vākyas* used in astronomical texts, and *saṅkrānti* or *saṅkramaṇavākyas* are one such class of *vākyas*. The *śrīrguṇamitrādivākyas* are one of the sets of *saṅkramaṇavākyas*. The *śrīrguṇamitrādivākyas* are a set of 12 *vākyas* that give the instant at which the Sun's transit occurs. Each *vākya* consists of five syllables, with three time units. That is, the instant is given in terms of a *week-day*, *nāḍikas* and *vināḍikās*. In this paper, having given the description of *śrīrguṇamitrādivākyas*, we shall provide the rationale for obtaining them based on the exposition given in the commentary written in the *Laghuprakāśikā* by Sundararāja.

Keywords: *Vākya*, *Vākyakaraṇa*, *Laghuprakāśikā,* Sundrarāja, *iterative process, śrīrguṇamitrādivākyas, saṅkramaṇavākyas*

1 INTRODUCTION

Based on the complexity involved in the computations and the choice of the epoch, Indian astronomical texts are divided into *siddhānta* and *karaṇa*. There is one more class of text, known as *vākya*, that can be traced back to the time of Vararuci (Venketeswara Pai et al., 2018). The work by Vararuci--known as the *Candravākyas* or Moon sentences--is the oldest-available work of the *vākya* system. These are also known as *gīrnaśreyādivākyas* which give the true longitude of the Moon, correct to a minute, for each day in an anomalistic cycle of 248 days (Venketeswara Pai, 2019; Venketeswara Pai et al., 2016; 2018).

A fully developed *vākya* system is outlined in the famous *karaṇa* text of the thirteenth century, the *Vākyakaraṇa*. The author of this text is not known. Since this school of astronomy was more popular in Tamil-speaking areas, the author probably hailed from the same region. Manuscripts of the work are available in various manuscript libraries in South India, especially Tamil Nadu. In the edition of the *Vākyakaraṇa*, Sastri and Sarma (1962) estimate that it was composed between CE 1282 and 1316.

The *Vākyakaraṇa* (CE 1282) only presents the lists of *vākyas* and the computational procedures for obtaining the longitudes of the planets using these *vākyas*. It is indeed the *Laghuprakāśikā* of Sundararāja (CE 1500) that gives short explanations for the algorithms mentioned in the *Vākyakaraṇa*. It also gives the rationale for some of the *vākyas* and algorithms described in the *Vākyakaraṇa* text.

The *Laghuprakāśikā* is a commentary on the *Vākyakaraṇa* composed by Sundararāja (CE 1500), who hailed from *Kāñcī* near Chennai. The work is based on the *Mahābhāskarīya* and *Laghubhāskarīya* of Bhāskara I who belonged to the Āryabhaṭa School, and the *Parahita* system of Haridatta (Venketeswara Pai et al., 2018; 2019).

In fact, while obtaining the *vākyas* pertaining to the Sun's transits (known as the *saṅkramaṇavākyas*) for any desired day, the text *Vākyakaraṇa* only mentions about the *śrīrguṇamitrādivākyas*. These *vākyas* give the time of the Sun's transit for the year which begins on Friday. The *Vākyakaraṇa* neither lists the *vākyas* nor explains the rationale behind them. It is the commentary, the *Laghuprakāśikā*, which lists the *vākyas* and gives the rationale for generating them with a very short explanation.

Using the *śrīrguṇamitrādivākyas*, one may be able to find the instants of zodiacal transits for any desired year. For this, one needs to find the week-day at the beginning of the year and this is referred to as *saṅkramaṇadhruva*. This *saṅkramaṇadhruva* can be found by knowledge of the *ahargaṇa* at the beginning of the year. The *Vākyakaraṇa* provides the algorithm to find the *ahargaṇa* in the beginning of the chapter, and the commentary *Laghuprakāśikā* gives the rationale for the algorithm. Hence, in the next section, we first provide the algorithm for finding the *ahargaṇa* at the beginning of any desired year and the rationale for the same as described in the *Vākyakaraṇa* and *Laghuprakāśikā* respectively.

2 OBTAINING THE NUMBER OF CIVIL DAYS ELAPSED SINCE THE BEGINNING OF THE *KALI*

धूसीकालयुतः शाकः   कल्यब्दइतिकीर्तितः ।
मातुलगुणः वर्षवांशेनसंयुतः।
पुनरब्दान्मानगुणात्सालप्रियविवर्जितात् ॥ २ ॥

तत्समाप्तैर्दिनैर्युक्तं शुक्रवारादिकंदिनम् ।
स्फुटार्कचक्रावधिम् ... ... ।

dhūsīkālayutaḥ śākaḥkalyabdaitikīrtitaḥ
kalyabdo mātulaguṇaḥ
varṣavāṃśenasaṃyutaḥ |
punarabdānmānaguṇāt sālapriyavivarjitāt ||
2 ||
tatsamāptairdinairyuktaṃśukravārādhikaṃdi nam|
sphuṭārkacakrāvadhikam ... ... |

The *śaka* [year] added to 3179(*dhūsīkāla*) is known as *kalyabda* or *kali* year. The product of *kalyabda* and 365 (*mātula*) is added to one-fourth of the years (*varṣavāṃśa*). Again, 1237 (*sālapriya*) has to be subtracted from the product of the years and 5 (*māna*), and divided by 576 (*tatsama*). [The result] thus obtained is added [to the previous result] in order to obtain the number of [civil] days since the beginning of the *kaliyuga*, which is being a Friday, to the true solar year.

The verse gives the expression for computing the *ahargaṇa*. If y be the number of solar years that have elapsed, then the *ahargaṇa, A*, at the end of the year can be written as

$A = 365y + y/4 + (5y – 1237)/576$, or

$$A = 365y + y/4 + 5y/576 – 1237/576. \quad (1)$$

The rationale for the above algorithm can be understood as follows. The parameters we require to derive equation (1) are the revolution number of the Sun and the number of civil days in a *mahāyuga*. As the *Vākyakaraṇa* is based on the *Laghubhāskarīya* (LB),[1] we consider the parameters as given in the LB. The values given in LB are 4320000 and 1577917500 respectively. Here, the HCF of 4320000 and 1577917500 is 7500. Dividing 4320000 and 1577917500 by 7500 (which is the HCF of the two), we get the values 576 and 210389 respectively. These are known as the *dṛḍhahāra* and the *dṛḍhaguṇakāra* respectively. That is,

the *dṛḍhahāra* = 4320000/7500 = 576, and

the *dṛḍhaguṇakāra* = 1577917500/7500 = 210389.

Dividing the *dṛḍhaguṇakāra* (210389) by the *dṛḍhahāra* (576), the quotient is 365 and the remainder is 149. That is,

$(210389/576)_{int} = 365$

$(210389/576)_{rem} = 149$,

where $(210389/576)_{int}$ and $(210389/576)_{rem}$ represent the integral part and the remainder respectively. Multiplying the elapsed number of years 'y' by the number of civil days in a *caturyuga* and dividing the result by the revolutions of the Sun, we get

$(1577917500 \times y)/4320000 = 365y + 149y/576$

$(1577917500 \times y)/4320000 = 365y + [(144 + 5)y]/576$

$$(1577917500 \times y)/4320000 = 365y + y/4 + 5y/576. \quad (2)$$

Equation (2) gives the mean *ahargaṇa*.

In order to find the true *ahargaṇa* from the mean, one has to subtract the number of civil days corresponding to the difference between the mean and true longitudes of the Sun—when the true Sun is at *mīnānta* (at the end of the solar year)—from equation (2). The number of civil days corresponding to the difference between the mean and true longitudes of the Sun given in the commentary is '*māyākāmidinendra*' (2-8-51-15, or 2.14756944 days). This can be under-stood as follows.

The true longitude of the Sun at the end of the year (*mīnānta*) is 360°. The difference between the mean and true longitudes of the Sun is given by

$$\delta\theta = \sin^{-1}[3/80\sin(\theta_0 – \theta_A)], \quad (3)$$

where the θ_0 and θ_A are the mean longitude and the longitude of the apogee.[2]

Here θ_0 is an unknown quantity, hence as a first approximation consider $\theta_0 = \theta$. That is, for the Sun at *mīnānta* θ = 360°. Therefore, applying $\theta_0 = \theta$ = 360° and θ_A = 78° in equation (3), we get

$\delta\theta = –2.10211$.

Now, the approximate mean longitude is obtained by applying $\delta\theta$ reversely to the true longitude. That is, the obtained result which is the difference in the true and mean longitudes is to be added to the true longitude. The result obtained will be the new mean longitude, or the *madhyama*:

$$\theta_0 = \theta + \delta\theta = 360 – 2.10211 = 357.89789. \quad (4)$$

Now, in a second approximation, applying θ_0 = 357.89789 in equation (3), we get $\delta\theta = –2.11709$ and this value is again added to the *sphuṭa* or the true longitude to obtain the next iterated value of the mean longitude. That is,

$\theta_0 = \theta + \delta\theta = 360 – 2.11709 = 357.88291$.

This process is repeated (iterated) until $\delta\theta$ attains a constant value. In the case above, after 7 iterations $\delta\theta$ reaches a constant value and it is 2.11719.

Therefore, the magnitude of the difference between the mean and true longitudes ($\delta\theta$) of Sun at *mīnānta* is 2.11719. The number of civil days corresponds to $\delta\theta$ = 2.11719 and is

$2.11719 \times 1577917500/1555200000 = 2.14812$

where 1577917500 and 1555200000 are the number of civil days and number of solar days in a *mahāyuga*. It should be noted that the computed value 2.14812 is close to the textual value 2-8-51-15 or 2.14756944, which is given as a *vākya* '*māyākāmidinendra*' in the commentary of the *Laghuprakāśikā*.

Therefore, after the correction, equation (2) becomes

$(1577917500 \times y)/4320000 - 2.14756944$

$= 365y + y/4 + 5y/576 - 2.14756944$

$= 365y + y/4 + 5y/576 - (2.14756944 \times 576)/576$

$$\approx 365y + y/4 + 5y/576 - 1237/576, \qquad (5)$$

hence the result.

2.1 An Illustrative Example

Find the *kalyahargaṇa* corresponding to the beginning of *Śaka* 1890 elapsed.

The number of *śaka* years elapsed at the beginning of the *Śaka* 1890 elapsed = 1890. The number of *kali* years elapsed at the beginning of *Śaka* 1890 elapsed = 1890 + 3179 = 5069.

The *ahargaṇa*, *A* at the beginning of the year is given by

$A = 365 \times 5069 + 5069/5 + (5 \times 5069 - 12375)/576$

$$= 1851494.10417. \qquad (6)$$

Hence, the *ahargaṇa* at the beginning of the *śaka* year 1890 elapsed, or the *kali* year 5069 elapsed, is 1851494.10417 days or 1851494 days, 6 *nāḏikās,* 15 *vināḏikās*, which can be represented as 1851494-6-15. Now, if we divide 1851494 by 7, the remainder is 1. Hence, the *Meṣasaṅkramaṇa* occurs at 6 *nāḏikās,* 15 *vināḏikās* on a Saturday for the *kali* year 5069 elapsed.

3 OBTAINING THE INSTANT OF THE SUN'S TRANSIT IN A DESIRED SOLAR YEAR

... ... सेनाभक्तावशेषितम् ॥ ३ ॥
श्रीर्गुणादिध्रुवं विद्याद्भान्तंवाक्यैस्तु दृश्यते ।

... ...senābhaktāvaśeṣitam || 3 ||
śrīrguṇādidhruvaṃ vidyād bhāntaṃ vākyaistu dṛśyate |

The remainder [obtained] by dividing [the end of the true solar year] by 7 (*sena*) would be the *dhruva* [which is to be added to] each of the [mnemonics (*vākyas*)] starting with *śrīrguṇamitra*. The ending moments (time instants and week-days) are shown (given/ represented) by these mnemonics.

The second half of verse 3 and the first half of verse 4 give the algorithm for obtaining the instant along with the week-day at which the Sun transits each *rāśi* or zodiacal sign. The basis for the algorithm is a set of *vākyas* which are known as *śrīrguṇamitrādivākyas*. These are 12 in number with *śrīrguṇamitra* as the first *vākya*. These are categorised into *saṅkrānti-vākyas*. Each mnemonic gives the week-day, *nāḍikā* and *vināḍikā* at which the *saṅkramaṇa* occurs. The commentary *Laghuprakāśikā* by Sundararāja gives all the 12 *vākyas* starting with *śrīrguṇamitra* (Sarma and Sastri, 1962: 12).

श्रीर्गुणमित्रा-भूर्विधिपक्षा-स्त्रीरतिशूरा-भोगवराते ।
भावचरोरिः-तेनवशत्वं-लोकजभीतिः-स्थूलहयोऽयम् ॥
अङ्गधिगारः-स्तम्भितनाभिः-नित्यशशीशो-यागमयोऽयम् ।
तावुरुपूर्वं सङ्क्रमवाक्यं तत्क्रमयोज्यं पादवशेन ॥

śrīrguṇamitrā-bhūrvidhipakṣā-strīratiśūrā-bhogavarāte |
bhāvacaroriḥ-tenavaśatvaṃ-lokajabhītiḥ-sthūlahayo'yam ||
aṅgadhigāraḥ-stambhitanābhiḥ-nityaśaśīśo-yāgamayo'yam |
tāvurupūrvaṃ saṅkramavākyaṃ tatkramayojyaṃ pādavaśena||

2-55-32 (*śrīrguṇamitrā*), 6-19-44 (*bhūrvidhi-pakṣā*), 2-56-22 (*strīratiśūrā*), 6-24-34 (*bhogavarāte*), 2-26-44 (*bhāvacaroriḥ*), 4-54-6 (*tenavaśatvaṃ)*, 6-48-13 (*lokajabhītiḥ*), 1-18-37 (*sthūlahayo'yam*), 2-39-30 (*aṅgadhi-gāraḥ*), 4-6-46 (*stambhitanābhiḥ*), 5-55-10 (*nityaśaśīśo*) and 1-15-31 (*yāgamayo'yam*) [are the *śrīrguṇamitrādi-saṅkrāntivākyas*]. The *saṅkramavākya* corresponding to each month when added to the *varṣādi-saṅkrama* [corresponding to any desired year] (*dhruva* at the *Meṣasaṅkramaṇa* of the year) would give the [*saṅkramaṇas*] starting from *Vṛṣabha* etc. [corresponding to the desired year.]

The *Śrīrguṇamitrādi-saṅkrāntivākya*s are a set of 12 *vākyas* that give the time-intervals between the entry of the Sun into a particular *rāśi* (zodiacal sign) and the entry into the *Meṣa rāśi* (Aries). A suitable multiple of 7 has to be added to obtain the actual time interval. Each *vākya* consists of five syllables with three time units. That is, the time interval is given in terms of the week-day, *nāḏikās* and *vināḏikās*. For example, the fifth *vākya* (*bhāvacaroriḥ*) represents the number 2-26-44, which means that if the *Meṣasaṅkramaṇa* is at 0-0-0 (at the mean Sunrise on a Friday), the *Kanyāsaṅkramaṇa* occurs on second day after Friday (i.e., Sunday), 26 *nāḏikās,* 44 *vināḏikās*, after the mean Sunrise. For any desired year if the *Meṣasaṅkramaṇa* is at x-y-z, where x-y-z is the *saṅkramaṇadhruva*, then this has to be added to the *vākyas* to obtain the *saṅkramaṇavākyas* for that year.

3.1 An Illustrative Example

Find *ravisaṅkramaṇas* for the *Śaka* year 1890 elapsed.

We saw that the *saṅkramaṇadhruva* corresponding to 1890 elapsed is 1-6-15. Adding this to all the *vākyas* would give the *saṅkramaṇas* corresponding to the *Śaka* year 1890 elapsed. We have tabulated them in Table 1.

3.2 The Rationale for *Śrīrguṇamitrādivākyas*

Before explaining the rationale, we quote a passage from the *Laghuprakāśikā* of Sundararāja which gives the explanation for the *vākyas*:

अत्रवासना – श्रीर्गुणादिवाक्येषु यावतिथं वाक्यं जिज्ञासितं तावतिथं वाक्यं ज्ञात्वा तत्पूर्वराश्यन्तमध्यमं च ज्ञात्वा तदन्तरालभूतमासिकसौरमध्यमभोगेन स्फुटमध्यमाहर्गणान्तरज्ञानेनोक्तत्रैराशिकसिद्धदिनादि | सप्तावतक्षणावशिष्टदिनादिकं संक्रमवाक्त्वेन विज्ञेयम् | पुनरप्यत्र बहुवक्तव्यमस्ति | ग्रन्थविस्तरभयान्नोच्यते | अतः परं गणितप्रक्रियाप्रकाशनमात्रमेव क्रियते | वासनात्वस्मदुक्तसुन्दरराजीयवाक्यकरणवासनाप्रकाशिकायां द्रष्टव्या |

atravāsanā -- śrīrguṇādivākyeṣu yāvatithaṃ vākyaṃ jijñāsitaṃ tāvatithaṃ vākyaṃ jñātvā tatpūrvarāśyantamadhyamaṃ ca jñātvā tadantarālabhūtamāsikasauramadhyamabho gena sphuṭamadhyamāhargaṇajñānenoktatrairāśi kasiddhadinādi | saptāvatakṣaṇāvaśiṣṭadinādikaṃsaṅkramav āktvena vijñeyam | punarapyatra bahuvaktavyamasti | granthavistarabhayānnocyate | ataḥ paraṃ gaṇitaprakriyāprakāśanamātrameva kriyate | vāsanātvasmaduktasundararājīyavākyakara ṇavāsanāprakāśikāyāṃ draṣṭavyā |

Table 1: *Saṅkramaṇas* corresponding to the *Śaka* year 1890 elapsed.

Name of the *Rāśi*		Instant of entry of the Sun
In *Devanāgari*	In Roman	
वृषभ	*Vṛṣabha*	4 – 01 – 47
मिथुन	*Mithuna*	7 – 25 – -59
कर्कटक	*Karkaṭaka*	4 – 02 – 37
सिंह	*Siṃha*	7 – 30 – 49
कन्या	*Kanyā*	3 – 32 – 59
तुला	*Tulā*	6 – 00 – 21
वृश्चिक	*Vṛścika*	7 – 54 – 28
धनुष्	*Dhanuṣ*	2 – 24 – 52
मकर	*Makara*	3 – 45 – 45
कुम्भ	*Kumbha*	5 – 13 – 01
मीन	*Mīna*	7 – 01 – 25
मेष	*Meṣa*	2 – 21 – 46

Here is a translation of the above passage:

Here is the explanation. Among the *śrīrguṇa-mitrādivākyas*, whichever *vākya* is desired, having found that *vākya*, and also having found the mean [longitude of the Sun] at the end of the previous *rāśi*, the *dinādi* (civil days from the *Meṣasaṅkramaṇa* to the desire *saṅkramaṇa*) is [found] by the rule of three applied by the knowledge of the *madhyama-bhoga* for the respective solar month obtained from the difference [in mean longitudes at the desired *saṅkramaṇa* from the beginning] and also by the knowledge of the difference between the true and mean *ahargaṇas*. The remainder, in terms of the day etc., obtained by dividing [the result] by 7 would be understood as the *saṅkramavākya*. Here again, so many things have to be told. It is not being told because of the fear of the content becoming very large (*granthavistarabhaya*). Therefore, only the necessary mathematical techniques have been dealt with. [More] detail has to be looked for in the "*sundararājīya-vākyakaraṇa-vāsanāprakāśikā*" which is [one of the works] composed by myself.

The above passage from the *Laghuprakāśikā* (LP) by Sundararāja gives the rationale for obtaining the *śrīrguṇamitrādisaṅkrāntivākyas* in brief. It is to be noted that the LP only gives the necessary mathematical procedure and not the detailed explanation for the same. Sundararāja claims that detailed explanation has been given in another work called the *sundararājīya-vākyakaraṇa-vāsanāprakāśikā*.

Now, a step-by-step procedure based on the commentary is given below.

First let us find the mean longitude of the Sun at the end of the *rāśi* previous to the *rāśi* whose *saṅkramaṇavākya* is desired. In other words, to obtain the *ṅkramaṇavākya* pertaining to any *rāśi* desired, first find the mean longitude of the Sun at the *saṅkramaṇa*.

If the *saṅkramaṇavākya* pertaining to i^{th} *rāśi* is desired, where i = 1, 2, ..., 12 for *Meṣa*, *Vṛṣabha*, ..., and *Mīna* respectively, then find the mean longitude of the Sun at the end of $(i – 1)^{th}$ *rāśi*. This is nothing but the mean longitude of the Sun at the beginning of the i^{th} *rāśi* itself.

For instance, the *saṅkramaṇavākya* desired is the one that pertains to the *Mithuna-saṅkramaṇa* (i = 3), then the mean longitude of the Sun at the end of the *Vṛṣabharāśi* [which is the previous *rāśi* of the *Mithuna*] (in this case, i = 3 and $i – 1 = 2$) has to be found. This is same as the mean longitude of the Sun at the beginning of the *Vṛṣabharāśi*. Let this mean longitude be denoted by θ_{0i}.

Now, let us find the *madhyamabhoga*, which is obtained by subtracting the mean longitude of the Sun at the beginning of the *Meṣarāśi* (θ_{01}) from the mean longitude of the Sun at the beginning of the desired *rāśi* (θ_{0i}). That is, the *madhyamabhoga* (θ_{mi}) is expressed as

$$\theta_{mi} = \theta_{0i} – \theta_{01}.$$

Using the rule of three, the number of civil days corresponding to the *madhyamabhoga* has to be found. This is nothing but the number of civil days between these two (1^{st} and i^{th}) *saṅkramaṇas*. The three parameters involved in the rule of three are θ_{mi}, number of solar days in a *mahāyuga* (1555200000, as per the *Bhāskarīya*) and the number of civil days in a *mahāyuga* (1577917500, as per the *Bhāskarīya*). Therefore, the number of civil days corresponding to the *Madhyamabhoga*, d_i, is given by

$$d_i = \theta_{mi} \times 1577917500/1555200000$$
$$= \theta_{0i} – \theta_{01} \times 1577917500/1555200000.$$

Therefore,

$$d_i = (\theta_{0i} \times 1577917500/1555200000) – (\theta_{01} \times 1577917500/1555200000). \quad (7)$$

In equation (7), θ_{01} is the mean longitude of the Sun when its true longitude is 0° or 360°. The numerical value corresponding to θ_{01} has already been obtained in Section 2 (see equation (4) for θ_0). Hence,

θ_{01} = 357.88291, or

= 2.11709. (8)

Hence, (equation 7) reduces to

$d_i = \theta_{mi} \times 1577917500/1555200000$

$= \theta_{01} \times 1577917500/1555200000 - (-2.11709) \times 1577917500/1555200000$

$= \theta_{01} \times 1577917500/1555200000 + (2.11709) \times 1577917500/1555200000$

Therefore,

$d_i = \theta_{01} \times 1577917500/1555200000 + 2.14812$. (9)

Now, in equation (9), 2.14812 corresponds to the difference between the mean and true *ahargaṇas*. Hence, the commentary in the *Laghuprakāśikā* says:

> ...
> ...**स्फुटमध्यमाहर्गणान्तरज्ञानेनोक्तत्रैराशिकसिद्धदिनादि**
>
> *... ... sphuṭamadhyamāhargaṇāntarajñānenoktatrairāśikasiddhadinādi.*
>
> Divide the *ahargaṇa* (d_i) corresponding to the *madhyamabhoga* by 7. The remainder will be the *saṅkramaṇavākya* (s_i) corresponding to i^{th} *rāśi*.

Therefore,

$s_i = [d_i/7]_{rem}$,

where $[d_i/7]_{rem}$ denotes the remainder.

Here, in equation (9), the *saṅkramaṇavākya* of any desired (i^{th}) *rāśi* can be obtained by the knowledge θ_{0I}. Therefore, it is necessary to know the procedure to obtain θ_{0I}. The *Laghuprakāśikā* gives a very short explanation for finding the mean longitude of the Sun from the true longitude at *saṅkramanas*. Now, what follows is the algorithm for finding the mean longitude from the true based on the description given in LP.

3.3 Obtaining the Mean Longitudes of the Sun at the Zodiacal Transits

While explaining the procedure to find the true *ahargaṇa* from the mean, Sundararāja gives the procedure to find the mean longitude of the Sun from the true, when the Sun is at the end of *Mīnarāśi*. However, one can use the same idea for finding the mean longitudes from the true longitudes at all *saṅkramaṇas*. Below is the passage given from the *Laghuprakāśikā* (Sarma and Sastri, 1962: 8).

> यदा **वर्षान्ते भास्करो मीनान्त एव वर्तेत**, **तदा** भास्करभुक्तराशिद्वादशकस्यैव स्फुटत्वेनाङ्गीकारात्स्फुटकर्मविपरीतप्रक्रियया **मध्यमावगतिः** | **तस्मादपि मध्यमात्भास्करोक्तप्रकारगणितस्फुटप्रक्रियया प्रातिलोम्येन गणनां कृत्वा** अविशेषयित्वा शुद्धमध्यमस्वरूपावगतिः |
>
> *Yadā varṣānte bhāskaro mīnanta eva varteta, tadā bhaskarabhuktarāśidvādaśakasyaiva sphuṭatvenāṅgīkārātsphuṭakarmaviparītaprakriyayā madhyamāvagatiḥ | tasmādapi madhyamātbhāskaroktaprakāragaṇitasphuṭa prakriyayā prātilomyena gaṇanāṃ kṛtvā aviśeṣayitvā śuddhamadhyamasvarūpāvagatiḥ |*
>
> At the end of the year when the Sun is situated at the end of *Mīna*, then it is agreed that the true longitude is nothing but the *bhukti* of the Sun through twelve *rāśis* and the mean longitude [corresponding to that] can be understood (obtained) by the reverse process. From this mean [longitude obtained here], based on the computational procedure as given by Bhāskara [I] for obtaining the true [longitude], the pure (*śuddha*) mean longitude can be obtained by the reverse procedure, having computed the [quantities] by iteration [till the constant value is obtained].

Although the above procedure is explained in the context of obtaining the mean longitude when the true longitude is 0° or 360°, the same procedure can be used for finding mean longitudes pertaining to other *saṅkramaṇas*.

Let the true longitude of the Sun at the end of i^{th} *rāśi* is $i \times 360°$ where $i = 1, 2, \dots, 12$ for *Meṣa, Vṛṣabha, ...*, and *Mīna* respectively. In other words, $(i-1) \times 30°$ is the true longitude of the Sun at its transit to the i^{th} *rāśi*. Then, the difference between the mean and true longitudes of the Sun as described by Bhāskara in the *Mahābhāskarīya* is given by

$\delta\theta_i = \sin^{-1}[3/80 \sin(\theta_{0i} - \theta_A)]$, (10)

where θ_{0i} and θ_A are the mean longitude and the longitude of the apogee of the Sun. Here, as θ_{0i} is an unknown quantity; hence, as a first approximation, consider $\theta_{0i} = \theta_i$. For instance, let us consider the Sun's transit into the *Mithunarāśi* (i = 3). The true longitude at the transit is

$(i - 1) \times 30 = 2 \times 30 = 60°$.

That is, for the Sun at the beginning of *Mithuna*

$\theta_{i.} = \theta_{3.} = 60°$.

Therefore, applying $\theta_{03} = \theta_3 = 60°$ and $\theta = 78°$ in equation (10), we get

$\delta\theta = -0.66397$.

Now, the approximate mean longitude is obtained by applying $\delta\theta$ reversely to the true longitude. That is, the obtained result which is the difference in true and mean longitudes is to be added to the true longitude. The result obtained will be the new mean longitude, or the *madhyama*.

$\theta_{03} = \theta_3 + \delta\theta = 60 + (-0.66397)$

= 60 – 0.66397) = 59.33603. (11)

Now, in a second approximation, applying θ_{03} = 59.33603 in equation 10, we get $\delta\theta$ = –0.68760 and this value is again added to the *sphuṭa* or the true longitude to obtain the next iterated value of the mean longitude. That is,

$\theta_{03} = \theta_3 + \delta\theta = 60 + (-0.68760)$

$= 60 - 0.68760 = 5931240.$

This process is repeated (iterated) $\delta\theta$ attains a constant value. In this case, after 10 iterations $\delta\theta$ reaches to a constant value and it is –0.68847°. Therefore, $\delta\theta$ corresponding to the Sun at the beginning of the *Mithuna* is –0.68847°. Therefore, the [accurate] mean longitude of the Sun at the beginning of the *Mithuna* is given by

$\theta_{03} = \theta_3 + \delta\theta = 60 + (-0.68847)$

$= 60 - 0.68847 = 59.31153°.$

In the same manner, we have computed the mean longitude of the Sun at each transit (*saṅkramaṇa*). The values obtained are tabulated in Table 2.

Table 2: Computed values of the mean longitudes from the true longitudes of the Sun at the end of each *rāśi*.

Name of the *Rāśi*		True Sun at *Saṅkramaṇa* (θ_i)	Mean Sun at *Saṅkramaṇa* (θ_{0i})
In *Devanāgari*	In Roman		
वृषभ	*Vṛṣabha*	30	28.36263
मिथुन	*Mithuna*	60	59.31153
कर्कटक	*Karkaṭaka*	90	90.46372
सिंह	*Siṃha*	120	121.47857
कन्या	*Kanyā*	150	152.06649
तुला	*Tulā*	180	182.08446
वृश्चिक	*Vṛścika*	210	211.55725
धनुष्	*Dhanuṣ*	240	240.64106
मकर	*Makara*	270	269.56910
कुम्भ	*Kumbha*	300	298.60157
मीन	*Mīna*	330	327.98081
मेष	*Meṣa*	360	357.88281

Now, substituting the θ_{0i}'s in the algorithm for finding *saṅkramaṇavākyas* given in Section 3.2, one can obtain the *saṅkramaṇavākyas* pertaining to all 12 Suns transits. We shall illustrate this with an example in the next section.

3.4 An Illustrative Example

Let us consider *Mithunasaṅkramaṇa* as an example. For *Mithuna*, the *i = 3*. Hence, to find the *saṅkramaṇavākya* for the *Mithuna*, one needs to find the mean longitude of the Sun when it is at the end of the previous *rāśi* (or, at the beginning of the *Mithunarāśi*; θ_{03}).

The mean longitude (θ_{03}) can be obtained by iterative procedure as mentioned in the algorithm. We have also tabulated the same, for all *saṅkramaṇas*, in Table 2. From the table, it is clear that

θ_{03} = 59.31153.

We can find the civil days, d_3 corresponding to the *madhyamabhoga*, θ_{m3}, by substituting the value of θ_{03} in equation (9). That is,

$d_3 = \theta_{03} \times 1577917500/1555200000 + 2.14812$

$= 59.31153 \times 1577917500/1555200000 + 2.14812$

$= 62.32604$ (12)

If we now divide d_3 by 7, the remainder will give the *saṅkramaṇavākya* (s_3), corresponding to the *Mithunasaṅkramaṇa*. Therefore, s_3 is given by

$s_3 = [d_3/7]_{rem}$

$= [62.32604/7]_{rem}$

$= 6.32604.$ (13)

Hence, the computed *Mithuna-saṅkramaṇa-vākya* is 6 days, 19 *nāḍikas* and 34 *vinādikas* (6|19|34) which is very much close to the *vākya, bhūrvidhipakṣā* (6|19|44). The difference of 10 *vināḍikas* is due to the computational factor. We have used the modern sine values for computing the mean longitudes; whereas, the sine value used by Sundararāja would definitely be an approximate one as compared to the modern value. We have computed all the *saṅkramaṇavākyas* and list them in Table 3.

4 NOTES

1. The second half of the first verse clearly states that *"bhāskarīyānusāreṇagaṇitaṃkriyatelaghu"*, which means the mathematical techniques devised based on the parameters given in the *Laghubhāskarīya*.
2. While explaining the rationale for the correction which is applied to the mean *ahargaṇa* to obtain the true one, the *Laghuprakāśikā* gives the method briefly for computing the mean longitude from the true longitude, which is given below.

   यदा वर्षान्ते भास्करो मीनान्त एव वर्तेत, तदा भास्करभुक्तराशिद्वादशकस्यैव स्फुटत्वेनाङ्गीकारात्स्फुटकर्मविपरीतप्रक्रियया मध्यमावगतिः । तस्मादपि मध्यमात्भास्करोक्तप्रकारगणितस्फुटप्रक्रियया प्रातिलोम्येन गणनां कृत्वा अविशेषयित्वा शुद्धमध्यमस्वरूपावगतिः ।

   Yadā varṣānte bhāskaro mīnanta eva varteta, tadā bhaskarabhuktarāśidvādaśakasyaiva sphuṭatvenāṅgīkārāt sphuṭakarmaviparītaprakriyayā madhyamāvagatiḥ | ... ... prātilomyena gaṇanāṃ kṛtvā aviśeṣayitvā śuddhamadhyamasvarūpāvagatiḥ |

   At the end of the year when the Sun is situated at the end of *Mīna*, then it is agreed that the true longitude is nothing but the *bhukti* of the Sun through twelve *rāśis* and the mean longitude [corresponding to that] can be understood (obtained) by the reverse

Table 3: Comparison between the tabulated and the computed values of the time-intervals between the entry into different zodiacal signs and the entry into the Aries sign.

Name of the *Rāśi*		Instant of Entry of the Sun			
In *Devanāgari*	In Roman	Given *Vākyas*			Computed Values
		In *Kaṭapayādi*		In Numerals	
वृषभ	*Vṛṣabha*	श्रीगुर्णिमत्रा/	*śrīrguṇamitrā*	2 – 55 – 32	2 – 55 – 30
मिथुन	*Mithuna*	भूर्विधिपक्षा	*bhūrvidhipakṣā*	6 – 19 – 44	6 – 19 – 34
कर्टक	*Karkaṭaka*	स्त्रीरतिशूरा	*Strīratiśūrā*	2 – 56 – 22	2 – 55 – 59
सिंह	*Siṃha*	भोगवराते/	*Bhogavarāte*	6 – 24 – 34	6 – 24 – -04
कन्या	*Kanyā*	भावचरोरिः	*bhāvacaroriḥ*	2 – 26 – 44	2 – 26 – 09
तुला	*Tulā*	तेनवशत्त्वम्	*Tenavaśatvam*	4 – 54 – 06	4 – 53 – 33
वृश्चिक	*Vṛścika*	लोकजभीतिः	*lokajabhītiḥ*	6 – 48 – 13	6 – 47 – 44
धनुष्	*Dhanuṣ*	स्थूलहयोऽयम्/	*sthūlahayo'yam*	1 – 18 – 37	1 – 18 – 16
मकर	*Makara*	अङ्गिधगारः/	*aṅgadhigāraḥ*	2 – 39 – 30	2 – 39 – 18
कुम्भ	*Kumbha*	स्तम्भितनाभिः	*stambhitanābhiḥ*	4 – 06 – 46	4 – 06 – 41
मीन	*Mīna*	नित्यशशीशो	*Nityaśaśīśo*	5 – 55 – 10	5 – 55 – 12
मेष	*Meṣa*	यागमयोऽयम्/	*yāgamayo'yam*	1 – 15 – 31	1 – 15 – 31

process. ... ... The pure (*śuddha*) mean longitude [from the true] can be obtained by the reverse procedure, having computed the [quantities] by iteration [till the constant value is obtained].

5 ACKNOWLEDGEMENT

The author would like to acknowledge the Indian National Science Academy (INSA) for financial support through the a project grant. The author would also like to acknowledge anonymous referees for their valuable comments and suggestions. Finally, the author thanks Professor Wayne Orchiston for his continuing support and encouragement.

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