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GEODETIC ARCS, PENDULUMS AND THE SHAPE OF THE EARTH

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Abstract: By measuring the length of an arc of meridian near Paris and the latitudes of its extremities, Jean Picard obtained for the first time in 1671 a good value for the dimensions of the Earth. Many such measurements of arcs of meridian were then made during almost three centuries in order to refine Picard's one and to determine the shape of the Earth and the value of its flattening, after they had demonstrated that it is approximately an oblate ellipsoid. Measurements of parallel arcs contributed in a lesser way to these results. Both kinds of arcs also made the frame of accurate geographic maps. Gravity was measured in many places of the world from the length of a pendulum oscillating in one second (the seconds pendulum), and contributed strongly to the determination of the shape of the Earth. After 1960, space geodesy and the GPS have made obsolete all this previous work, which is now only of historical interest. This paper summarizes the history of this past activity, which has involved a considerable number of astronomers and geodesists, civilian or military, often working in difficult conditions, sometimes within important international collaborations.

Keywords: geodesy, triangulation, pendulum, arc of meridian, arc of parallel, figure of the Earth, geoid.

1 INTRODUCTION: GEODESY AND ASTRONOMY

The reader might be surprised to find in the *JAHH* a paper on geodesy. He/she should realize, however, that geodesy was done mainly by astronomers until the middle of the nineteenth century, with some important exceptions (Great Britain and India, where it was done by militaries). While measurements of lengths or angles in the field have little to do with astronomy, astronomical observations are required to determine latitudes, longitudes and azimuths, and one of the purposes of geodesy was until recently to determine the dimensions and shape of the Earth, clearly an astronomical problem.

Before the advent of space geodesy and the GPS, the main technique of geodesy was triangulation, invented by the Dutchman Gemma Frisius (1508–1555), perhaps together with his pupil Mercator (1512–1294). It was used to produce the first correct map of the Netherlands. Tycho Brahe (1546–1601) obtained by triangulation the distance between the coast of Denmark and the island of Hven, where his observatory was located. The French astronomer Jean Picard (1620–1682) described in his famous book *Mesure de la Terre* (Picard, 1671) the practical bases of the method that was to be used for three centuries without important changes. The principle of triangulation is actually very simple, although its implementation can be complicated.

Box 1: The Principle of Triangulation (see Figure 1)

First, the length of a base AB is measured with rulers, chains or invar steel wires; in general this length is of the order of 10 kilometers. Then, considering a target C, for example a steeple, one measures the angles *a* and *b*, which requires that reciprocal sighting is possible between A and C, and B and C. The triangle ABC is now completely defined, and the length of its side BC can be calculated. BC can now be used as a base for a new triangle BCD, and so on. Some triangles are redundant, allowing checks. The measurement errors accumulate when there are many triangles, but fortunately the angles can be measured very accurately. One has to take into account the altitude difference between the vertices of the triangles, and work on their projection. Of course, signals must be erected for sightings if no target like a steeple is available (Figure 2).

Astronomical observations are required to determine the azimuth of the base or of any side of a triangle, setting in this way the orientation of the whole rigid set of triangles. For this, the direction of the north is determined by observations of fundamental stars, most often the Pole Star in the northern hemisphere. Other astronomical observations, generally of stars passing near the zenith in order to minimize the effect of atmospheric refraction, are required to determine latitudes. A correction for deviations of the vertical by the attraction of nearby mountains might be necessary.

A lively popular description of these operations is given by Airy (1856).

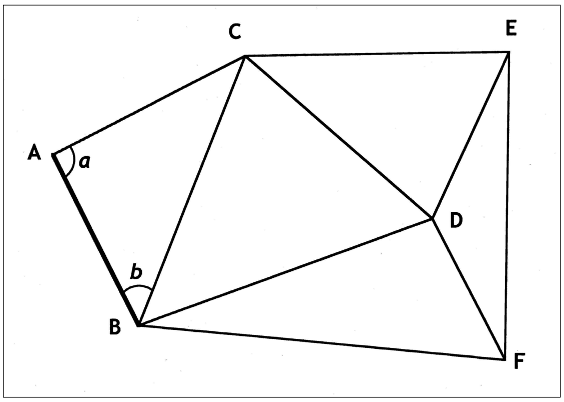


Figure 1: Principle of triangulation (diagram: James Lequeux).

Soon, it was found of interest to measure the length of arcs of meridians. This simplifies the construction of geographical maps, while the comparison of the length of the arc with the difference of latitude of its extremities allows the calculation of the length of a degree and, assuming that the Earth is an ellipsoid of revolution, the local radius of the Earth. Comparing the radii or the lengths of a degree of latitude obtained at different latitudes gives the flattening of the Earth (Figure 3) and the length of the meridian, hence the dimensions of the ellipsoid.

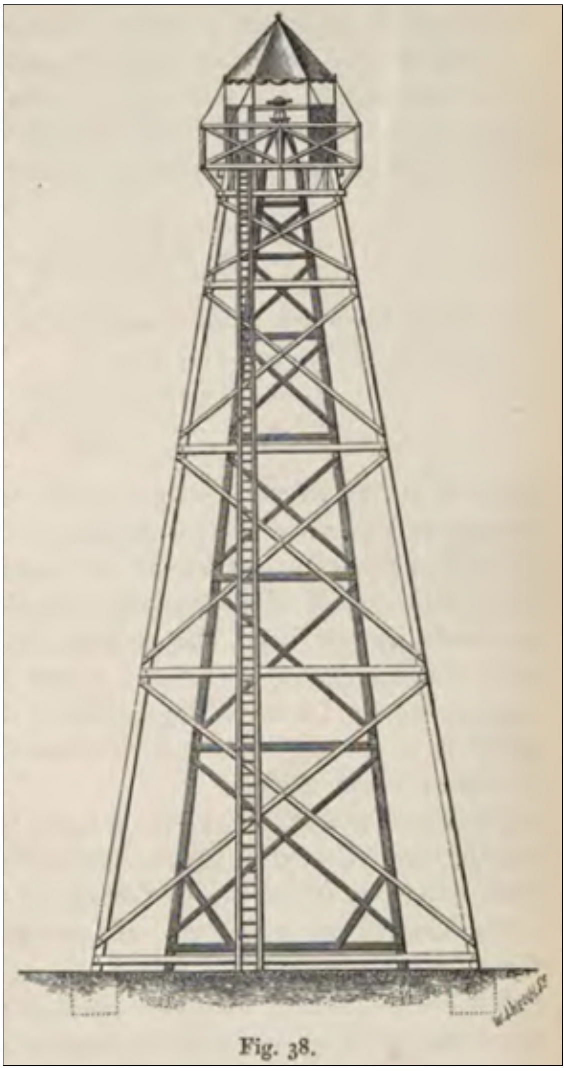


Figure 2: A 21-m high scaffold used as a geodetic point. Note the inner independent scaffold supporting the instrument in order to avoid disturbances by the motions of the observer (after Clarke (1880: 181), Paris Observatory Library).

2 THE FIRST MEASUREMENTS OF MERIDIAN ARCS

After the famous measurement by Erathosthenes (276–194 BC) of an arc in Egypt, which gave for the first time a correct value for the size of the Earth, there have been a few measurements of the length of a degree of meridian, in China, in Persia and in France (Batten and Smith, 2006). In the present paper, we will only deal with the more accurate measurements obtained by triangulation. Willebrord

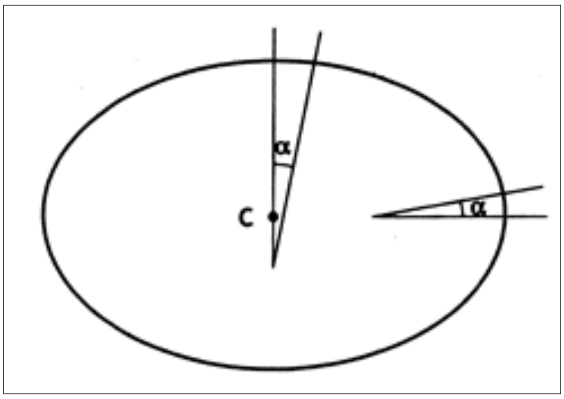


Figure 3: The length of the arc corresponding to the same difference of latitude, hence the local radius of the arc, depends on latitude because the Earth is not spherical (its flattening is very exaggerated here). The difference of latitude α is the angle between the verticals at both ends of the arc, which generally do not cross at the center of the Earth. For a flattened Earth, the local radius is larger near the poles and the arc longer for a given value of α (diagram: James Lequeux).

Snellius (ca. 1580–1626), the one who discovered the law of refraction, was apparently the first to use this technique for a measurement of the length of a portion of meridian, between Alkmaar and Bergen-op-Zoom in the Netherlands. Giovanni Battista Riccioli (1598–1671) made a similar measurement in Italy around 1645. Soon after the Academy of Sciences was founded in 1666, it put Picard in charge of measuring the length of a degree of the meridian of Paris, near the city. The basis of his triangulation, about 11 kilometers, was measured in joining end-to-end wooden rulers of two toises (about four meters). The triangulation covered 153.5 kilometers. The latitude was obtained at both ends by measuring the height of stars above the horizon either with a vertical quadrant or with a new instrument, the zenith sector (Picard, 1671: Plate 3); all the angles of the triangulation were measured with a horizontal quadrant (Picard, 1671: Plate 1). It was then possible to calculate the length corresponding to a difference of latitude of one degree. Picard obtained 57,060 toises, corresponding to 111.3 kilometers. Then, multiplying this number by 360, he could determine the circumference of the Earth, which he assumed to be spherical. This was the first time that the Earth was measured accurately: this played an important role for Newton in his work on the law of universal attraction.

Sometime later came the question of the shape of the Earth, an important question for map making. Was it flattened like a pumpkin or elongated like a rugby ball? Newton predicted that the Earth should be flattened due to the centrifugal force created by its rotation when it was still fluid. The Academy of Sciences decided to extend Picard's measurements from Dunkirk to Perpignan, at the extreme northern and southern points of France. This was achieved in 1718 by Jacques Cassini (Cassini II (1677–1756); see Cassini, 1720). Comparing the northern and the southern parts of the meridian, he decided that the Earth was elongated. A dispute between his followers and those of Newton arose, so that the Academy decided to send two geodesic expeditions, one to Lapland (in northern Sweden), as close as possible to the pole and the other one to Peru (in what is now Ecuador) near the terrestrial equator. The Lapland expedition was directed by Pierre-Louis Moreau de Maupertuis (1698–1759) in 1736–1737 (Outhier, 1744). The Peru expedition of Louis Godin (1704–1760), Pierre Bouguer (1698–1758) and Charles-Marie de La Condamine (1701–1774) started in 1735, but encountered so many problems (including earthquakes) that it was finished only in 1743. As a consequence, it was almost forgotten: on 13 November 1737, Maupertuis announced alone in front of a large audience that the Earth was flattened, and the famous writer Voltaire (1694–1778) declared that he "… had flattened the Earth and the Cassini's".

The results of the Peru expedition were not reported to the Academy until 14 November 1744 (Bouguer, 1744), and they confirmed the flattening of the Earth (Bouguer, 1749). It was shown later that the Peru results were very reliable, while those in Lapland suffered from excessive haste and were far from perfect. For one of the many popular accounts of these expeditions, see Ferreiro (2011).

To be sure, after the report by Maupertuis, the Academy decided to measure again the meridian in France. This was done by Cesar-François Cassini de Thury (Cassini III, 1714–1784), the son of Cassini II, and Nicolas–Louis de la Caille (1713–1762); they found that the initial measurement was slightly erroneous and that the Earth was indeed flattened (Cassini de Thury, 1740). Other measurements of parallels and meridians were also made, providing the frame for the construction of the first detailed and accurate map of France. This map, the *Carte de Cassini*, was realized under the supervision of Cassini III and completed by his son Jean-Dominique Cassini (Cassini IV, 1748–1845). The measurements took 34 years, from 1750 to 1784, requiring about 20 men in the field and an equal number to engrave the maps (Pelletier, 2013). The result was a major progress in cartography. However, there were not yet elevations in the map.

Figure 4: Jean-Baptiste Joseph Delambre (Wikimedia Commons, Clic-gauche).

3 THE FRENCH-SPANISH MERIDIAN ARC (12° 22′, MEASURED FROM 1792 TO 1807)

The end of the eighteenth century saw many more triangulations, especially in France and England, including several geodesic linkages between the two countries (see later). In 1791, the new Assembly (*Assemblée Constituante*), elected during the French Revolution, decided to create a universal decimal system of units, which would replace the incredible diversity of measures used in France and elsewhere. In a search for universality, the new unit of length, the meter, was to be linked to a fixed quantity common to the entire world: the length of a meridian of the Earth, which was supposed to be an ellipsoid of revolution. There would have been 10 million meters by definition in a quarter of this meridian. It was decided to measure again a part of the Paris meridian, because the previous measurements were considered insufficient. Jean-Baptiste Joseph Delambre (1749–1822, Figure 4) and Pierre-André Méchain (1744–1804, Figure 5)

Figure 5: Pierre-André Méchain (Wikimedia Commons, Hurle).

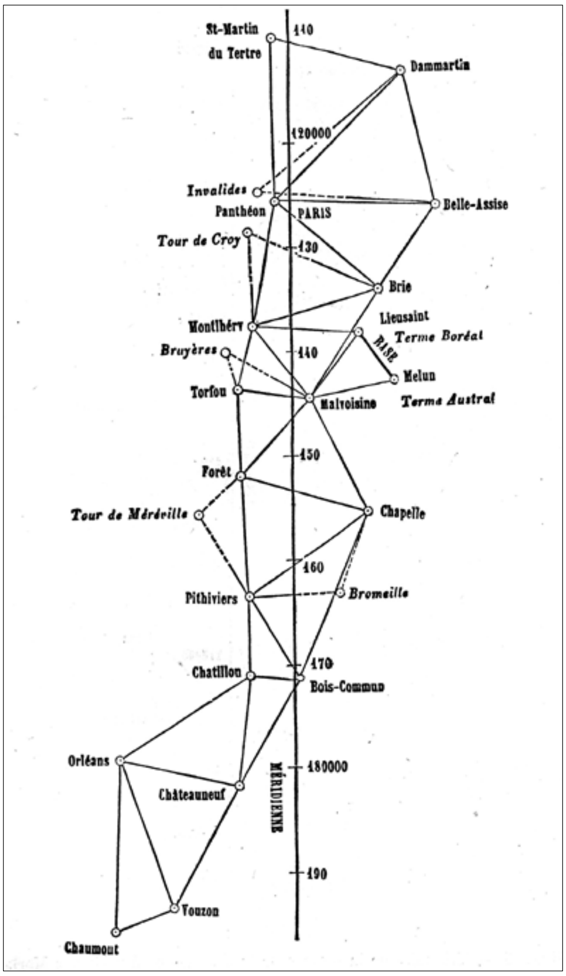


Figure 6: A part of the triangulation of Delambre and Méchain, near Paris. The 11-km base was from Melun to Lieusaint, on a straight road (after Arago (1854 –1857), Lequeux Collection).

were in charge of a new triangulation between Dunkirk and Barcelona (Figure 6). In the meantime, provisional meters were constructed from the determination by Cassini III and La Caille, and distributed throughout France so that people would become familiar with them.

To prepare for the triangulation, Charles de Borda (1733 – 1799) asked Étienne Lenoir (1744 – 1832) to build four repeating circles ac-

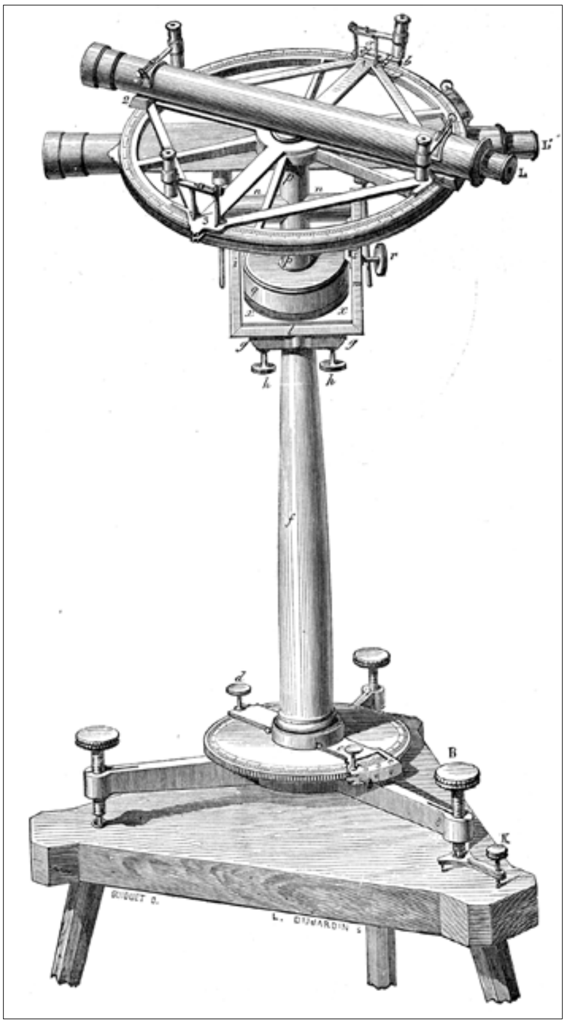


Figure 7: Borda's repeating circle. It was used in a horizontal position to measure angles of the triangulation, or in a vertical position to measure elevations of stars hence latitudes. This one, very similar to those of Lenoir, was built by Jean-Nicolas Fortin (1750 –1831) and used by Biot and Arago in the extension of the French meridian arc (after Arago (1854 –1857), Lequeux Collection).

cording to his design, four rulers of two *toises* each (a bit less than four meters) for measuring the bases, and *réverbères* (reflectors) with parabolic mirrors and oil lamps for observation of geodetic points at night. The repeating circles (Figure 7), whose principle (Figures 8 and 9) was due to the German astronomer Tobias Mayer (1723–1762), were universal instruments that could measure angles to better than one arc second. The rulers (Fig-

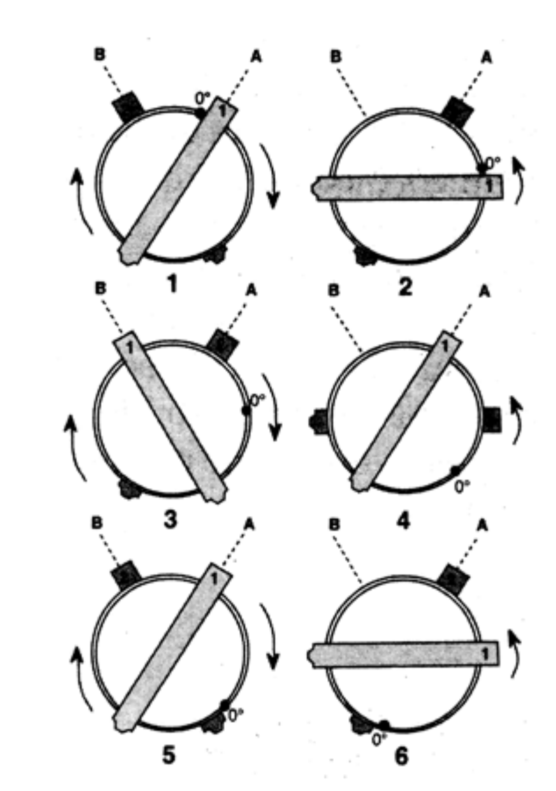


Figure 8: Running the repeating circle in a horizontal position for triangulation. (1) To measure the angle between directions A and B telescope 1 is directed to A while telescope 2, behind the circle, is directed to B; they are both clamped to the circle. Point 0° is the origin of the graduations. (2) The whole set is rotated such that telescope 2 looks to A. (3) Telescope 1 is unclamped from the circle and pointed to B. (4) Both telescopes are clamped and the whole is rotated such that telescope 1 looks to A. (5) Telescope 2 is unclamped and pointed to B. The telescopes are again clamped and the configuration is identical to (1), but the circle has turned by twice the angle between A and B. The cycle is resumed, (6) corresponding to (2), etc. After a number of cycles, the accuracy is improved by the many readings and the errors due to the eccentricity and the graduation imperfections of the circle are reduced (adapted from Arago (1854 –1857), Lequeux Collection).

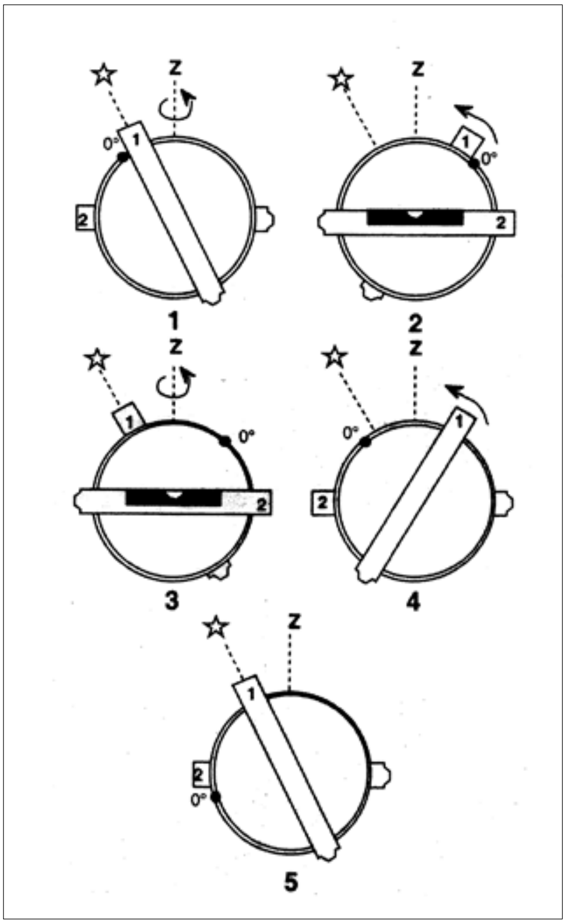


Figure 9: Running the repeating circle in a vertical position for measuring a zenith distance. (1) Telescope 2 is clamped to the circle and set in a horizontal position thanks to an attached spirit level, and telescope 1 points to the star and is clamped to the circle. (2) The circle is turned by 90°. (3) Telescope 1 is unclamped, pointed to the star and clamped again. (4) The circle is turned by 90° around the vertical axis. (5) Telescope 1 is pointed to the star; the situation is similar to (1) but the circle has been rotated by twice the zenith distance. At all steps, an assistant adjusts the horizontality of telescope 2 by acting on screws on the foot of the instrument (see Figure 6). Adapted from Arago (1854 –1857), Lequeux Collection).

ure 10) were also especially well designed.[1] The measurement of a base was as usual a major operation: that of the 11-kilometer base from Lieusaint to Melun took 36 days. Another base was measured near Perpignan near the southern border of France. Its measured length differed only by 30 centimeters from the value calculated from all the triangulations from Paris to Perpignan, a remarkable agreement. The latitude of the ends of the triangulation in Dunkirk and Barcelona, and at several intermediate places, was measured with the repeating

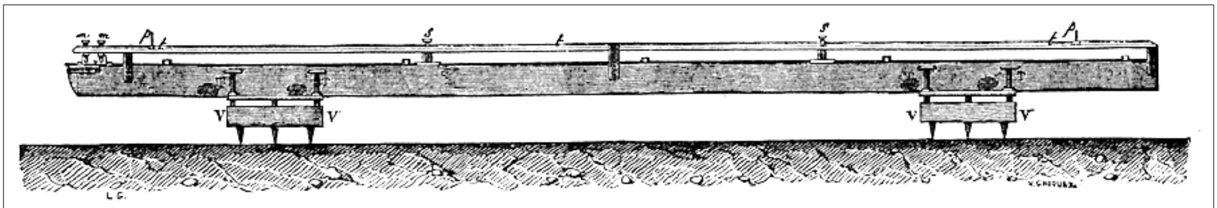


Figure 10: Borda's ruler for measuring the bases of the triangulation, lateral view. It was made of a brass ruler, superimposed on a platinum one, fixed to each other at one end in order to measure the relative expansion of the two rulers and to correct the measurements to a reference temperature. In order to preserve their ends, two successive rulers were not in direct contact. A small graduated ruler slided in the platinum ruler and touched the end of the next ruler. The rulers were set horizontally and aligned through rods *p* (after Arago (1854 –1857), Lequeux Collection).

circles in vertical position. This was criticized later because these circles suffered from a recurrent unidentified problem, and zenith sectors were generally preferred.

The new triangulation (1792–1798) was conducted with remarkable skill and rigor despite many difficulties due to the agitated period of the Revolution (Méchain and Delambre, 1806–1810); for an interesting contemporary account in English, see Playfair (1822(4): 223–260); for popular accounts, see e.g. Adler (2002) or Murdin (2009).

To calculate the total length of the Paris meridian from Pole to Equator, Delambre and Méchain had to assume a value for the flattening of the Earth: they took 1/335 from measurements of gravity at different latitudes. This is slightly too small, but the difference is rather unimportant at latitudes around 45°. Following all these operations and the painful reductions made principally by Delambre, the Commission of Weights and Measures fixed in 1799 the length of the meter as 443.296 lignes of the *toise du Pérou* (there are 864 lignes in a toise, so that the length of the toise was 1.949036 meter). The *toise du Pérou*, an iron bar built in 1735, was the one used by the astronomers who measured the length of a degree near the Equator, and was the standard for the Borda rulers.

However, it was initially foreseen to measure a longer arc of meridian: it would have been symmetrical with respect to the 45° parallel, so that the effect of the poorly known flattening of the Earth would have been minimal. When all the operations were finished, the Board of Longitudes gave attention to the problem. Méchain insisted to be in charge of the extension south of Barcelona, to the Balearic Islands. This would have offered him the opportunity to solve a problem that tormented him.[2] Thus, he returned to Barcelona in 1803, but he died from malaria on 20 September 1804 in Castellón de la Plana, without having time to do much.

A year and a half after Méchain's death, the Board of Longitudes decided to resume the measurements from Barcelona to the Balearic Islands (Lequeux, 2016: 164), and sent Jean-Baptiste Biot (1774–1862) to Spain, assisted by a very young scientist, François Arago (1786–1853). After a rapid beginning when they checked Méchain's measurements, they were stuck for almost six months at the *Desierto de las Palmas* due to poor visibility and to an incorrect orientation of the lamp and reflector at Ibiza (the measurements had to be done by night, with light signals produced by *réverbères*). They finally succeeded in linking Ibiza to the continent, a very difficult operation because of the distance (up to 170 kilometers), then continued to Formentera in April 1807. Biot came back to Paris in May; Arago was left alone with his two Spanish assistants, and nevertheless succeeded in linking Mallorca to Ibiza and Formentera.

Biot and Arago must be commended for having succeeded in completing their mission in very difficult circumstances, as there was a war between France and Spain. The precision of their results is remarkable. Pierre-Simon Laplace (1749–1827) and Marie-Charles de Damoiseau (1768–1846) estimated the probable error on the distance from Perpignan to Formentera, 466 kilometers in a straight line as only 8 meters. The extension of the meridian arc to the Balearic Islands confirmed the value given previously to the meter; using now the value of 1/305 for the flattening of the Earth derived by Laplace from the theory of the Moon,[3] the Board of Longitudes obtained a length of the meter equal to 443.2958 lignes of the *toise du Pérou*, almost identical to the 1799 value. There was no need to build new standard meters.

However, in 1836 the geodesist Louis Puissant (1769–1843) discovered a subtle error made by Delambre in calculating the distance between the parallels of Barcelona and Formentera. Correcting for this calculation and including some new measurements, the distance between the parallels of Majorca and Formentera was found too short by 70 toises. Should one again change the length of the meter? The answer was: No! Thus the 1799 unit was kept. The beautiful idea of the *Assemblée Constituante* according to which the meter would have been linked to the dimensions of the Earth was over. We might consider that the saga of Delambre, Méchain, Biot and Arago was of no use whatsoever. But they stay engraved in our collective memory.

4 THE ENGLISH MERIDIAN ARC (10° 56′, MEASURED FROM 1791 TO 1853)

Contrary to what occurred in France, the British Ordnance trigonometrical survey did not directly measure a meridian arc, but calculated its length after a full survey of Great Britain (including the Shetland islands) had been completed (Clarke, 1858). This triangulation originated in an operation undertaken in 1783 in order to establish a geodetic connection between the observatories of Greenwich and Paris, and progressively extended to the whole of the islands, first under General William Roy (1726–1790), and then Major William Mudge (1762–1820): see Widmalm (1990) and Playfair (1822:181–220).

The instruments used for this survey were quite different from the French ones. The angles were measured with large theodolites. The two first ones (Figure 11) were built by the famous constructor Jesse Ramsden (1735–1800). They were cumbersome with their horizontal circle 3 feet in diameter, but very accurate. They were not repeating instruments, but the angles could be measured with different portions of the circle, minimizing in this way the errors due to the eccentricity and the graduation imperfections. There was also a Ramsden theodolite with a 2-feet circle, a 18-inch one built by Edward Troughton (1753–1835) and his associate William Simms (1793–1860), used after 1826, and several smaller theodolites.

The first base across Hounslow Heath was measured in 1784 with deal (pine or fir) rods, and in parallel with a 100-ft steel chain built by Ramsden and supported at many places. The rods proved ineffective because of their variations with humidity and were replaced by glass tubes, but finally the chain itself was found more accurate; for this reason the 1791 survey started by re-measuring the 8.35 km base with a new 100 ft chain. Chains were used until a new base instrument was invented in 1825 by Major Thomas Colby (1784–1852, Figure 12), successor of Major Mudge, who served as Director General of Ordnance Survey from 1820 to 1846. This instrument (Figure 13), built by Simms, had some similarities with the Borda rulers, except that it gave automatically the right length thanks to a lever attached to the extremities of the parallel iron and brass rods.

The latitudes were initially measured with a zenith sector constructed by Ramsden (Figure 14), which was destroyed by the great London fire of 1835: the Ordnance office was then housed in the famous Tower of London. This cumbersome instrument was then replaced by a new one called Airy's sector, because George Biddell Airy (1801–1892), the Director of the Royal Observatory at Greenwich, had been supervising its construction. It was also cumbersome, but easier to use as the plumb-line was replaced by a bubble level.

In 1821, about one third of England and Wales had been mapped under the direction of William Mudge. It was hard work: Thomas Colby walked 586 miles (943 km) in 22 days on a reconnaissance in 1819. In 1824, Colby, who had succeeded Mudge, and most of his staff moved to Ireland, while the rest of the staff continued the measurements in England. The survey of Ireland was completed in 1846.

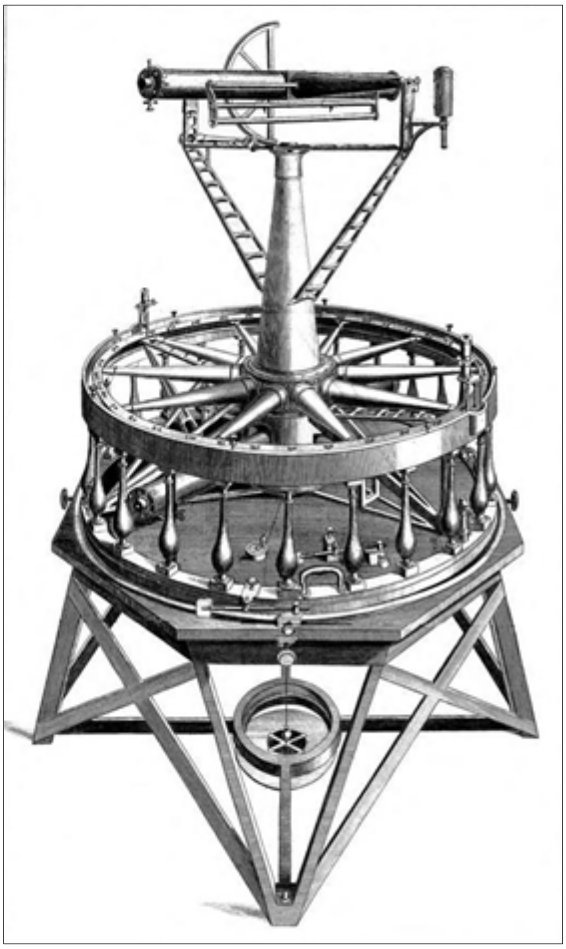


Figure 11: Ramsden's 3-feet theodolite (built in 1787); Details in Roy (1790) (Wikimedia Commons).

Figure 12: Drawing of Thomas F. Colby by William Brockedon (Wikimedia Commons).

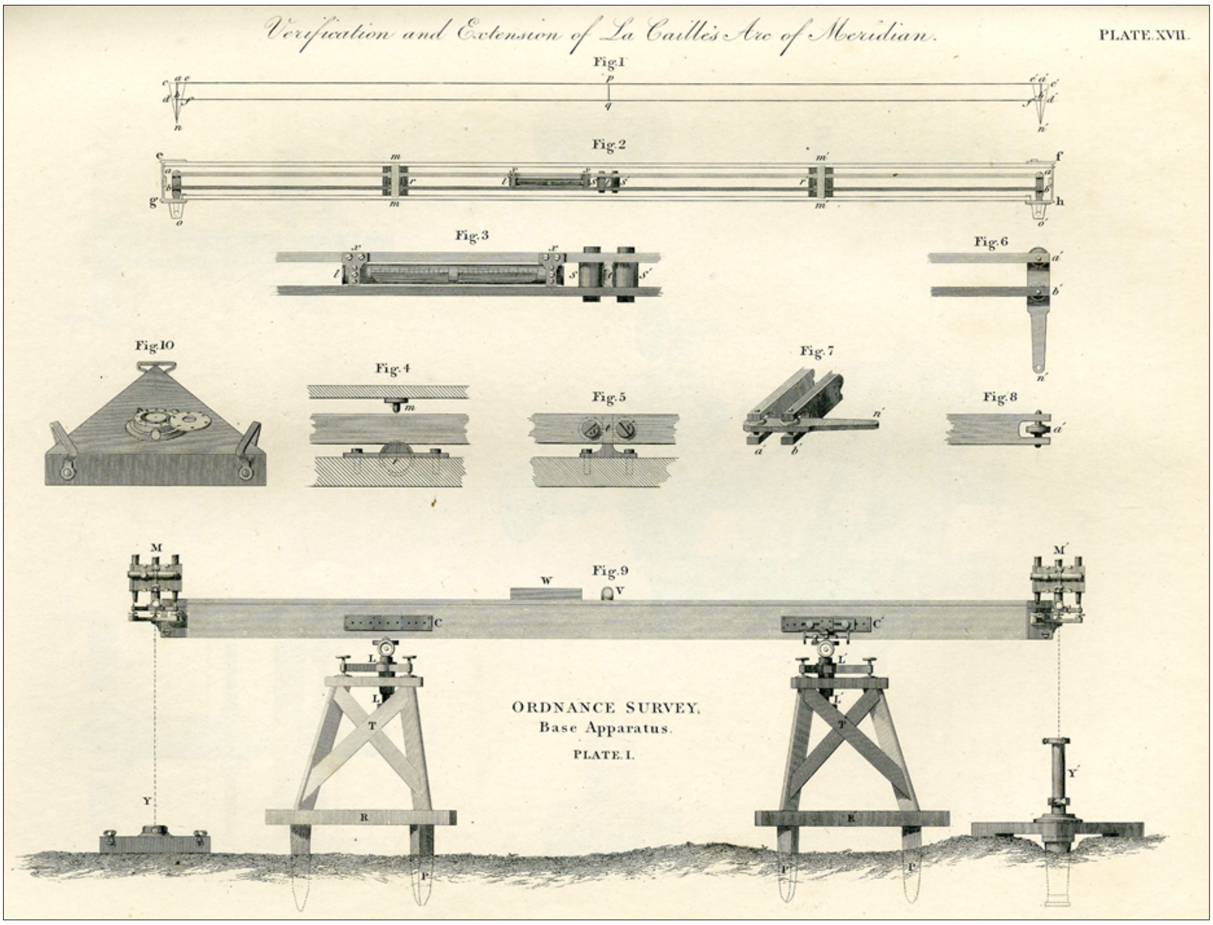
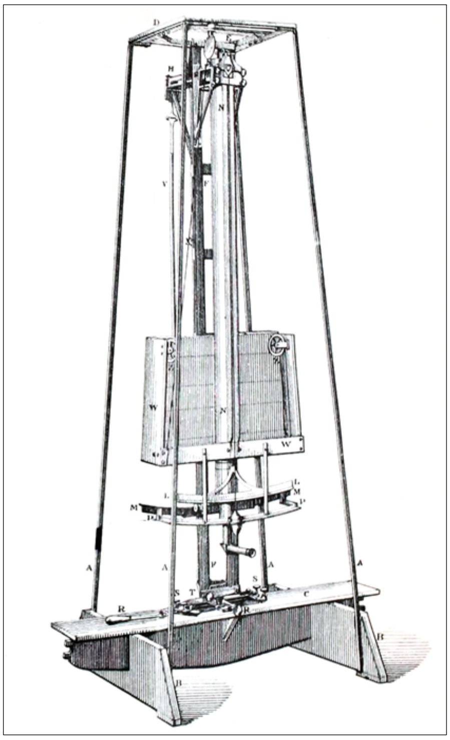


Figure 13: Colby's base apparatus (called a compensation bar). It is formed by two parallel bars about 3 meters long, one in brass and the other in iron (Figs. 1 and 2), which are secured to each other in the middle by the two cylinders S' and S'' (Fig. 2, detail Fig. 3). At each free end there is a lever, an and a'n' (Figs. 1, 6 and 7), which tilts more or less with temperature due to the different expansion of the bars, but such as the point n or n' near its end has a temperature-independent position. A gold needle is inserted there, and is aligned vertically by an optical device with a mark on the ground whose position is adjustable. This mark is the starting point for the next position of the compensation bar (after Maclear (1866: 1), Paris Observatory Library).



It caused suspicions and tensions with the local people, a difficulty already encountered in France by Delambre and Méchain. In 1840 the survey had covered all of Wales and all but the six northernmost counties of England, and was complete in 1853. It is illustrated by the map of Figure 15.

The final reductions were made by Alexander Ross Clarke (1828–1914) (Clarke, 1858; 1859). He has been especially careful to correct the numerous latitude measurements for deviations of the vertical due to nearby mountains, which can be important. These deviations had been suspected by Pierre Bouguer

Figure 14: Ramsden's zenith sector. This instrument measured the zenith distance of reference stars passing close to the zenith at their meridian crossing, in order to determine latitude accurately. It consisted of a telescope 8 feet long and 4 inches in aperture, showing distinctly stars of 3^d magnitude during daylight in clear weather. The large plate attached to the telescope contained micrometrical divisions by which a tenth of an arc second could be measured. The zenith distance was read behind the plumb-line with a microscope (after Strahan(1903), Wikimedia Commons).

Figure 15: Map of the triangulation of Great Britain (after Clarke, 1858, Volume 2). This shows the triangulation as completed in 1853 (Paris Observatory Library).

(1698–1758) who attempted to measure the deflection of a pendulum near the Chimborazo in 1737. In 1774, Nevil Maskelyne (1732–1811) had undertaken an expedition to Schehallien, a rather symmetrical mountain in Scotland, and now spelled Schiehallion (Maskelyne, 1775).

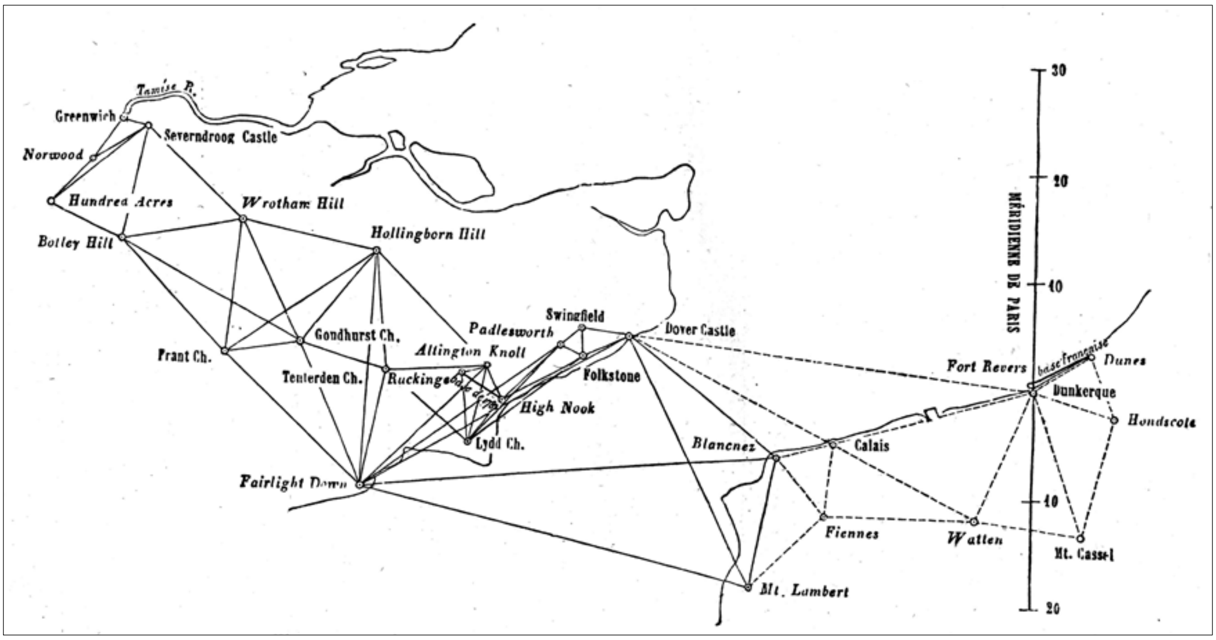


Figure 16: The 1821–1823 triangulation from France to England. Note the French base near Dunkirk (Dunkerque) and the British one between Ruckingham and High Nook (after Arago (1854 –1857), Lequeux Collection).

He measured the latitude difference between two places on either side of the mountain, by the astronomical method and by triangulation respectively. He found 11.6 arc seconds as the sum of the two deviations on each side, quite a large amount. This result was used to estimate the mean density of the mountain, after Henry Cavendish (1731–1810) had measured the constant G of gravitation. Clarke did it the other way, by estimating the deviation of the vertical from his estimate of the distance, shape and density of the nearby mountains.

From the triangulation of Great Britain, the length of an arc of meridian of 2° 50′ with a median latitude of 52° 02′ N was first derived, then after completion an arc of 10° 56′ between the parallels of the northernmost point of the Shetland islands and the southernmost point of Cornwall, corresponding to 1,217 km. It appeared that this arc could be extended to the Balearic Islands by combining it with the French arc. For this, a geodesic linkage was necessary between France and England.

5 THE NEW GEODESIC LINKAGE OF GREAT BRITAIN AND FRANCE AND THE ‘ANGLO-GALLIC’ ARC

In 1787, a first linkage between the Paris and Greenwich Observatories was performed; this marked, as said before, the beginning of the Ordnance trigonometric survey of the United Kingdom, which was initially considered as the extension of this operation. However, and in spite of the novel instruments that were used, the 1787 linkage was not a full success, because the partial results did not always agree. Relations between France and England having improved after 1815, the two countries agreed that new measurements had to be made. Already, Arago and Alexander von Humboldt (1769–1859) had visited London with Biot in 1817 to compare the lengths of their seconds pendulums with those of Captain Henry Kater (1777–1837), with whom they became acquainted. Biot and Arago had also collaborated in 1818 with Mudge to determine the latitude of Dunkirk, the common point of the French and British arcs.

In 1821, the Academy of Sciences and the French Board of Longitudes designated Arago and Claude-Louis Mathieu (1783–1875) to work on the linkage, while the Royal Society chose Kater and Colby. The measurements began on 24 September 1821 but stopped three days later because of bad weather. Then, according to Kater “… our much esteemed companion M. Arago …” returned to Paris. Operations resumed on 12 August 1822, to stop again in November, and resumed anew in July 1823, this time without Arago. Figure 16 shows a map of the triangulation. The instruments were about the same as used in 1787: on the English side, Ramsden’s original theodolite was replaced in 1823 by a similar one belonging to the Ordnance Survey, and on the French side a repeating circle was used, then a theodolite by Gambey (Figure 17). The most significant improvement was the replacement of the *réverbères* (oil lamps with parabolic reflectors) by *lamps with compound lenses* (Figure 18), which were the first lighthouse multi-part lenses invented by Augustin Fresnel

(1788–1827); they had a diameter of three feet and could be seen "… at a distance of 48 miles like a star of the first magnitude."

The detailed results of the French party were never given by Arago to the English one; perhaps, the data were never reduced. However, the observations were sufficient to determine accurately enough the latitude difference between the parallels of Dunkirk and Greenwich. Thus, in 1853, a meridian arc of 22° 10′ (between Saxavord, at the northern tip of the Shetlands, and Formentera) became available, corresponding to a total arc length of 2,465 km.

The France-England linkage by triangulation was done once more in 1861, rather badly because there were fewer and fewer competent people in France and the three repeating circles remaining at the *Dépôt de la Guerre* (War Storehouse) were in poor condition. The means on the other side of the Channel were now superior. This last linkage has been of no practical use.

6 THE INDIAN MERIDIAN ARC (22° 19′, MEASURED FROM 1805 TO 1841)

When the British finished conquering India at the end of the eighteenth century, they realized that they should have accurate geographic data and maps for military and administrative purposes, and also in order to collect taxes. There were then only disparate longitude and latitude measurements and rudimentary maps.

In November 1799, Colonel William Lambton (1753–1823, Figure 19) submitted to the authorities a plan for triangulation and cartography of India, modeled on what was being done in France. Lambton set to work in 1802. A first base was measured in April-May at Mont St. Thomas, not far from the meridian of Madras (present-day Chennai), and Lambton measured there, in one year, a first arc of meridian of 1° 35′. Then he began the measurement of another arc of the meridian, 2° 37′ west of that of Madras, which was to become a part of a much longer meridian arc. He also directed a triangulation which covered a large part of the south of India. From 1805, 10 bases were measured by him and his assistants near this great meridian arc, with an average length of 9.5 km. Finally, by 1818, Lambton had triangulated the meridian from Cape Comorin, at the southern tip of India (latitude 8° 10′), to Bidar (or Beder) north-west of Hyderabad (latitude 18° 03′), i.e. a difference in latitude of 9° 53′ 45.2″ and a length of 1,095 km (Lambton, 1818). According to Lambton, it was the largest meridian arc ever measured in one piece, but perhaps he had forgotten about the

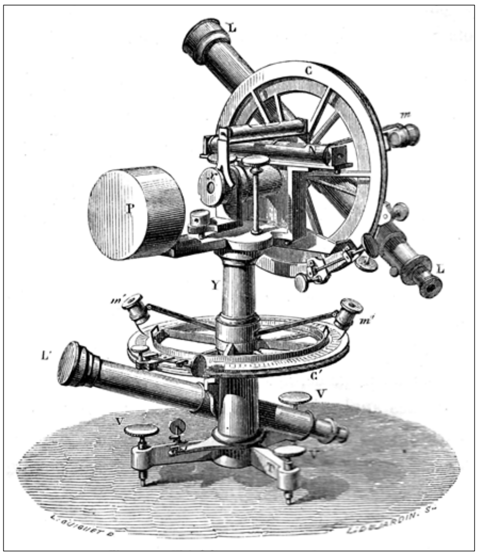


Figure 17: A theodolite with two circles and two telescopes, built by Froment, mid-nineteenth century. It is similar to the Gambey one used for the linkage between France and England in 1821–1823 (after Arago (1854–1857)).

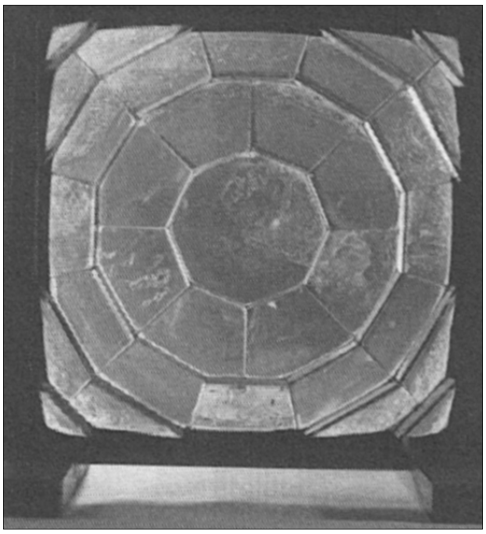


Figure 18: One of the first Fresnel lighthouse lenses, as used in the geodetic linkage France-England. It is made of curved glass prisms glued together (Paris Observatory Library).

Figure 19: William Lambton (Wikimedia Commons, Shyamal).

Dunkirk-Formentera arc (12° 22'). He had excuses though, because inconsistencies had just been discovered in Delambre's analysis.

Lambton compared his measurements with those of meridian arcs in France, England and Sweden, and deduced a flattening of the Earth of 1/310. Then he proposed to extend the Indian arc to the North. This was done by George Everest (1790 – 1866, Figure 20), who succeeded him as Superintendent of the Great Trigonometrical Survey of India (for a biography of Everest, see Smith 1999). In 1823, Everest had completed the measurements of the great meridian arc between Bidar and Sironj (24° 07'). However, he fell ill in 1824 and spent five years in England. Returning to

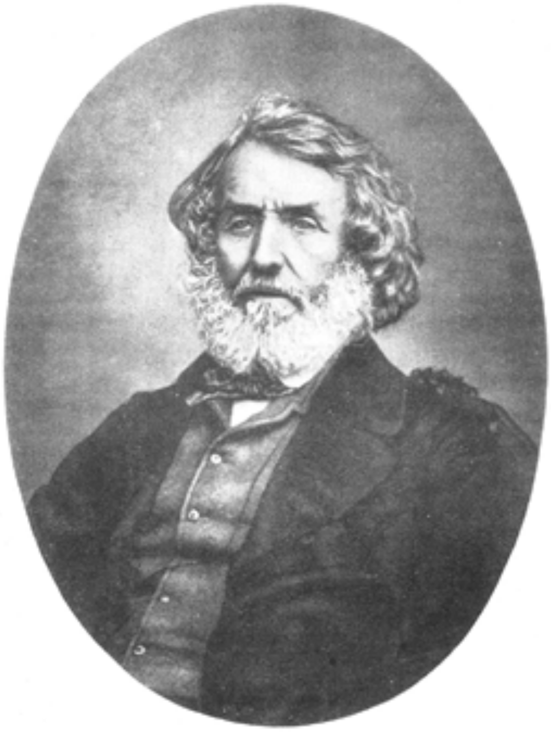


Figure 20: George Everest (Wikimedia Commons, Hephaestos).

India, he repeated the previous measurements, which he was not satisfied with, and between 1832 and December 1841 completed the measurements of the great arc, which now extended 22° 19', from Cape Comorin to Banog, near Mussoorie at the foot of the Himalayas, at a latitude of 30° 29′, a length of almost 2,500 km (Everest, 1847; Fillimore, 1945; 1950; 1954 and 1958; Markham, 1878; see, also, Roy, 1986).

By combining his measurements with the Dunkirk–Formentera arc, Everest determined a flattening of the Earth of 1/300.8, a value that was to be used for all English reductions until at least 1870; but he realized that the Earth is not really an ellipsoid of revolution. Between 1843 and 1861 his collaborator and successor, Andrew Scott Waugh (1810 – 1878), measured the altitudes of 79 Himalayan peaks, and gave to the highest the name 'Everest' ... who never saw it!

The principles of the Lambton and Everest triangulations are similar to those we have described in England, and the instruments were more or less the same. The ten Lambton bases were measured with chains of 66 or 100 feet resting on a horizontal wooden support or even sometimes put directly on the ground, and stretched by a weight. Figure 21 illustrates the beginning of a base measurement made with a chain. Everest, who measured several new bases, introduced instead of these chains Colby's *compensation bar* (Figure 12).

The instruments were, for the measurement of latitudes, a Ramsden's zenith sector (Figure 14) used from 1801 to 1825, and for triangulation and azimuths a large theodolite (Figure 22) of William Cary (or Carey, 1759 – 1825), copy of a Ramsden's 1791 theodolite. It was used from 1802 to 1866, with repairs after an accident in 1808 and a renovation in 1835. There was also a theodolite half the size, two vertical circles by Troughton, a transit telescope, chronometers, etc. Everest also used two new large 3-ft theodolites, one built by Troughton and the other by Henry Barrow (1790 – 1870) in Calcutta, a 12-inch theodolite by Troughton & Simms, and four vertical circles 18 inches in diameter. For the astronomical observation of latitudes, Everest used two double vertical circles 3 feet in diameter, also made by Troughton & Simms, which he placed in especially constructed observatories.

Lambton had worked during the day, having had great difficulty in seeing the geodetic points identified by flags or various signals, but Everest used at night more sophisticated means: *réverbères* from Troughton & Simms with Argand lamps, and above all blue powder signals lit at the top of masts every four minutes during observations. It was also necessary to construct towers in bamboo, wood and sometimes bricks to allow long-distance sightings. It is clear that Everest worked with the greatest of care: his methods marked a clear progress in geodesy

The great Indian arc served as a starting point for a general triangulation followed by a cartography which, at the time of Lambton, already covered most of southern India, and extended later to the whole colony (Figure 23). However, work on the meridian arc was continually subject to criticism and even hostility from certain administrators and even scientists, so that Lambton, on numerous occasions,

Figure 21: Measurement of the Calcutta baseline in 1832. This shows surveyors stretching a chain on coffers supported by pickets. The chain is protected from thermal expansion by a shade, and is aligned using a boning telescope. At the back is a survey tower used as a geodetic point, where the instruments were housed (after Fillimore (1958: Volume 4), Wikimedia Commons).

had to demonstrate its quality and practical interest. Everest had fewer problems because the project had made good progress when he took it over, but he was criticized for being more interested in measuring the meridian arc than anything else. Nevertheless, he allowed himself to envisage an extension of this arc to the north of Siberia, a project that has not been carried out.

7 THE STRUVE MERIDIAN ARC (25° 20′, MEASURED FROM 1816 TO 1855)

During a trip to the north of Norway, a few years ago, I came across a monument erected on a low hill located at a place named Flugenæs, in the industrial area north of the city of Hammerfest, the northernmost of Europe (Figure 24). As can be seen from the Latin inscription on this monument, it marks the north end of a 25° 20′ meridian arc. I learned then that its realization had been entrusted by Tsar Alexander I to the astronomer Friedrich Georg Wilhelm von Struve (1793–1864, Figure 25) and to the Russian General Carl Friedrich Tenner (1783–1859), both of German origin. Wilhelm von Struve was the first in four generations of astronomers that extended until the twentieth century (this reminds me of the Cassini dynasty in France): for a biography of von Struve and his son Otto, see Batten (1988). When the Tsar ordered the work in 1816, because he wished to establish precise military

Figure 22: The great Cary's theodolite. Its horizontal circle is 36 inches (91 cm) in diameter and its vertical circle 18 inches. The instrument with its carrying case weighed 500 kg. Note in the background the portrait of Lambton, reproduced here in Figure 19 (after Fillimore (1958: Volume 4), Wikimedia Commons).

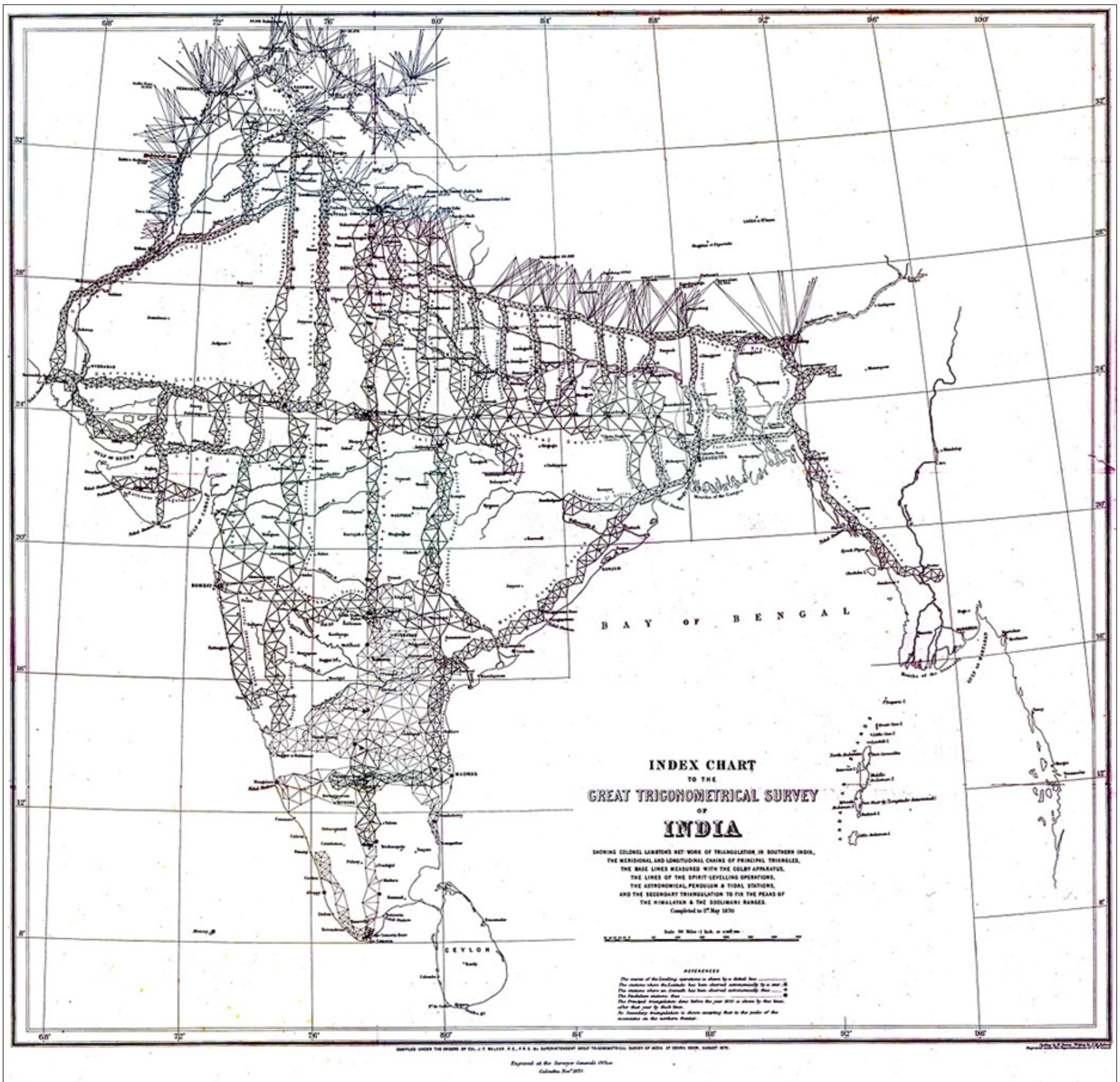


Figure 23: The triangulation of India in 1870. In addition to the great meridian arc starting from Cape Comorin, the southermost point of India, many triangulations were made with somewhat lower precision. They were the backbone of a complete 1/50,000 map of India. The lines at the top of the map show the sights that allowed determination of the positions and altitudes of the Himalayan peaks (after *Survey of India*, Wikimedia Commons, Shyamal).

maps of which the meridian arc would represent an element of the framework, Struve was a Professor at Dorpat (today Tartu, in Estonia). He directed the Dorpat Observatory from 1816 to 1839, at which date he founded the Pulkovo Observatory near Saint Petersburg and moved to that city.

While the original purpose of this arc was military, Struve and Tenner soon considered using it for a new measurement of the shape and dimensions of the Earth. The flat nature of the terrain and the absence of mountains which could cause deviations from the vertical seemed to them particularly suitable. In 1827, a first triangulation was completed, and published in 1831 in the German language. It extended between 52° and 60° in latitude, along the meridian of Dorpat (Tartu), whose Observatory (latitude 58° 25′) was one of the main stations. Then an extension towards the north was carried out with the collaboration of the Directors of the Observatories of Stockholm, Nils Selander (1804–1870) and of Oslo, Christopher Hansteen (1784–1873). Tenner was responsible for an extension south. The final arc, officially completed in 1852 but in fact in 1855 due to numerous verifications, extended from northern Norway (latitude 70° 40′) to the Black Sea (45° 20′), covering 25° 20′ or 2,822 km (Figure 26). The triangulation contained 10 bases, 275 main stations forming 258 triangles, and 13 points where geographic coordinates and azimuth were measured.

The operations are described in detail in a work of 1860 in French, published in Saint Petersburg and signed by the four people mentioned above; unfortunately it is not digitized, but see Clarke (1880: 34) for the results. The story of the measures is told in a very interesting way by Struve (1852), a work also in French. A good English summary of this book is given by Mazurova (2014).

Much physical evidence remains of this arc, which led to their protection by UNESCO, which classified it as a World Heritage site in 2005 (http://whc.unesco.org/en/list/1187/). At the time of the measurements, the arc extended only over two countries: Sweden-Norway to the north and Russia to the south. Currently, it crosses no less than 10 different nations: Norway, Sweden, Finland, Russia, Estonia, Latvia, Lithuania, Belarus, Ukraine and Moldova!

Two of the bases of triangulation were measured with l'*appareil de Delambre à languettes mobiles* (the Delambre device with movable tabs), which is none other than Borda's ruler (Figure 10). The other bases used an instrument from the Pulkovo Observatory *à levier de touche*, derived from Borda's ruler, which served in Russia until the twentieth century. These devices were indirectly calibrated on the *toise du Pérou*, which was the standard for most triangulations of the time.

A *universal instrument* (a kind of theodolite) built by Georg Friedrich von Reichenbach (1771–1826) and his collaborator and successor Traugott Lebrecht von Ertel (1777–1858), then another by Ertel alone, served for the triangulation of the Struve arc, as well as repeating circles built in Stuttgart by Baumann, others from Troughton, and various theodolites by Reichenbach, Ertel and Johann Georg Repsold (1771–1830). The latitudes and altitudes were measured with a meridian instrument of Peter Dollond (1731–1820), two identical vertical circles of Reichenbach and Ertel, and by two other vertical circles, including one by Repsold. Time was deduced from observations with various German instruments, in particular an instrument of passages by Carl Brauer. So most of the instruments came from German manufacturers, the rest being English. As a large part of the arc was covered with forests, many towers and pyramids up to 10 m in height had to be raised, as well as signals reaching sometimes 50 meters.

Struve had envisaged the extension of his arc further to the south, and his son Otto Wilhelm von Struve (1819–1905) published in 1868 an article describing a reconnaissance made under his direction with a view to an extension via Romania, Bulgaria, and Turkey un-

Figure 24: The author at the foot of the Struve arc monument in Hammerfest (Norway). Here is the translation of the Latin inscription: "Northern end of the arc of meridian measured from the Arctic Ocean to the Danube, through Norway, Sweden and Russia, by order and under the auspices of the very august King Oscar 1st and the very august Emperors Alexandre 1st and Nicolas 1st; recalls the uninterrupted work of three groups of surveyors from 1816 to 1852. Latitude 70° 40′ 11.3″." These three groups are the Russian group of Tenner and Struve, the Swedish group of Selander and the Norwegian group of Hansteen. A similar monument is located at the southern end of the arc.

Figure 25: Wilhelm von Struve (Tallin Museum, Wikimedia Commons).

til the island of Kos, with a possible extension to Crete. The junction of Struve's arc with Crete did indeed take place between the two World Wars, but via Yugoslavia and mainland

Figure 26: The Struve Meridian Arc (UNESCO). The 34 remaining geodetic points are indicated by red dots; they are all classified by UNESCO.

Greece (Smith, 2004). On his side, the Cape of Good Hope astronomer David Gill (1843–1914) had proposed in 1879 the measurement of an arc of meridian at longitude 31° 16′ East (the longitude of the Struve arc is 23° 39′ E), from Port Elizabeth in South Africa to Egypt, or 7,120 km. This measurement was effectively carried out in the next 75 years, with the exception of a portion of 1,000 km in Sudan which was only completed in 1954. It remained to link the two triangulations between Crete and Egypt, which would have required practically impossible sightings of 300 to 400 km. Various solutions, more or less fanciful, were proposed, until in 1953 the US Air Force used a radio technique called HIRAN (the high-precision version of SHORAN = SHOrt RAnge Navigation, which measured round-trip travel times of radio pulses) between Egypt, Crete and Rhodes to finally make this connection. However, it does not appear that anything has been deduced about the shape of the Earth from this immense arc.

8 THE MACLEAR SOUTHERN MERIDIAN ARC (4° 37′, MEASURED FROM 1841 TO 1848)

Although the extent of this arc is much smaller than that of the previous ones, it is worth discussing because it was the only arc measured in the Southern Hemisphere before the great Gill arc I just mentioned. In 1752 La Caille (1755) had already triangulated a meridian arc of 1° 13′ in South Africa, but his measurements of latitude were distorted by the attraction of mountains at both ends, so that he deduced that the Earth was pear-shaped! This result being suspect, several geodesists including Everest proposed the measurement be redone, which finally was accomplished by Thomas Maclear (1794–1879, Figure 27), Director of the Royal Observatory at the Cape of Good Hope. Maclear verified that the length of the arc measured by La Caille was excellent and that it was only his latitude measurements that were wrong. He took the opportunity to extend his triangulation to a total length of 4° 37′. Circumstances were much more favorable than in India or Russia as the ground was almost flat and was bare everywhere, but Maclear, less experienced than La Caille, Lambton and Everest, needed 6½ years for these measurements, whereas La Caille had taken only 2 months to make his.

To measure its base, 42,818.75 ft long, near the old La Caille base, Maclear obtained two Colby compensation bars (Figure 13). He needed 88 days to measure the base with no less than seven other astronomers and officers and 18 others. The angle measurements were made with a repeating circle by George Dollond (1774–1852), then a 20-inch repeating theodolite by John Fuller (1757–1834) derived from the Ramsden theodolite, and a Reichenbach & Ertel theodolite. Heliostats were installed at geodetic points for sightings. Latitudes were determined with an historical zenith sector built by George Graham (1673–1751) and rehabilitated for the occasion by Simms (Figure 28): this was the one with which James Bradley (1693–1762) had studied the aberration of light and nutation in 1727. The measurements (Maclear, 1866) were of good quality.

9 SOME OTHER MERIDIAN ARCS

In addition to the great arcs I have discussed above, a few other meridian arcs deserve being cited. In chronological order, we find the following arcs.

9.1 M1

As mentioned earlier, Bouguer, Godin (who did very little) and La Condamine measured a 3° arc from 1735 to 1743 in Peru (now Ecuador) (Bouguer, 1744 and 1749). The base, 12 km in length, was measured with wooden rulers two toises (3.9 m) long. The angles were measured with *quarts-de-cercle* (90° sectors). A second base was measured at the end of the triangulation and its length was found to differ by less than 2 feet from the calculated one after measurement of 32 triangles. The latitudes were determined with zenith sectors, 12 ft and 8 ft in radius respectively. As Godin had kept the former sector for himself, the latter was constructed locally, its telescope being equipped with a home-made micrometer! Bouguer knew about the aberration of light recently discovered by Bradley, but not the nutation that he discovered around 1737. Fortunately, they had measured almost simultaneously the latitude of the extremities of the arc using the same stars, so that the effects of aberration and nutation were negligible. Overall, the measurements were very reliable and were to be used for the determinations of the shape of the Earth discussed in the next section. This was not the case for the measurements of the Lapland arc, which were superseded later by the Swedish arc discussed below.

9.2 M2

Between 1750 and 1753 the Jesuits Christoforo Maire (1697–1767) and Rugerius Josephus Boscovich (1711–1787) measured a meridian arc of two degrees between Rome and Rimini (Maire and Boscovich, 1755). They used a 3-ft radius *quart-de-cercle* to measure angles and a 9-ft radius sector for latitudes, both built by Ruffo in Verona. Two bases were

Figure 27: Thomas Maclear (courtesy: Brian Warner).

Figure 28: Bradley's zenith sector, 12½ ft. in radius, used by Maclear to determine latitudes. As with many instruments from the seventeenth and eighteenth centuries, the vertical direction was given by a plumb line protected by a tube, and the graduations were read behind the plumb line (after Maclear (1866: Volume 1), Paris Observatory Library).

measured with wooden rulers. The measurements and reductions appear to have been done very carefully, including corrections for deviations of the vertical.

9.3 M3

The astronomer Charles Mason and the geodesist Jeremiah Dixon measured in 1766–1768 a 1½° meridian line at the boundary of the states of Maryland and Delaware (Maskelyne 1768; Mason & Dixon 1768): this was a part of the famous Mason-Dixon survey (see https://en.wikipedia.org/wiki/Mason%E2%80%93 Dixon_line). For latitudes, they used a 6-ft radius sector built by John Bird (1709–1776), and their bases were measured with chains and fir rods. No other meridian arc has been directly measured in the USA to my knowledge, only arcs of parallel and a long oblique arc. For a good history of geodesy in the USA, see Dracup (n.d.).

9.4 M4

Jöns Svanberg (1771–1851) and colleagues, who had doubts about the 1,736 triangulation of Maupertuis, measured 180 km (1° 37′) of meridian in Lapland in 1801–1803 (Svanberg, 1805). Strongly influenced by the French, they ordered from Lenoir a repeating circle similar to those used in the Delambre-Méchain measurements, that they preferred to the zenith sector for determining latitudes. The base, 14.5 km long, was measured using 6-m iron rulers, with of course a correction for their thermal expansion.

9.5 M5

A meridian arc of 1.5 degrees was measured in the 1810's in Denmark between Lauenburg and Lysabbel. I could not find any information on this measurement, except in Bessel (1837) who gives the results.

9.6 M6

In 1820 Carl Friedrich Gauss (1777–1855) measured a meridian arc of 2 degrees (225 km) between Göttingen and Altona in Germany (see Schumacher, 1821 for the base apparatus built by Repsold). Details can be found in Airy (1845: 212), who wrote that it was "… undoubtedly, one of the most accurate determinations ever made." For this triangulation, Gauss used as targets *heliotropes* of his invention, sorts of heliostats reflecting the light of the sun to the remote observer.

9.7 M7

A meridian arc of 1° 7' (126 km) had been measured around 1760 in Italy by Giovanni Battista Beccaria (1716–1781), between Mondovi and Andrate, near Turin. As the results were contested, a new base was measured in 1821–1822 by Giovanni Plana (1781–1864) and Francesco Carlini (1783–1862); the triangulation was done again with repeating circles of Gambey, by them and by the surveyors of the French government under Colonel Brousseaud, as a complement for their measurement of a large arc of parallel (see below). Some details can be found in Brousseaud and Nicollet (1829) and in Airy (1845: 212). The latter wrote that

> … the whole of this measure, and in particular the determination of latitudes, seem to have been made with so much care, that we must allow its excellence.

However the measurements were strongly affected by the deviations of the vertical due to the Alps.

9.8 M8

Friedrich Wilhelm Bessel (1784–1846) and Johann Jacob Baeyer (1794–1885) measured in 1834 an oblique arc with an extent in latitude of 1° 30' (168 km) around Königsberg (presently Kaliningrad), in Eastern Prussia (Bessel and Baeyer, 1838). Their rather short base of 1823 m was measured by a two-metal ruler built in Hamburg by Repsold (1770–1830), inspired by that of Colby. The angles were measured with an Ertel theodolite. The zenith distances of stars for latitude determination, and the determination of time, necessary for obtaining longitudes, were done with a transit instrument. This measurement was a clear improvement with respect to the preceding ones.

9.9 M9

In 1879, the Paris meridian was extended to Algeria by François Perrier (1833–1888, Figure 29). This was a heroic operation, because sightings of 270 km were necessary (Figure 30): France was then in bad terms with Morocco, which could not be crossed by the geodesists. Fortunately, Spain had just completed a high-quality triangulation under General Carlos Ibáñez e Ibáñez de Ibero (1825–1891), which allowed the junction with the French one. For the work, Perrier employed Johann Brunner (1804–1862) to construct a new instrument, the *repeating azimuthal circle*, so good that it was used in France until 1945. For latitudes, he used Brunner's meridian circles. Borda's rulers were replaced by new ones due to Ignazio Porro (1801–1875), also built by Brunner. For sightings, he used large

Figure 29: François Perrier as a Colonel in 1883, photograph by Eugène Pirou (Wikimedia Commons).

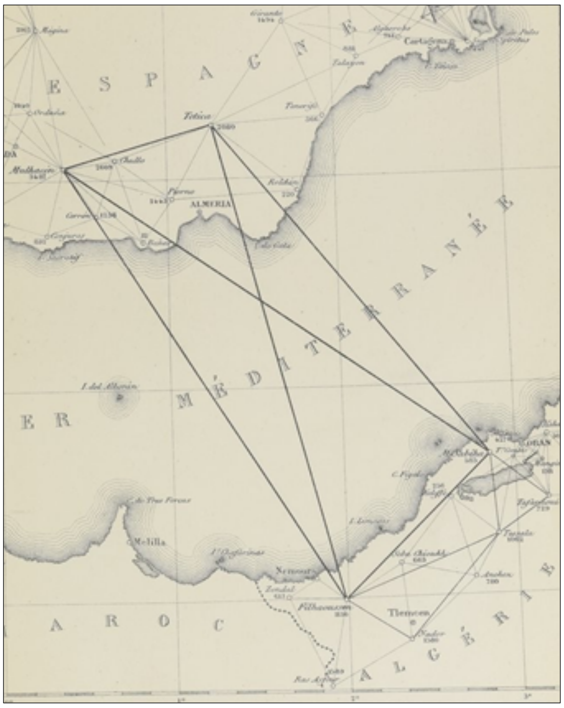


Figure 30: The geodesic junction between Spain and Algeria (adapted from Ibáñez and Perrier (1886), Bibliothèque Nationale de France).

Brunner heliotropes with a 30 × 30 cm mirror, constructed by Louis François Bréguet (1804–1883), and at night, an optical collimator lit by an arc. Ibáñez and Perrier (1886) give a detailed description of the observations, with beautiful plates (Figure 31). Together with the Anglo-French arc, the new measurements give a 27° meridian arc, from the Shetlands to Algeria.

9.10 M10

Much later, a 4° 10′ meridian arc was measured as far north as possible, in the Spitzberg around 78° 42′ latitude (Figure 32). This was not a new idea, as the English geodesist Captain Edward Sabine (1788–1883) had proposed it in 1825 (Sabine, 1825: 360). The measurements were done from 1899 to 1902 by a Swedish party (northern part) and a Russian one (southern part). This expedition was also a means to know more about this relatively unexplored country: a detailed account in French is given by Hansky (1902), with numerous photographs but no detail about the geodetic measurements. As could be expect-

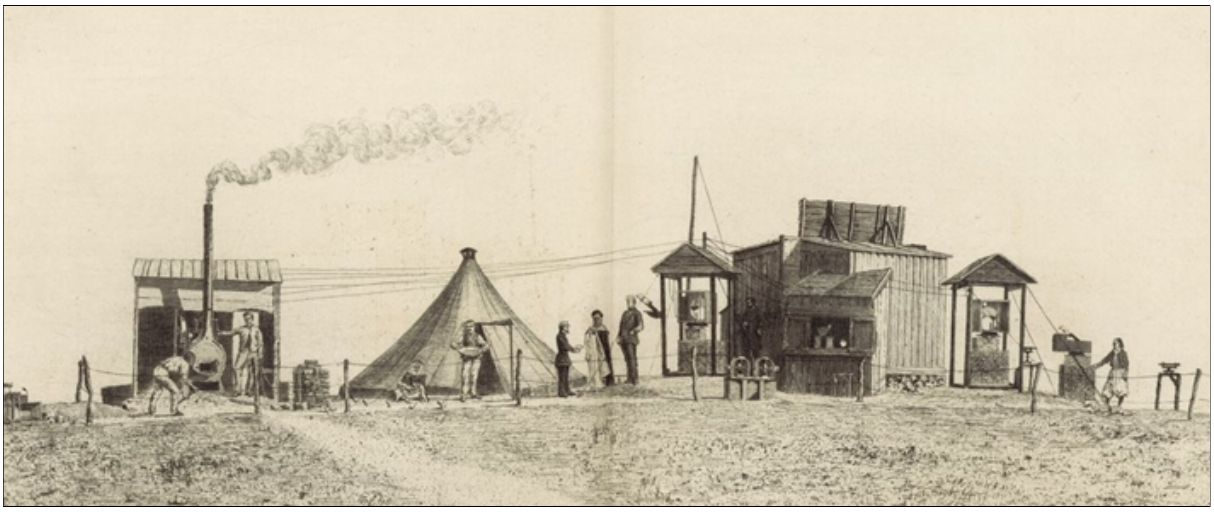


Figure 31: The geodesic station in M'Sabiha, Algeria. From left to right, the steam engine electric generator, the reference pillar, a tent, a heliotrope, two meridian circles on a stand, a shelter for the arc collimator and behind a shelter probably for latitude observations, another heliotrope, and a box containing the azimuthal circle (after Ibáñez and Perrier (1886), Bibliothèque Nationale de France).

Fig. 4. — *Le Spitzberg et le réseau géodésique pour la mesure d'un arc de méridien.*

Figure 32. The triangulation of Spitzberg (after Hansky (1902), Bibliothèque Nationale de France).

ed, the participants encountered many difficulties, including polar bears. The Russian part of the work has been published with a luxury of details, but unfortunately not digitized (Anonymous, 1904–1911). The 6-km base was measured with a Struve ruler as already used for the large Struve arc, and also with a new apparatus due to the Swedish head of the expedition, Edvard Jäderin (1852–1923), which consisted of 25-m long wires of *Guillaume metal* (invar) with a low expansion index. Invar wires have later been used for most base measurements in the world. For the measurements of angles and latitudes, both parties used mainly large universal instruments built near Berlin by Carl Bamberg-Friedenau.

9.11 M11

In 1898, the International Geodesic Association insisted on the need to measure again "… with all the methods of modern geodesy …" the Bouguer-La Condamine-Godin arc in Peru,

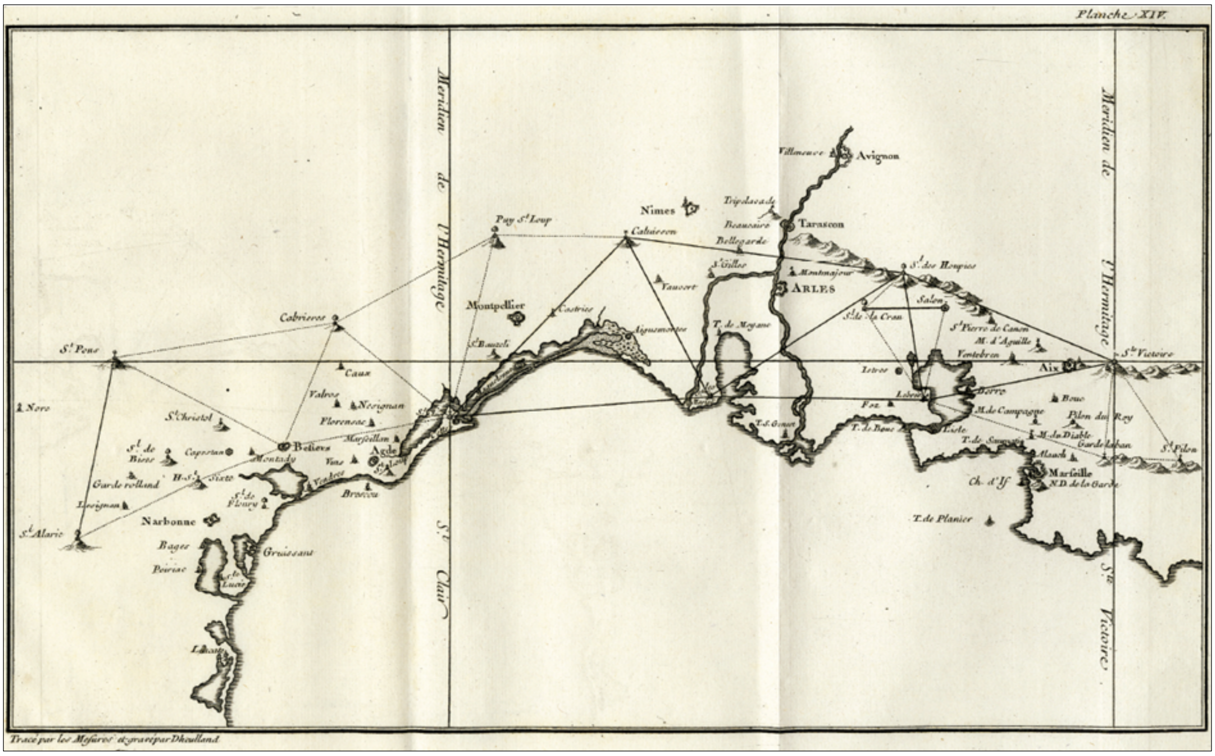


Figure 33: The triangulation of the parallel arc at latitude 43° 32′ N (after Cassini de Thury (1740), Library of the Paris Observatory).

extending it to 6° if possible. This was done by a French party of the *Service Géographique de l'Armée* from 1898 to 1906. The arc length was 5° 53′ 33.8″ or 651.764 km. No less than 49 latitudes were measured with a meridian circle of Brunner, a theodolite and a new *prism astrolabe* of Auguste Claude and Ludovic Driencourt (1858–1940), built by Amédée Jobin (1861–1945). This was to account as well as possible for the deviations of the vertical due to the numerous high mountains of the region. The expedition suffered from important financial difficulties, and a legend claims that the platinum reference meter they carried had to be temporarily deposited at the Quito pawnshop! Pierre Tardi (1897–1972), who made the final reductions (Tardi, 1956), noted the good agreement of the results with those of the eighteenth century scientists, who obviously have been very careful.

10 PARALLEL ARCS

It is also possible to use arcs along parallels to determine the flattening of the Earth. If one succeeds in measuring both the length of the arc and the difference of the longitudes of its extremities, the length of the small circle corresponding to the parallel can be calculated. Then, assuming the Earth to be an ellipsoid of revolution, the flattening can be derived by combination with meridian measurements.

10.1 P1

The first arc of parallel with both triangulation and astronomical determination of longitudes was measured by Cassini de Thury and La Caille, in December 1739 and January 1740 (Cassini de Thury, 1740). The triangulation (Figure 33) included a new base measurement near Salon-de-Provence. The length of the 43° 32′ N parallel between the meridian of Mont Saint-Clair near Sète and that of Montagne Sainte-Victoire near Aix-en-Provence (the mountain painted so often by Cézanne) was found to be 41,618 toises, or 81,115 m. For measuring the longitude difference between the two sites, Cassini stood on Mont Saint-Clair and La Caille on Montagne Sainte-Victoire. Each had an excellent astronomical clock of Julien Le Roy (1686–1759) that they set on the meridian transit of reference stars to obtain the local sidereal time. Ten pounds of gun powder were fired on the terrace of the church of Saintes-Maries-de-la-Mer and each observer read the time of the explosion on his clock: the difference between the times was equal to the difference of longitudes (the velocity of light is too large to have affected the result). It was very probably the first time that fire signals were used to measure longitudes. Four independent measurements gave longitude differences within 1.5 seconds of time. Comparing the angular length of the parallel arc determined in this way with its length, the astronomers concluded that the Earth was flattened by 1/168, a figure similar to that found from the costly triangulations in Lapland and Peru, which could have been avoided!

10.2 P2, P3

In the course of the English survey, several arcs of parallel have been measured (Airy, 1845). The only one fully worthy of confidence was that measured by General William Roy between Beachy Head and Dunrose, with a length of 1° 27′. Another one, much longer (6° 22′ from Dover to Falmouth), was apparently not as good according to Airy. The longitude difference was measured by transporting chronometers.

10.3 P4

During the new geodetic survey of France initiated by Laplace in 1811, a parallel was measured across France from Marennes, near Bordeaux, and combined with several other determinations to reach Padua in Italy, a total length of 1,011 km for a difference in longtudes of 12° 59′ (Brousseaud and Nicollet, 1826). This took place from 1821 to 1824. This arc of parallel near latitude 45° was extended to Fiume (presently Rijeka, in Croatia), for a total length of 15° 32′ 26.76″ or 1,210,673 m (Arago, 1854–1857(3): 338). The longitudes of many points among this parallel were measured by using fire signals, and anomalies appeared when crossing the Alps, due to deviations of the vertical or to local deformations of the geoid.

10.4 P5

A very long parallel arc at 52° latitude, from Valentia in Ireland to Omsk in Siberia, 69° in longitude, was measured in the 1860s. This measurement was suggested by Wilhelm von Struve, who conducted preliminary negotiations for it before illness forced him to return home. Only the part between Valentia and Mount Kemmel in Belgium, measured in 1861, appears to have been discussed in studies of the shape of the Earth (James, 1863).

11 DETERMINATIONS OF THE DIMENSIONS AND THE SHAPE OF THE EARTH

As can be expected, most of the geodesists who measured meridian or parallel arcs compared their determinations with previous ones to obtain figures for the dimensions and flattening of the Earth. We cannot review all these works, but only discuss those that marked the history of geodesy: Airy, Bessel and Clarke. In his *Mécanique Céleste*, Laplace (1798) was the first to attempt to combine several meridian arcs (France, M1 Peru, La Caille, Mason and Dixon, Boscovich, Lapland by Maupertuis and a bad arc in Austria), but he failed to find a satisfactory solution due to defects in several of these arcs (Delaunay et al., 1864).

Tables 1 and 2 give respectively a list of the principal meridian and parallel arcs measured up to the beginning of the twentieth century, ordered according to latitude. Meanwhile, Figure 34 displays the different meridian and parallel arcs discussed in this paper.

In general, the geodesists attempted to fit the arcs to a two-axes ellipsoid whose minor axis coincides with the rotation axis of the Earth. One of the difficulties they encountered was the homogenization of the units. While most of the measurements referred to the *toise du Pérou*, the English used initially the fathom. Bessel took 1 toise = 1.06576542 fathom and gave his results in toises. I converted them to meters using the definition "… the meter is worth 443,296 lines from the toise of Peru …" which gives 1 m = 0.51307407 toise. Later, Everest and Maclear gave their lengths in feet, as well as Airy; to assemble Tables 1 and 2, I converted them to meters by taking 1 ft = 0.3048 m exactly.

Airy (1845) (Figure 35) based his determination of the figure of the Earth on a discussion of fourteen meridian arcs and four arcs of parallel (see Tables 1 and 2): the resulting quantities are

Semi-major axis: a = 6,377,548 m
Semi-minor axis: b = 6,356,241 m
Flattening: $f = (a-b)/a$ = 1/299.3

However he used several arcs of poor quality.

Bessel (Figure 36) was perhaps more aware of the limits of geodesy, and argued that it was difficult to obtain latitudes to better than one arc second due to the anomalies of the vertical. After a first publication (Bessel, 1837), he corrected his results for the error discovered by Puissant in the Formentera-Barcelona triangulation. His definitive ellipsoid (Bessel, 1841), which long served as the basis for European geodesy, has the following characteristics:

Semi-major axis: a = 6,377,397 m
Semi-minor axis: b = 6,356,079 m
Flattening: f = 1/299.2,
the length of the quarter of the terrestrial meridian being 10,000,856 meters.

Bessel's ellipsoïd is very close to the modern WGS84 one (f = 1/298.257223). The agreement between Airy's and Bessel's ellipsoids is remarkable, but should be considered more as a coincidence than as a proof of their accuracy.

Clarke (Figure 37; 1858) successively proposed two ellipsoids to represent the geoid. The first one has the following parameters:

Table 1: Arcs of meridian used by Airy (A), Bessel (B), Clarke in 1858 (C1) and Clarke in 1880 (C2).

Arc	Longitude	Mean latitude	Angular length	Linear length (m)[1]	Length of a degree (m)	User
Maupertuis[2]	21° 15′ E	66° 19′ N	00° 57′ 30.4″	107 238.4	111 888	A
M4 Sweden	21° 15′ E	66° 20′ N	01° 37′ 19.6″	180 827.7	111 477	A, B
Struve 1	26° 43′ E	58° 18′ N	03° 35′ 05.2″	399 209.4	111 362	A
Struve total	26° 43′ E	58° 00′ N	25° 20′ 08.5″	2 821 789.0	111 376	C1, C2
Struve 2	26° 43′ E	56° 04′ N	08° 02′ 28.9″	895 315.2	111 339	B
M8 Bessel	20° 31′ E	54° 58′ N	01° 30′ 29.0″	167 962.1	111 377	B, C1, C2
M5 Denmark	10° 32′ E	54° 08′ N	01° 31′ 53.3″	170 417.0	111 277	B, C1
Great Britain 1	00°	52° 35′ N	03° 57′ 13.1″	439 812.1	111 242	A
M6 Gauss	09° 55′ E	52° 32′ N	02° 00′ 57.4″	224 458.3	111 342	A, B, C1, C2
Great Britain 2	00°	52° 02′ N	02° 50′ 23.5″	315 891.9	111 235	B
Anglo-Gallic	01° E	49° 44′ N	22° 09′ 44.0″	2 464 862.9	111 219	C1, C2
Cassini-La Caille	02° 20′ E	46° 52′ N	08° 20′ 00.3″	926 776.4	111 212	A
M7 Plana-Carlini[4]	07° 50′ E	44° 57′ N	01° 07′ 31.1″	126 387.5	112 313	A
France-Spain[3]	02° 20′ E	44° 51′ N	12° 22′ 12.7″	1 374 591.9	111 119	A, B, C1, C2
M2 Boscovich	12° 30′ E	42° 59′ N	02° 09′ 47.0″	240 157.7	111 027	A
Mason and Dixon[5]	75° 47′ W	39° 12′ N	01° 28′ 45.0″	164 012.9	110 882	A
India total	77° 42′ E	18° 50′ N	21° 20′ 47.2″	2 363 976.1	110 743	C1, C2
India 2	77° 42′ E	16° 08′ N	15° 57′ 40.7″	1 766 161.5	110 653	B
India 1	77° 42′ E	12° 32′ N	01° 34′ 56.4″	175 048.8	110 626	B, C1
M1 Peru[6]	79° W	01° 31′ S	03° 07′ 01.0″	344 776.6	110 614	A, B, C1, C2
Maclear	19° 05′ E	32° 03′ S	04° 36′ 48.6″	511 568.9	110 885	C2
La Caille[4]	19° 05′ E	33° 19′ S	01° 13′ 17.5″	135 790.2	111 164	A

1 Reduced to sea level.
2 Poor measurement, used only by Airy.
3 Corrected value. Airy used the uncorrected one.
4 Latitudes affected by attraction by mountains, uncorrected, used however by Airy.
5 Results somewhat dubious.
6 Figures from Bouguer (1749). Airy uses slightly different figures from a later reduction by Delambre.

Table 2: Arcs of parallel used by Airy.

Arc	Latitude	Mean longitude	Angular length	Linear length (m)	Length of a degree (m)
P2 Beachy Head-Dunrose	50° 44′ N	00° 29′ W	01° 26′ 47.9″	102 443.0	70 814.4
P3 Dover-Falmouth	50° 44′ N	01° 42′ W	06° 22′ 06.0″	449 511.4	70 585.4
P4 Marennes-Padua	45° 43′ N	05° 23′ E	12° 59′ 03.8″	1 011 014.3	77 863.8
P1 Mont St Clair-Montagne Ste Victoire	43° 32′ N	04° 40′ E	01° 53′ 19.0″	153 321.1	81 181.9

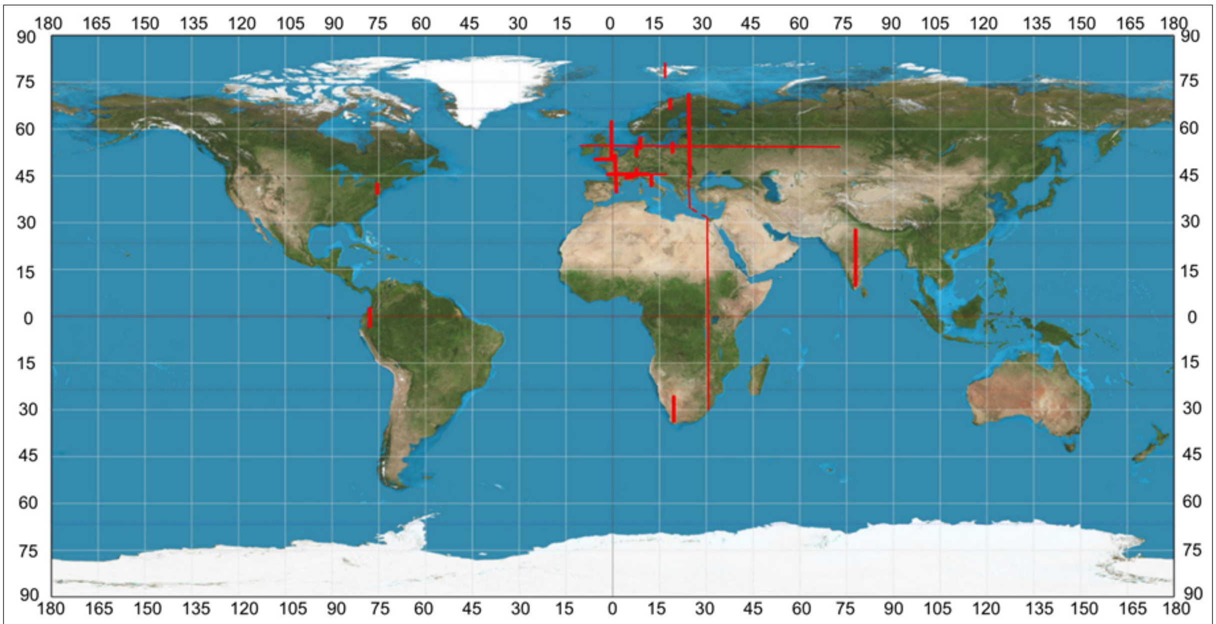


Figure 34. The arcs of meridian and parallel discussed in this paper. Those not used for the determination of the figure of the Earth are represented by thin lines. Note the extreme concentration of the measurements in Europe.

Figure 35: George Biddell Airy (Wikimedia Commons, after Lock & Wellcome).

Figure 36: Friedrich Wilhelm Bessel in 1839, painting by Christian Albrecht Jensen (1792–1870) (Wikimedia Commons, Ny Carlsberg Glyptotek).

Figure 37: Alexander Ross Clarke in 1861 (Wikimedia Commons, Southampton Archives Service).

a = 6,377,993 m
b = 6,356,578 m
f = 1/297.8

The second one (Clarke, 1880), for which I had to take 1 ft = 0.304797265 m, has the parameters:

a = 6,378,312 m
b = 6,356,572 m
f = 1/293.0,
the length of the quarter of the Earth's meridian being 10,001,866 meters

Hervé Faye (1814–1902) also proposed an ellipsoid in 1880, but it has not survived time (see e.g. Faye, 1905: 260). This is also the case for those of William Harkness (1837–1903) in 1891, and others. The ellipsoid proposed in 1909 by Hayford, with f = 1/297, was adopted in 1924 by the International Union of Geodesy and Geophysics as the official reference one.

The second Clarke ellipsoid, which was adopted as a reference before Hayford's one, and the latter are not better representations of the geoid than that of Bessel or others, because the Earth is not really an ellipsoid of revolution, and also because the determinations of latitudes suffer from local deviations of the vertical, even in the case of the English survey for which these deviations were supposedly well corrected for. This was discussed in depth by the Russian general Theodor Friedrich de Schubert (1779–1865): see Airy (1860) and de Schubert (1860). The French geodesist Pierre Tardi was still rather pessimistic when considering the results of the M11 1898–1906 triangulation around the equator (Tardi, 1956). He wrote:

> In order to be used in a calculation whose aim would be to obtain better elements for the terrestrial ellipsoid, the astronomical results of this mission should be corrected for the local attractions very far around each station, taking into account the isostatic compensation of the masses. In the present status of our cartographic knowledge concerning the South-American continent and the southern Pacific Ocean, this calculation would be impossible to perform. However, we may hope that in a near future our geodesic knowledge of the totality of the terrestrial globe will allow to calculate these corrections. It would then be of extreme interest to confront the results obtained in this way (after reductions) with those in course of publication concerning the African meridian arc from the Cape to Cairo. Such a comparison might help to elucidate the still unresolved problem of the probable tri-axiality of the terrestrial ellipsoid. (my English translation).

Figure 38: An astronomer's cabinet, illustration in Richer (1679). Note the Huygens-type clock giving the seconds in the background (Paris Observatory Library).

Indeed, the attempts to fit a three-axes ellipsoid to the measurements have not been convincing. While the measurements of the large arcs have represented considerable progress for geodesy and for knowledge of the shape and dimensions of the Earth, one had to wait for space geodesy and the GPS to obtain a final, accurate description of the geoid.

12 DETERMINING THE FLATTENING OF THE EARTH WITH PENDULUMS

In 1672, Jean Richer (1630–1696) was sent by the French Academy of Sciences to Cayenne (French Guyana, near the Equator) to perform astronomical observations. He brought two accurate pendulum clocks designed by Christiaan Huygens (1629–1695) and made by Isaac Thuret (ca. 1630–1706) (Figure 38), as well as a seconds pendulum adjusted in Paris. He remarked that he had to shorten this pendulum in Cayenne (Richer, 1679: 66). The minutes of the Academy for 1674 (Académie des Sciences, 1666–1686: 177) note:

> The seconds pendulum, which is in Paris of 3 pieds 8 lignes ½ long, is in Cayenne shorter by 1 ligne 1/4. Without this variation, a universal measure would have been found, all the nations would have determined the same length by taking a pendulum that would have beaten exactly the second of mean solar time. (my English translation).

This shows that the idea of a universal unit of length had arisen much before the French Revolution.

We find later in these minutes that Picard and his collaborators have noted that the seconds pendulum has about the same length as in Paris in Denmark, London and den Haag, as well as in Montpellier, Sète and Lyons in France. No explanation was given for the variations of the length of the seconds pendulum, but it was understood that they meant that the weight of the bodies was smaller at the equator than at the poles. Then, systematic measurements were done by scientific expeditions throughout the world, for example in 1682 in Gorée near Dakar and in Guadeloupe: in these low-latitude places, the length of the seconds pendulum was found to be 2 lignes shorter than in Paris. The existence of a systematic effect with latitude was then fully recognized. This was the beginning of gravimetry.

In order to understand the importance of this fact, a short reminder is in order. The half-period of oscillation t of a pendulum of length l is given by:

$$t = \pi\,(l/g)^{1/2}\,(1 - \theta^2/16),$$

where g is the acceleration of gravity and θ is the maximum deflection of the pendulum in radians. Thus, measuring the length l of a seconds pendulum allows to calculate g. It is necessary to correct the value of g to zero elevation if the observation is made at a large elevation. Apparently, this was done for the first time in Peru by Bouguer (1749: 357), who noted the difficulties in making this correction.

Now, a theorem by Alexis Clairaut (1713–1765) states that if the variation of gravity with latitude ϕ has the form $g \propto (1 + F\sin^2\phi)$, the ellipticity e of the Earth's surface, supposed to be an ellipsoid with its minor axis along the axis of rotation, is $e = (5m/2 - F)$, with m = 0.0034672 being the ratio of the centrifugal force to the gravitational force at the equator. This allows calculation of the flattening of the Earth.

It would be boring to describe the very numerous measurements of gravity made after the time of Richer. Those made before 1840 or so are detailed by Airy (1845), and later ones by Clarke (1880). It suffices to say that it is practically impossible to fit the length of a pendulum so that it beats exactly one second:

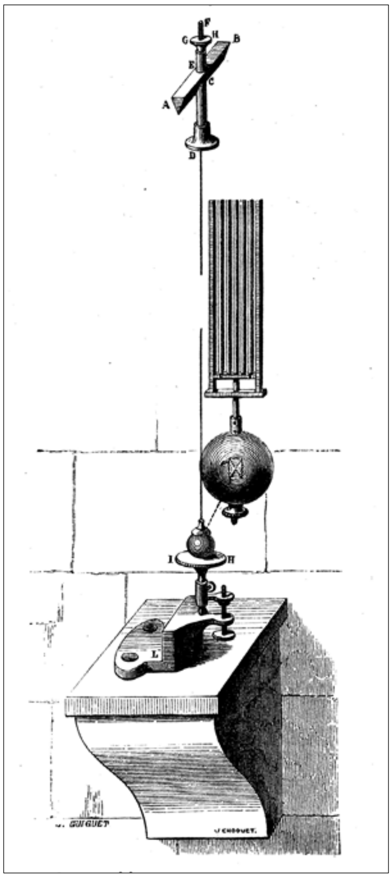


Figure 39: Borda's pendulum for gravimetry. This pendulum, the upper and lower parts of which are represented, was supported by the knives AB sitting on a plane surface. The lower ball was made of platinum, and sat on a plate that was lowered for operation. One compared the oscillations of the pendulum with the almost synchronous ones of the pendulum of a clock, seen at the back. The period of this clock was determined by astronomical observations. Then the period of the pendulum could be determined very accurately (after Arago (1854 –1857), Lequeux Collection).

Figure 40: Friedrich Robert Helmert, a German, was one of the leading geodesists of his time, and an outstanding mathematician (Wikimedia Commons).

its beats should be compared to those of a good pendulum clock. Then, once its period is accurately determined, it is easy to correct its length to obtain that of the seconds pendulum. Figure 39 shows as an example a pendulum designed by Borda and used in many French expeditions.

It would also be of limited interest to recall the many determinations of the flattening of the Earth done in this way during the eighteenth to twentieth centuries: Strasser (1957) enumerates with details no less than 107 determinations from 1800 to 1950! I will cite only Laplace (1798, N° 42): 1/335, revised to 1/310 in 1825; Edward Sabine (1825: 352): 1/289; Airy (1845: 231): 1/283; Clarke (1880: 350): 1/292.2 ± 1.5; Friedrich Robert Helmert (1843 – 1917; Figure 40): 1/298.3 in 1906, obtained with his *method of condensation* to reduce the data; Hayford in 1909: 1/297, correcting for isostasy but using measurements only in America. Note that the pendulum measurements were distributed on the surface of the terrestrial globe, including oceans, much more evenly than the arc measurements.

Modern determinations by space geodesy have confirmed Helmert's value. The present reference ellipsoid WGS84, used in particular for the GPS, has the following parameters:

a = 6,378,137.0 m
b = 6,356,752.3 m
f = 1/298.257223563.

13 CONCLUDING REMARKS

It looks satisfying at first glance that the two methods used in the past to obtain the flattening of the Earth agreed on the same value within the errors. However it should be kept in mind that they do not refer to the same thing, as clearly stated for the first time by Gauss in 1828. Triangulation measurements result in a mathematic surface that is supposed to fit best the data, the reference ellipsoid, while gravity measurements refer to a physical surface, a gravitational equipotential surface that coincides approximately with the surface of the oceans. In 1873, the latter surface was called the geoid by Johann Benedict Listing (1808–1882), a pupil of Gauss. Both methods suffer from local deviations of gravity in intensity and direction, due to the inhomogeneity of superficial and internal masses.

Starting about 1960, space geodesy progressively superseded all the previous work. The geoid is now measured with high precision, within a few centimeters for the oceans by radar altimetry and within a meter or so for continents by an analysis of the perturbations

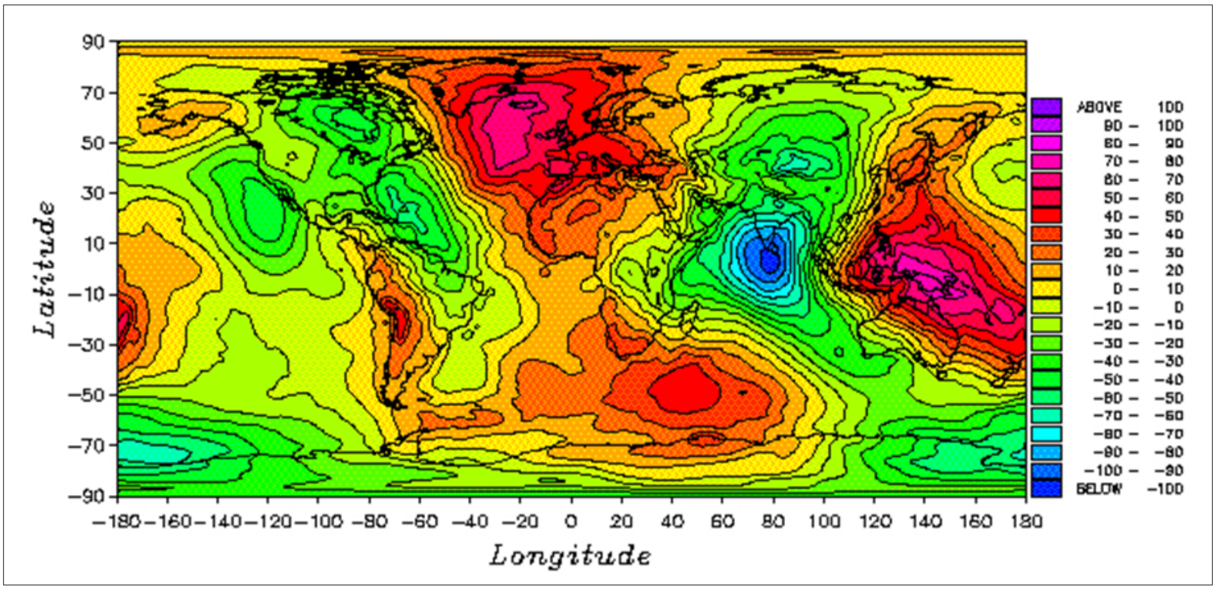


Figure 41: Map of the deviations of the geoid with respect to the WGS84 reference ellipsoid. The depressions are in green and blue and the humps in red and purple. Note that while Western Europe is in a hump India is in a depression, explaining the discrepancies in the length of a degree noted by Everest (Bureau Gravimétrique International, 2001).

of satellite orbits. The deviations of the geoid with respect to the WGS84 ellipsoid are limited to about 100 meters (Figure 41), contrary to the 1000 meters claimed in 1930–1950 on the basis of intensive measurements of gravity both on ground and at sea: isostatic equilibrium reduces the deviations, as discussed as early as 1901 by Helmert, who considered rightly that they could not be larger than 200 to 250 meters.

One might consider that, because its results have been superseded by modern means, the immense and painful work done by geodesists during more than three centuries has been a waste of time. But this is how scientific research goes ... The triangulations have served as bases for a continuously improving cartography of the Earth, and gravimetry has been of great help for geologists to understand the nature and structure of our planet.

14 NOTES

1. The rulers are preserved at Paris Observatory. Two of the circles are respectively at the Marseilles Observatory and at the National Geographic Institute at Saint-Mandé, near Paris.
2. Méchain had measured the latitudes of two places in Barcelona, and had found that their difference was smaller by 3 arc seconds than expected from the direct measurement of distance by triangulation. Later investigations determined that the error came from the repeating circle that was used for the measurements (see Lequeux, 2016: 163).
3. Precession and nutation arise from the action of the Sun and the Moon on the terrestrial ellipsoid, which also causes inequalities in the motion of the Moon. Values for the Earth flattening can be derived from a study of these phenomena: one finds 1/305.5. However, this is not the real flattening of the geoid because they depend on the internal structure of the Earth and on the presence of the liquid outer nucleus. For this reason, they are of high interest in studies of the internal structure of the Earth.

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16 REFERENCES

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